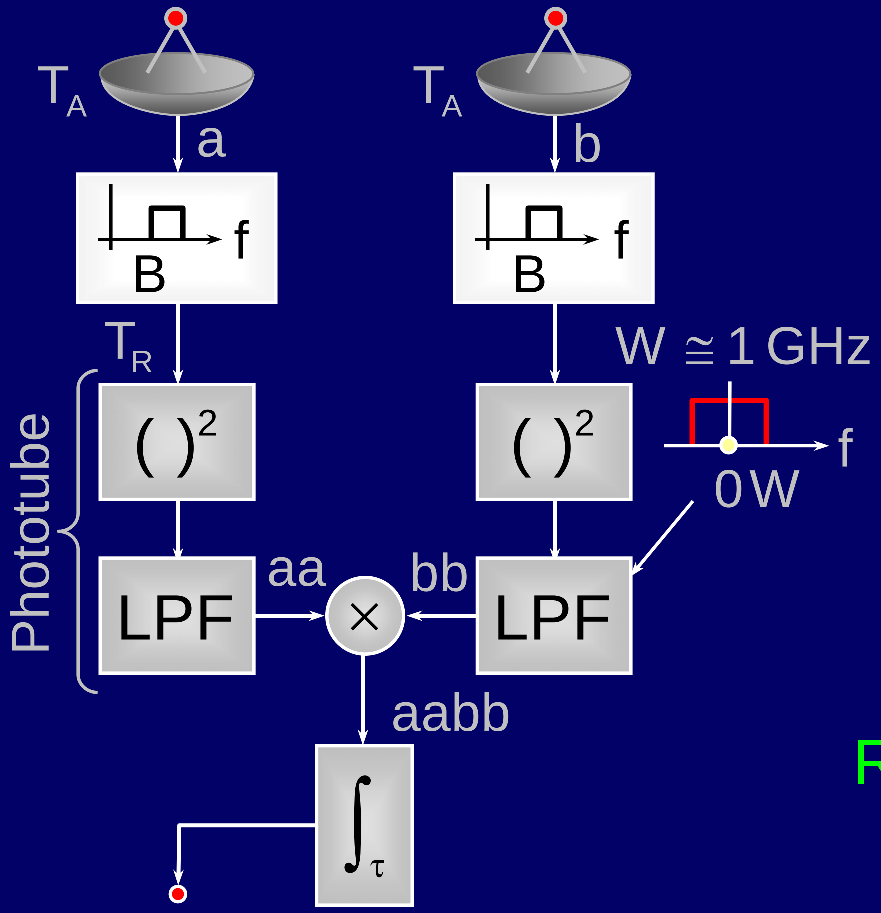
Phaseless Interferometry

Hanbury-Brown and Twiss

Visible interferometer at
Narrabri, Australia

$$\left[\text{For } T_R >> T_A, \frac{v_{o\ rms}}{\langle v_o \rangle} \cong \frac{T_R^2}{T_A^2\sqrt{2W\tau}}\right]$$



$\propto \left|\phi_E\left(\bar{\tau}_\lambda\right)\right|^2 \Rightarrow$ source size, etc.

One might (wrongly) think photodetectors would lose all phase information and ability to measure source structure at λ/D resolution.

Recall: $E\left[aabb\right] = \overline{a^2}\,\overline{b^2} + 2\overline{ab}^2$,
where $\overline{ab}$ is $\phi_E\left(\bar{\tau}_y\right)$ here.

