6.777J/2.372J Design and Fabrication of Microelectromechanical Devices
Spring Term 2007

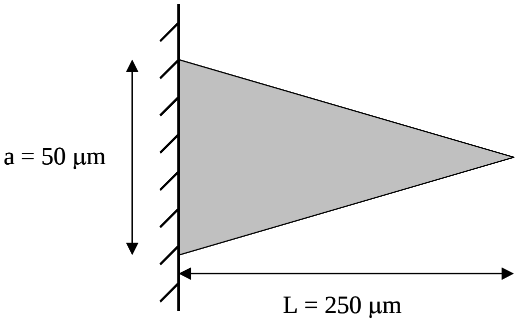
Massachusetts Institute of Technology

PROBLEM SET 3 (13 pts)

Issued: Lecture 6 Due: Lecture 9

Problem 9.14 (2 pts): Bending of an AFM Cantilever

A silicon nitride (LPCVD, stoichiometric) AFM cantilever has a thickness of 0.5 μm and the triangular shape (top view) shown below. It is subjected to a point load F at the tip, pointing into the paper. Assuming small deformations, find the tip deflection in terms of F, and calculate an effective spring constant for this beam. For a load which produces a tip deflection of 2 μm, calculate the maximum stress at the support.



Problem 9.10 (4pts): Thermal Stress Induced Wafer Curvature

Wafer-scale fusion bonding is commonly used to create complex silicon structures. However, two wafers can be bonded together only if their bonding surfaces are about the same shape. If the shapes are too different, the energy required to deform the wafers exceeds the energy gained by bringing them into contact, and the wafers will not bond. Because stressed films deform substrates, film thicknesses are limited on wafers that will be wafer-bonded.

Consider a PECVD silicon dioxide film that is deposited on a 650 μm thick silicon wafer. The wafer is then annealed at 1100 °C and cooled to room temperature. Assume that the oxide is stress-free when it is above the oxide softening temperature, which we will take to be 950 °C. Also assume biaxial plane stress in the film, and approximate the substrate as isotropic.

- **(a)** Using constitutive relationships, calculate the thermal stress σ_r in the film for the case where the film thickness is much less than the substrate thickness.

Experience shows that the wafer's radius of curvature must be greater than about 100 m to ensure that the wafer can be bonded to a second, flat wafer. In the next two parts, we will determine the maximum thickness of the oxide film to ensure bondability.

- **(b)** The wafer curvature is determined by moment balance over the thickness of the wafer. Find an expression for the net moment per unit width as a function of the geometry, materials properties, and thermal stress in the film.
- **(c)** Write an expression relating the film thickness to the radius of curvature. What is the maximum film thickness to ensure bondability for the case described above?

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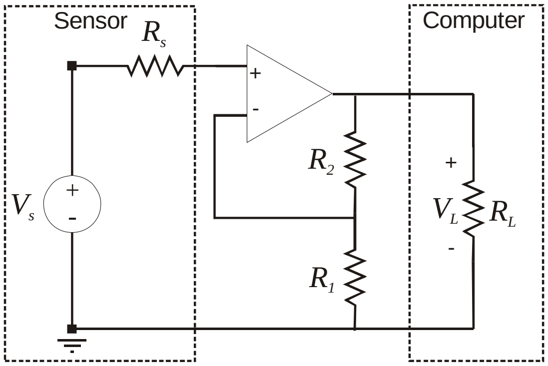
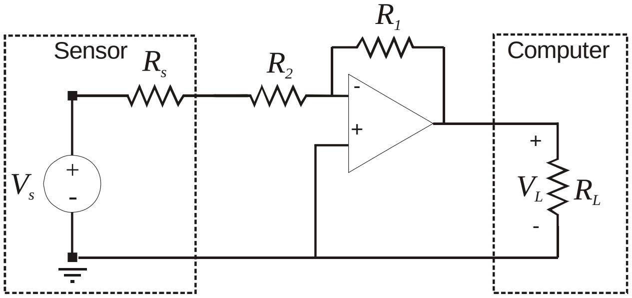
Problem 14.12 (2 pts): Circuit loading

In this problem we will explore the differences among different op-amp topologies. Assume we are trying to measure the signal V_s from a linear sensor and would like to amplify that before we measure it with a computer-based data acquisition card. Thevenin's theorem states that any linear electrical sensor can be represented by a voltage source and a series impedance (in this case, a resistance R_S). Your data acquisition board, like all circuits, has some input resistance denoted by R_L.





