MASSACHUSETTS INSTITUTE OF TECHNOLOGY

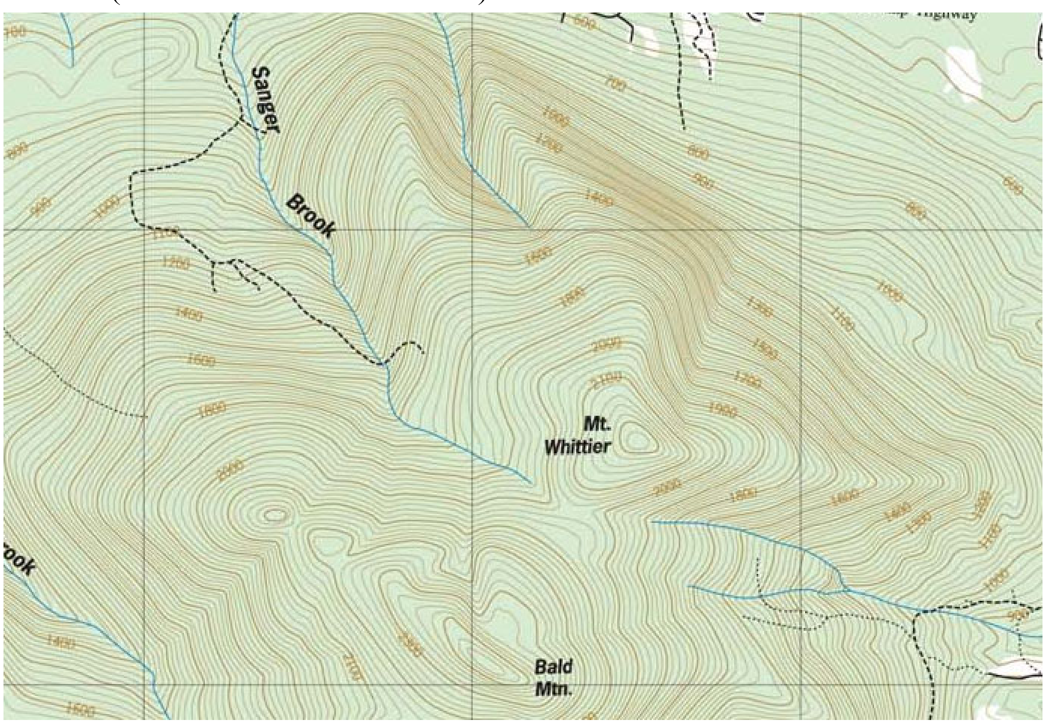
DEPARTMENT OF COMPUTER SCIENCE AND ELECTRICAL ENGINEERING

6.801/6.866 MACHINE VISION QUIZ I

Handed out: 2020 Oct. 20th **Due on: 2020 Oct. 29th**

Problem 1: Contour maps are easy to interpret in part because they provide a shading effect - that is, brightness is a function of surface orientation. This is because spacing between dark contour lines varies with the slope of the surface.

In the following example steeper areas appear darker. Suppose that black contour lines (reflectance 0) of width w are drawn on white paper (reflectance 1). The map is drawn at a scale s (that is, distances in the real world are s times as large as they are on the map). Finally, h is the vertical interval between neigboring contours (measured in the real world).



(a) How does the average reflectance of a region of the map depend on the slope m (magnitude of the gradient) of the corresponding portion of the surface in the real world?

(b) What is the reflectance map $R(p,q)$ (for the contour map) in this case? (Check that $R(p,q)$ is not less than zero, or greater than one, for *any* gradient).

(c) Is there a limit on the slope m above which there is no further variation of average reflectance with slope?

Problem 2: Consider the surface of a rocky planet or moon, the reflecting properties of which are modelled by Hapke's formula

$$\sqrt{\cos\theta_i / \cos\theta_e}$$

where θ_i is the angle between the surface normal $\hat{\mathbf{n}}$ and the direction $\hat{\mathbf{s}}$ towards the distant light source, while θ_e is the angle between the surface normal $\hat{\mathbf{n}}$ and the direction to the distant viewer — taken to be parallel to $\hat{\mathbf{z}}$ in our case. **Hint:** In the following it is useful to work in gradient-space ((p, q)) — where the light source direction can be specified by (p_s, q_s).