Phaseless Recovery of Source Structure

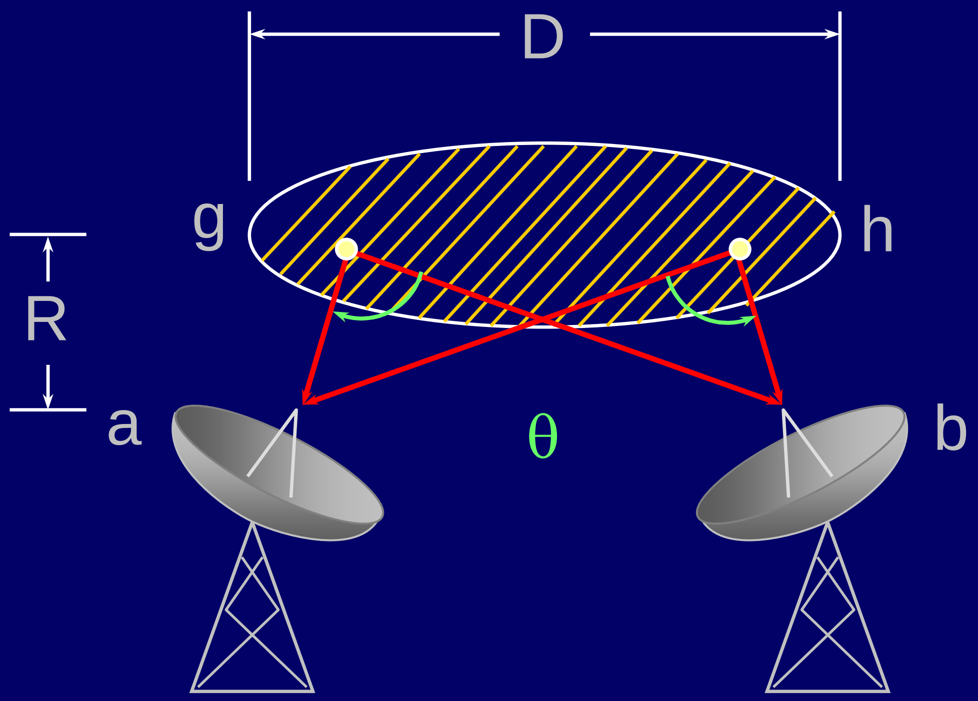
Recall: $E\left[aabb\right] = \overline{a^2}\,\overline{b^2} + 2\overline{ab}^2$, where $\overline{ab}$ is $\phi_{\underline{E}}\left(\overline{\tau}_y\right)$ here.

Recall:

$$\begin{tikzcd}
\underline{E}(x,\ y) \arrow[r, leftrightarrow] \arrow[d] & \underline{E}\left(\overline{\psi}\right) \arrow[d] & \\
\underline{\phi}_{\underline{E}}\left(\overline{\tau}_\lambda\right) \arrow[r, leftrightarrow] \arrow[d] & \left|\underline{E}\left(\overline{\psi}\right)\right|^2 \arrow[r, Rightarrow] \arrow[d] & \mathrm{I}\left(\overline{\psi}\right) \\
\left|\underline{\phi}_{\underline{E}}\left(\overline{\tau}_\lambda\right)\right|^2 \arrow[r, leftrightarrow] & R_{\left|\underline{E}\left(\overline{\psi}\right)\right|^2}\left(\Delta\overline{\psi}\right) &
\end{tikzcd}$$

Purely real if source is even function of position, allowing perfect source reconstruction

Phaseless Interferometer Interpretation: Independent Radiators



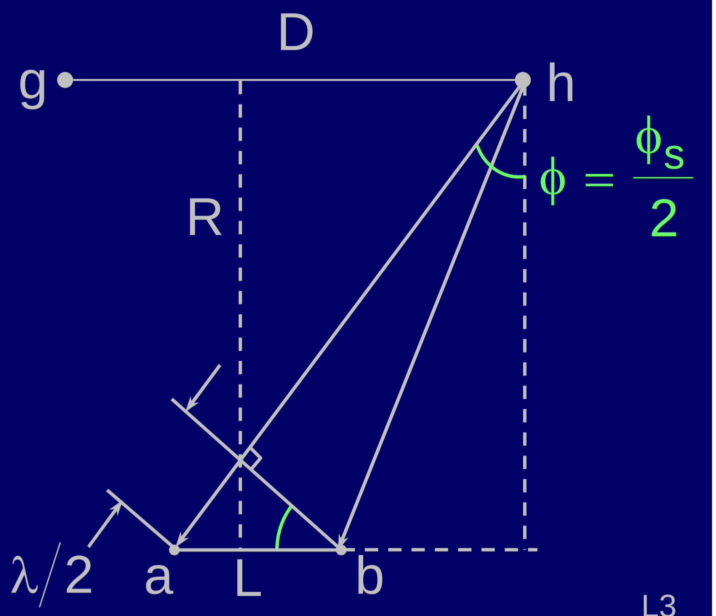
Source, independent thermal radiators g and h

