Massachusetts Institute of Technology

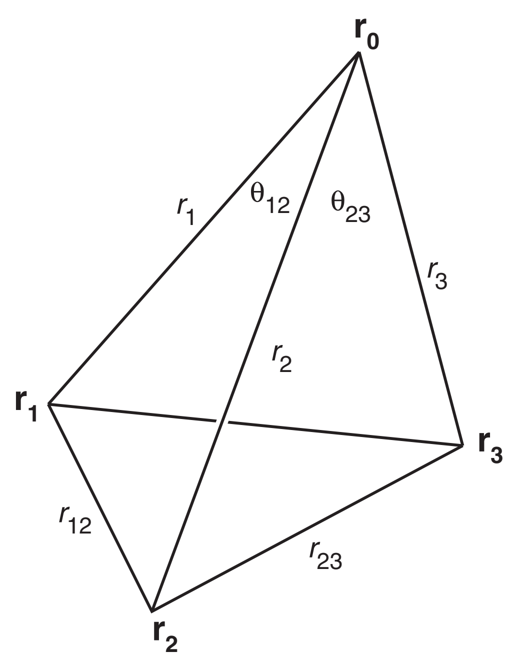
Department of Computer Science and Electrical Engineering

6.801/6.866 Machine Vision QUIZ II

Problem 1: Exterior Orientation. Suppose you are flying above terrain for which a 3-D digital terrain model (DTM) is available and that you have identified several landmarks in an image. The problem of exterior orientation is that of finding out where you are — and how the camera is oriented (This is also the "Location Determination Problem" (LDP) addressed in the "Random Sample Consensus" (RANSAC) paper by Fischler and Bolles). Here we consider the situation where three landmarks can be identified in the image (the minimal case for exterior orientation).

(a) Let $\mathbf{r}_i$ (for $i > 0$) be the position (in a world coordinate system) of the i-th landmarks on the ground, while $\mathbf{r}_0$ is the (unknown) position of the center of projection (COP) of the camera. What is the distance $r_{i,j}$ between landmarks i and j?

(b) If we know the camera's interior orientation (calibration), then we can construct rays $\mathbf{a}_i$ (in the camera coordinate system) towards the landmarks by connecting the COP to corresponding image points. Determine the angles $\theta_{i,j}$ between rays $\mathbf{a}_i$ and $\mathbf{a}_j$ from the COP towards landmarks i and j.



(c) Next, we find the lengths of the lines connecting the landmarks to the COP, i.e. $r_i = \;\; \mathbf{r}_i - \mathbf{r}_0 \;\;$: Write down an equation in the unknown lengths r_i and r_j, the known distance $r_{i,j}$ and the known angle $\theta_{i,j}$. Hint: may need to look up some triangle equalities involving trigonometric terms.

(d) Suppose that three landmarks are needed to solve for the position of the COP. What is the largest number of solutions that three equations of the type discussed in part (c) could have by Bézout's theorem? How many different solutions of the equations can be obtained by just flipping the sign of the three quantities r_i?

(e) Now suppose that we have solved the equations and obtained the three lengths r_i for $i = 1$, 2, and 3. Describe geometrically how knowing these three distances allows us to determine the position $\mathbf{r}_0$ of the COP (in the world coordinate system). How many solutions can there be? (Note: you do not need to actually solve the equations). If there is more than one solution, do all the solutions make physical sense (in terms of the original problem)?

(f) We have now figured out where you were when you took the image ($\mathbf{r}_0$). It remains to find the camera orientation (in the world coordinate system). Let $\mathbf{b}_i = (\mathbf{r}_i - \mathbf{r}_0)$ be vectors from the COP to the landmarks (in the world coordinate system) — which we can now compute since we have determined $\mathbf{r}_0$. We have already defined the directions $\mathbf{a}_i$ of the corresponding rays in the camera coordinate system in part (b). Explain why

$$R = (\hat{\mathbf{a}}_1\ \hat{\mathbf{a}}_2\ \hat{\mathbf{a}}_3)(\hat{\mathbf{b}}_1\ \hat{\mathbf{b}}_2\ \hat{\mathbf{b}}_3)^{-1}$$

is the rotation that takes world coordinates into camera coordinates (where the circumflexes denote unit vectors). Is R orthonormal?

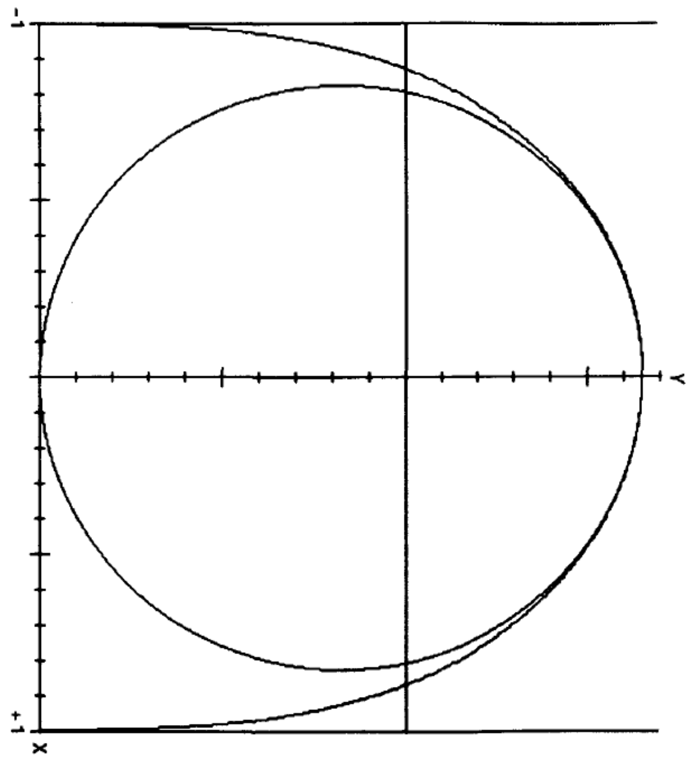
Problem 3: This question is about the Extended Gaussian Image (EGI) representation for object shape, specialized to solids of revolution. Consider a planar curve with the curious property that its curvature is proportional to the distance from some line (see figure on next page). That is $K_G = Ar$, where $K_G = d\eta/ds$, A is a constant, and r is the perpendicular distance from the line. At the points where the curve touches the line it is perpendicular to the line.