(a) Show that the slope m of the surface can be determined — but only in a particular direction — from brightness E. What is the special direction in which the slope can be determined? Show that that slope is given by the formula

$$m = (E^2 - \cos\theta_s) / \sin\theta_s$$

were θ_s is the angle between the direction to the light source ($\hat{\mathbf{s}}$) and the direction to the viewer ($\hat{\mathbf{z}}$). What happens when $\theta_s = 0$?

(b) What do you expect a sphere of this kind of material to look like when illuminated from the same direction as it is being viewed from?

(c) Why are there spatial variations in brightness in the image below of the (almost) full moon? **Hint:** Hapke's formula really applies only to the ancient lava found in the maria ("oceans").

Problem 3: Vector notation can reduce the size of expressions encountered when dealing with motion vision problems. Let us revisit the recovery of camera motion and/or the orientation of a planar surface from an image sequence. The equation $aX + bY + cZ = d$ applies to a planar surface.

(a) Show that the equation of the plane can be written in the form

$$\mathbf{R} \cdot \mathbf{n} = 1,$$

for some vector $\mathbf{n}$, where $\mathbf{R} = (X, Y, Z)^T$. Give an expression for the unit surface normal (in terms of a, b, c, and d). What is the perpendicular distance of the plane from the origin?

(b) Suppose the plane is in front of an imaging system. Show that under perspective projection

$$\frac{1}{f}\mathbf{r} \cdot \mathbf{n} = \frac{1}{\mathbf{R} \cdot \hat{\mathbf{z}}}$$

where f is the principal distance of the imaging system, and $\mathbf{r} = (x, y, f)^T$ is the image of the point $\mathbf{R} = (X, Y, Z)^T$ in the world.

(c) Suppose now that the plane is moving with velocity $\mathbf{t} = (U, V, W)^T$ with respect to the camera (or equivalently that the camera is moving with velocity $-\mathbf{t}$ w.r.t to plane). Differentiate the perspective projection equations

$$x/f = X/Z \quad \text{and} \quad y/f = Y/Z$$

to obtain expressions for the motion field components $u = dx/dt$ and $v = dy/dt$ in terms of the components of the translational motion vector $U = dX/dt$, $V = dY/dt$ and $W = dZ/dt$.

(d) Show that the brightness change constraint equation

$$uE_x + vE_y + E_t = 0$$

leads to an equation of the form

$$E_t + (\mathbf{s} \cdot \mathbf{t})(\mathbf{r} \cdot \mathbf{n}) = 0,$$

in the case of the planar surface, for some vector $\mathbf{s}$. Write the vector $\mathbf{s}$ in terms of components of the image brighness gradient (E_x, E_y).

(e) If $\mathbf{t}$ were known, how would you recover $\mathbf{n}$? (Hint: Consider a least squares approach using all of the image data).

(f) If $\mathbf{n}$ were known, how would you recover $\mathbf{t}$? (Hint: Consider a least squares approach using all of the image data).

Problem 4: In the case of pure rotation of the camera about its center of projection, the following error integral is to be minimized in order to determined the angular velocity $\omega = (A, B, C)^T$:

$$(E_t + \mathbf{v} \cdot \omega)^2 \, dx \, dy.$$

(where $\mathbf{v}$ is a vector that depends on image position (x, y) and the brightness gradient $E(x, y)$ measured at (x, y).) Show that the ω that minimizes the integral is the solution of a set of linear equations (or write it in matrix-vector form).