a,b are uncorrelated if $\Delta\phi_a - \Delta\phi_b \tilde{>} 2\pi$

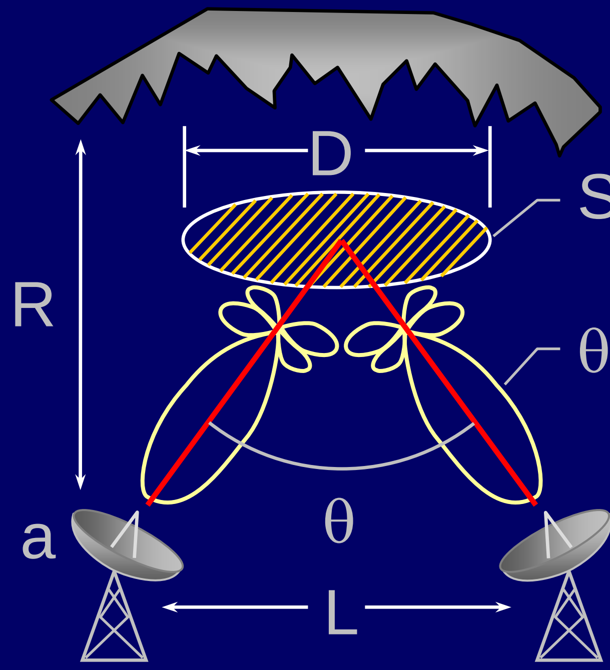
$[\Delta\phi_a$ is $\Delta\phi$ at "a" for rays g,h]; or if

$$\frac{\phi_s}{2} = \phi \tilde{>} (\lambda/2)/L.$$

Thus a,b decorrelated if $\phi_s \tilde{>} \lambda/L$.



Phaseless Interferometer Diffraction-Limited Source



Source "Aperture"

$\theta_B \cong \lambda / D$

$\theta \cong L / R$

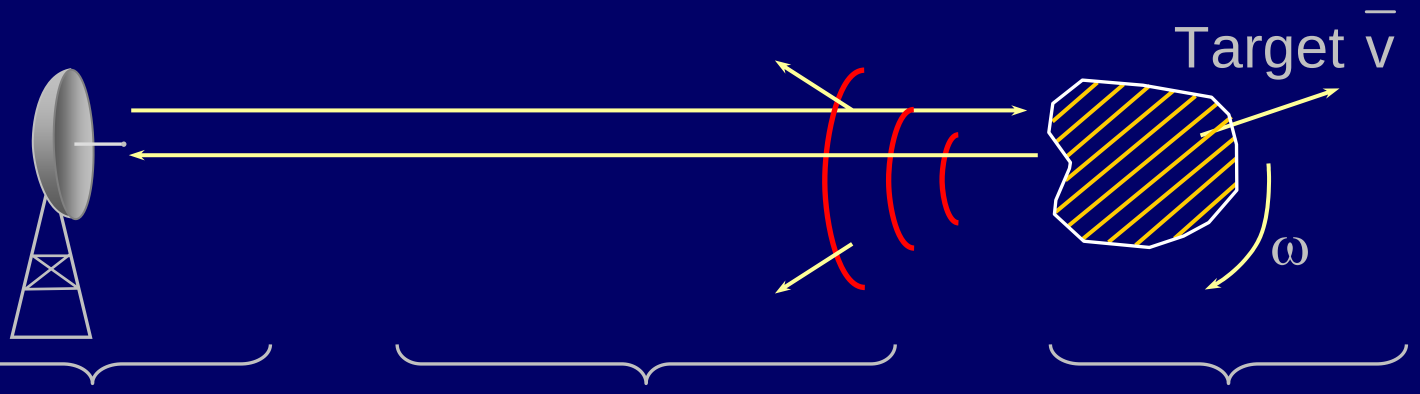
If $\theta \overset{\sim}{>} \theta_B \cong \lambda/D$, then a and b are ~uncorrelated