Now imagine spinning the curve about the line. Compute the extended Gaussian image (EGI) of the resulting solid of revolution.

(a) Does this correspond to the EGI of an object we discussed in class?

(b) Would a method based on matching EGIs confuse these two objects?

(c) Are there any other convex objects with the same EGI?



Problem 5: This question is about motion fields induced by camera motion, their dependence on distance and on the size field of view. We obtained expressions for the motion field by differentiating the perspective projection equations and knowing the velocities of points in the environment.

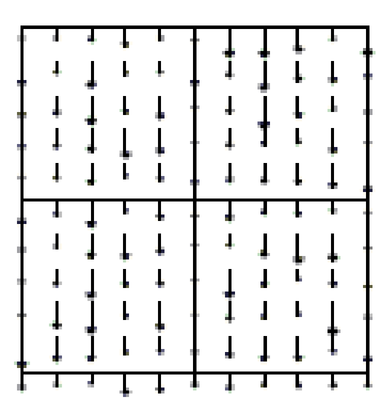
Each of the motion fields illustrated on the next page was generated by moving a camera relative to a fixed environment. In each subfigure, all but *one* motion parameter was zero. The camera motion can be described by $\mathbf{t} = (U, V, W)^T$ and $\boldsymbol{\omega} = (A, B, C)^T$ where U, V, W are the components of translational motion, while A, B, and C are the components of the rotational motion in the camera-centric ccordinate system.

(a) For each of the six patterns state which of the six parameter was non-zero.

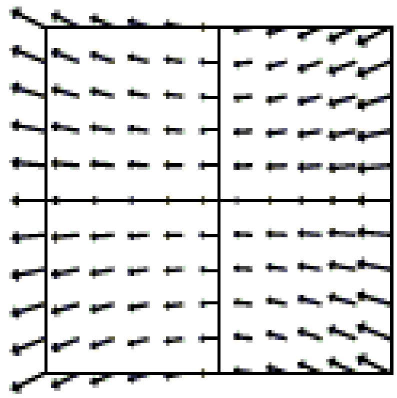
(b) Which of the six flow fields would change if the shapes of objects in the scene were changed?

(c) Which pairs of components of the camera motion would get harder to distinguish if the field of view (FOV) of the camera was greatly reduced?

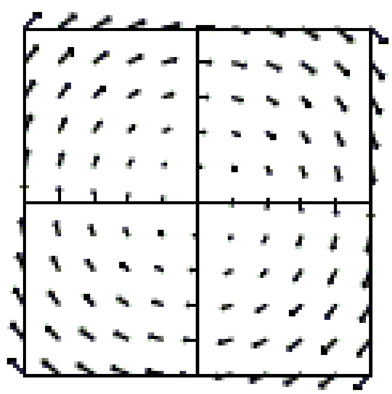
(d) If possible, estimate the field of view θ of the camera in degrees (the ratio of one half the width of the image to the principle distance is $\tan \theta/2$). If it is not possible to recover the width of the field of view, explain why.



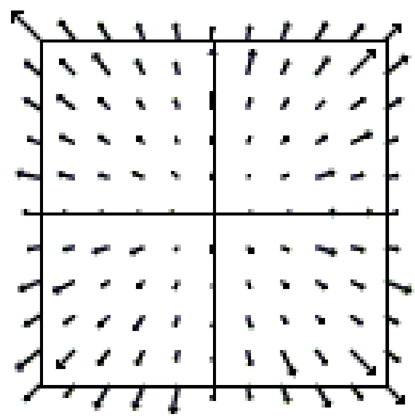
1



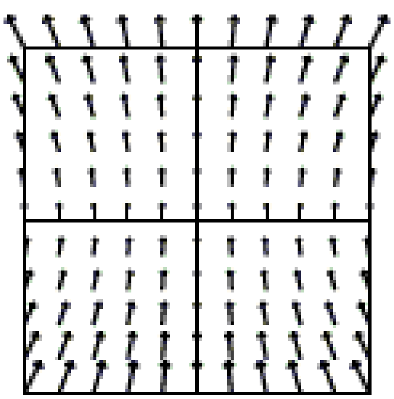
2



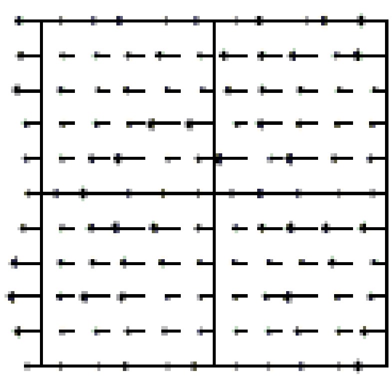
3



4



5



6