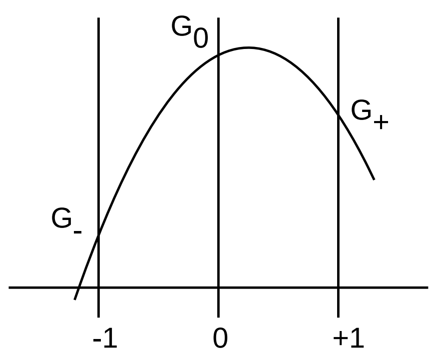
Hint: Remember that $\mathbf{a} \cdot \mathbf{b} = \mathbf{a}^T \mathbf{b} = \mathbf{b}^T \mathbf{a}$ and that

$$\frac{d}{d\mathbf{a}} \mathbf{a} \cdot \mathbf{b} = \mathbf{b},$$

and note that rotation here is in 3-D (i.e. not in the image plane).

Problem 5: To obtain edge positions and directions in an image, we can start with estimates of the gradient magnitude and gradient directions at points on a grid with one pixel spacing (this grid may be aligned with the pixel grid itself or offset by 1/2 pixel in each direction). We estimate the subpixel position using the gradient magnitude at three points in the direction of the brightness gradient near a directional local maximum in gradient magnitude. (Refer to US patent 6,408,109 for details needed below).

Suppose now that there is a vertical edge in the image passing near the origin (so that the direction of the brightness gradient is everywhere parallel to the x-axis). Let the estimates of the brightness gradient magnitude be G_-, G_0, and G_+ at $x = -1$, $x = 0$, and $x = +1$ respectively. (In answering the following, make sure to state where you need to use the assumption that $G_0 > G_+$ and $G_0 \geq G_-$).



(a) For now, assume that the gradient magnitude curve as a function of position has a parabolic shape. That is $G(x)$ can be approximated by $ax^2 + bx + c$, for some unknown parameters a, b, and c. Where is the extremum of the parabola? When is the extremum a maximum? Find expressions for the parameters a, b, and c in terms of the gradient estimates G_-, G_0, and G_+.

(b) Show that the maximum in brightness gradient may be found at $x = s$, where

$$s = \frac{1}{2}\frac{G_+ - G_-}{(G_0 - G_+) + (G_0 - G_-)}$$

Show that $-\frac{1}{2} \leq s \leq +\frac{1}{2}$. That is, the distance of the maximum from the point where the gradient has value G_0 is less than or equal to one-half of the spacing between the pixels where the gradient magnitude has the values G_-, G_0, and G_+ (which may be equal to the pixel spacing — or $\sqrt{2}$ times that for diagonal directions).

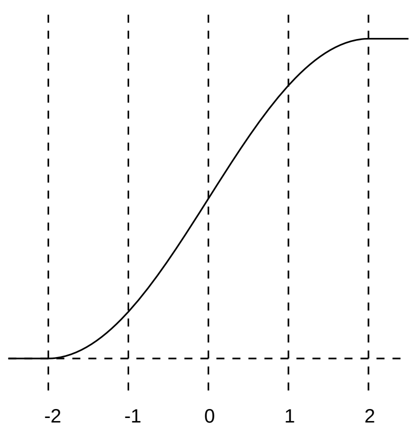
In order to determine the accuracy of the estimated subpixel edge position, we need to know what kernels are used to estimate the brightness gradient. Suppose that we use the simple kernel $[-1, +1]$, which estimates the first derivative in the x-direction at points half way between samples (i.e. if we consider the samples to relate to the centers of square pixels, then the derivative estimates would be for points on the boundaries betwen those square pixels). Further, assume that the brightness profile of a vertical edge passing through the origin is $E(x)$.

(c) Now suppose that a vertical edge is offset so that it lies at $x = s$ (rather than at $x = 0$), for $0 \leq s \leq 0.5$. Our task is to estimate s from the known values of brightness gradient magnitude. Show that

$$G_+ - G_- = E(-s + \tfrac{3}{2}) - E(-s + \tfrac{1}{2}) - E(-s - \tfrac{1}{2}) + E(-s - \tfrac{3}{2})$$

and

$$(G_0 - G_+) + (G_0 - G_-) = -E(-s + \tfrac{3}{2}) + 3E(-s + \tfrac{1}{2}) - 3E(-s - \tfrac{1}{2}) + E(-s - \tfrac{3}{2})$$



In order to determine the accuracy of the estimated subpixel edge offset, we further need to know the shape of the edge transition. As a particular example, suppose that a vertical edge in the image centered on the line $x = 0$ has a smooth edge transition of the form

$$E(x) = \begin{cases} 0 & \text{for } x < -2 \\ \frac{1}{2}\ 1 + \sin(\frac{\pi}{4}x) & \text{for } -2 \leq x \leq +2 \\ 1 & \text{for } x > +2 \end{cases}$$

(where the units for x are pixels).