Therefore decorrelated if $D\theta \overset{\sim}{\geq} \lambda$

or if $DL/R \overset{\sim}{\geq} \lambda$ since $\theta \cong L/R$

or if $\phi_S \overset{\sim}{\geq} \lambda/L$ since $\phi_S \cong D/R$

Radar Equation



Issues: Signal design
Processor design
Antenna

Propagation, absorption,
refraction, scintillation,
scattering, multipath

Scattering

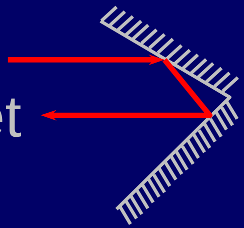
$$P_{rec} = \overbrace{\underbrace{\frac{P_t}{4\pi R^2}}_{Wm^{-2}\text{ at target}} \bullet G_t \bullet \frac{\sigma}{4\pi R^2}}^{Wm^{-2}\text{ at transmitter}} \bullet A_t = P_t \left(\frac{G\lambda}{4\pi R^2}\right)^2 \frac{\sigma}{4\pi} \text{ Watts}$$

σ "scattering cross-section" is equivalent capture cross-section for a target scattering isotropically

Radar Scattering Cross-Section

σ "scattering cross-section" is equivalent capture cross-section for a target scattering isotropically

Note: Corner reflector can have $\sigma >>$ size of target

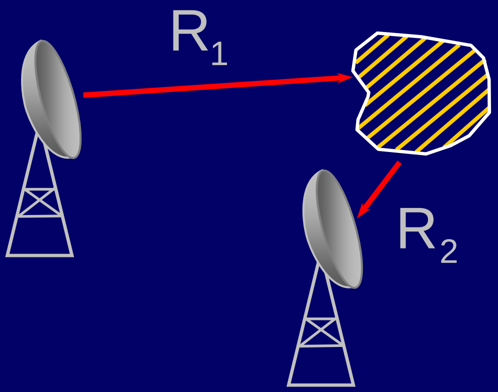


Biastatic radars:

If target is unresolved, $P_{rec} \propto 1/R_1^2 R_2^2$

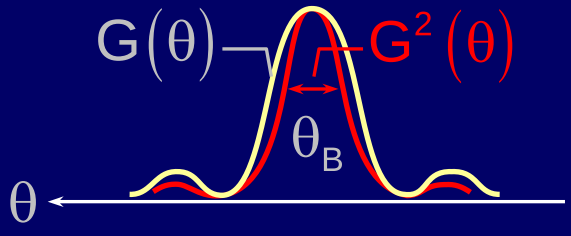
If target is resolved by the transmitter,

$P_{rec} \overset{\sim}{\propto} 1/R_2^2$



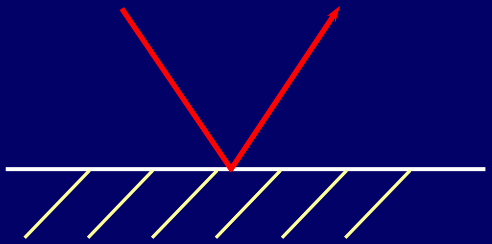
Note resolution enhancement:

$P_{rec} \propto R^{-4} G^2$ where $G^2(\theta)$ has a narrower beam than $G(\theta)$