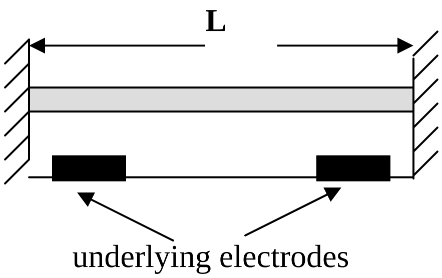
(a) One way to amplify the signal V_S is to use a non-inverting amplifier (Figure, left). If we use this circuit, what is V_L as a function of V_s and the four resistors, assuming that the op-amp is ideal?

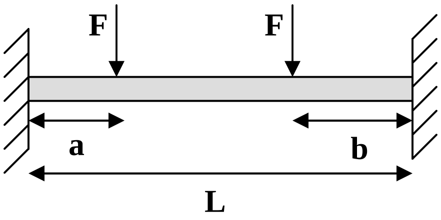
(b) A second way to amplify the signal is to use the inverting amplifier (Figure, right). Repeat part (a) for this circuit.

(c) As R_S changes, the gain of each circuit may change, leading to unpredictable behavior, which we'd like to avoid. What is the gain of each circuit when $R_S = 0$? This is the ideal gain. Determine analytically the (actual gain)/(ideal gain) for each circuit. Give the constraints on each circuit such that the actual gain approaches the ideal gain. Which circuit is less sensitive to changes in R_S?

Problem 9.13 (3pts): Leveraged Bending with Electrostatic Actuation

As we will learn in lecture 9, pull-in limits the useful analog travel of a voltage-controlled moveable capacitor plate to one third of the initial gap between the plates. One trick that MEMS designers sometimes use to get greater controllable travel at the center of the beam is leveraged bending (illustrated in the left hand figure below). In this scheme, the bottom capacitor plate is limited to the ends of the beam instead of extending to the center. Because no voltage difference is applied between the center of the beam and the substrate, the center can deflect more than one third of the initial gap before pull-in occurs.





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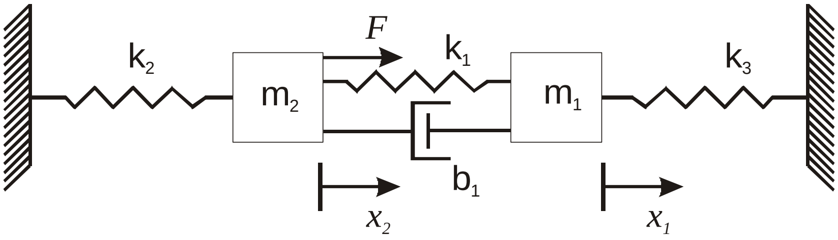
Consider a fixed-fixed silicon beam of width w, length L, and height h that is actuated by leveraged bending. If the electrodes are narrow enough, we may approximate the load as two point forces applied at distances a and b from the ends of the beam, as shown in the right hand figure above. (We would typically design the system so that a = b, but fabrication nonuniformities may lead to some asymmetry.)

Assuming small deformations, calculate the deflection of the beam at the center. You are welcome to use any analytic method you like (explicit integration of the beam equation, use of tabulated beam bending solutions, etc.)

Problem 5.9 (2 pts): Circuit representation of a lumped mechanical system

A mechanically coupled micromechanical resonator can be lumped modeled as shown in the figure. Derive an equivalent circuit in the $e \rightarrow V$ convention and find the transfer function $x_1(s)/F(s)$.



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