(d) Show that in this case (with the derivative estimator described above):

$$G_+ - G_- = \sin(\tfrac{\pi}{4}s) \;\; \cos\tfrac{\pi}{8} - \cos\tfrac{3\pi}{8}$$

and

$$(G_0 - G_+) + (G_0 - G_-) = \cos(\tfrac{\pi}{4}s) \;\; 3\sin\tfrac{\pi}{8} - \sin\tfrac{3\pi}{8}$$

(e) Show that the estimate of the subpixel edge offset then is

$$s \;\, = \frac{1}{2}\frac{\tan(\frac{\pi}{4}s)}{\tan(\frac{\pi}{8})}$$

Verify that the above produces the correct offset for $s = -1/2$, 0 and $1/2$.

(f) By setting the derivative of $(s \;\; - s)$ w.r.t. s equal to zero, show that that the largest difference occurs for $s \; = 0.2927498\ldots$ Just how large is the difference? How could you compensate for the bias (i.e. the difference between the true offset s and the estimate s calculated as above)?

Hint: $\sin(3x) = 3\sin x - 4\sin^3 x$ and $\cos(3x) = 4\cos^3 x - 3\cos x$.

(Note, by the way, that the above is the result for a *vertical* edge in the image and that the bias will be different for edges of different orientations — in part because the distance between the points where the gradient equals G_-, G_0, and G_+ depends on edge direction).

Problem N+1: Zebra Stripes Protect Against Biting Flies. This is a more open-ended, "researchy" question, a place where you can earn some extra points. Maybe we can co-author a paper explaining this interesting effect if we get some good ideas! First, read (at a minimum) the abstract and conclusions of the paper:

> “Benefits of zebra stripes: Behaviour of tabanid flies around zebras and horses” by Tim Caro, *et al* in *PLoS ONE*, Vol. 14, No. 2, 2019 Feb. 20 Open access, available at:
> https://journals.plos.org/plosone/article?id=10.1371/journal.pone.0210831

From the Abstract (emphasis mine):

> “In separate, detailed video analyses, tabanids approached zebras faster and *failed to decelerate* before contacting zebras, and proportionately more tabanids simply touched rather than landed on zebra pelage in comparison to horses”

From the Conclusions (emphasis mine):

> “In summary, multiple lines of evidence indicate that stripes prevent effective landing by tabanids once they are in the vicinity of the host but did *not* prevent them approaching from a distance.”

Note that the paper has figures and graphs that may be useful. See also popular press accounts such as the one in *Discovery Magazine:*
https://www.discovermagazine.com/planet-earth/zebra-stripes-protect-against-flies-now-we-know-how

> “Frame by frame analyses of our videos showed that flies slowly decelerated as they approached brown or black horses before making a controlled landing. But they *failed to decelerate* as they approached zebras. Instead they would fly straight past or literally bump into the animal and bounce off.”

and https://www.futurity.org/zebra-stripes-flies-flight-1989432/

> “We found that zebras and horses received a similar number of approaches from horseflies, probably attracted by their smell—but zebras experienced far fewer landings. Around horses, flies hover, spiral and turn before touching down again and again. In contrast, around zebras flies either flew right past them or made a single quick landing and flew off again.”

There are also some short videos s.a.: https://youtu.be/JyDa8SQ0l3I

(a) In the context of this course, can you propose an explanation for these observations?

(b) What properties, parameters, and measurements of zebras, horses, horse flies and tse-tse flies would be helpful in supporting your proposed answer?

(c) Do the above mentioned reports of the work already foreshadow your conclusion? If so, how?

(d) How can we test your proposed explanation — and eliminate alternatives?