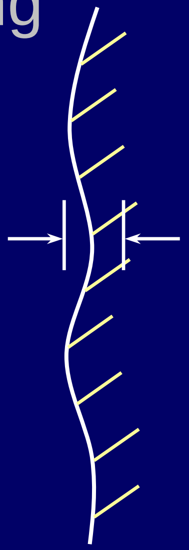


Target Scattering Laws

1) Specular

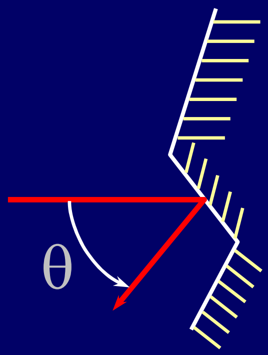


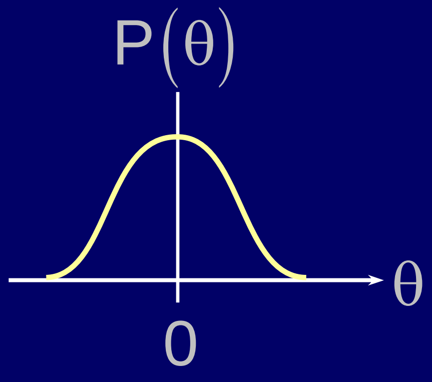
2) Scintillating



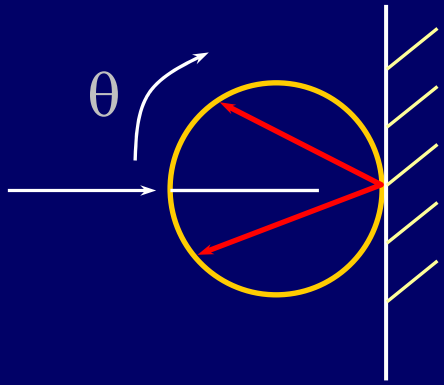
$\delta << \lambda \Rightarrow \sim$ nulls

3) Faceted





4) Lambertian

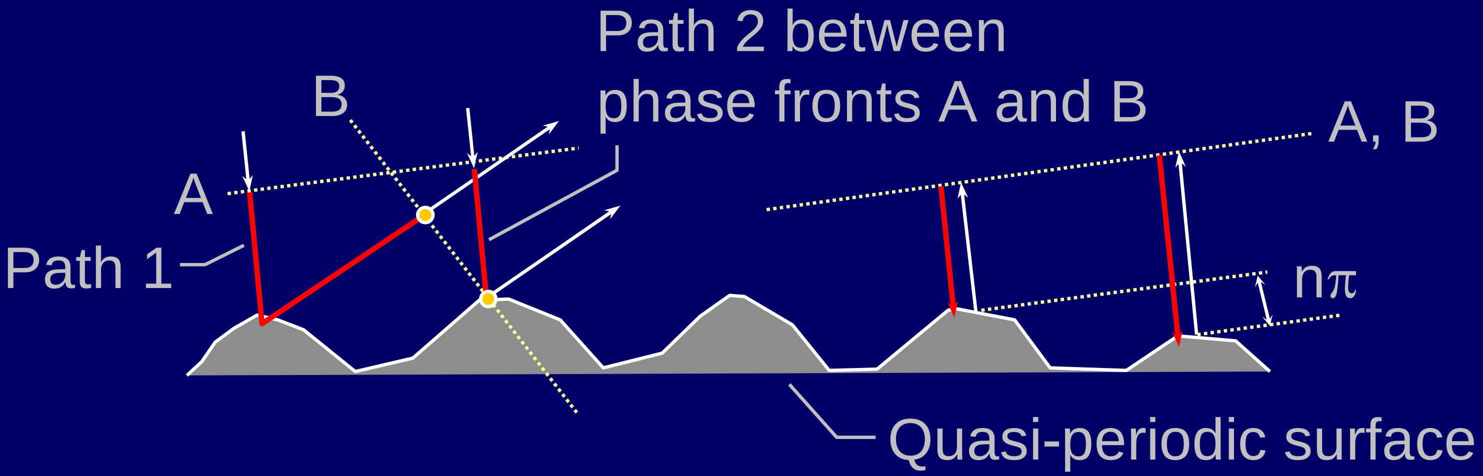


$\cos\theta \propto$ power scattered
(geometric projection only)

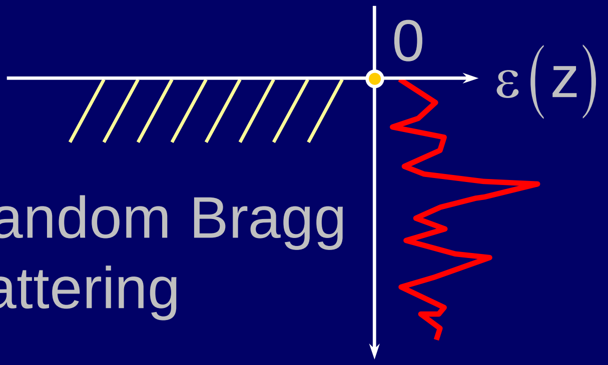
Target Scattering Laws

5) Random Bragg Scattering (frequency selective)

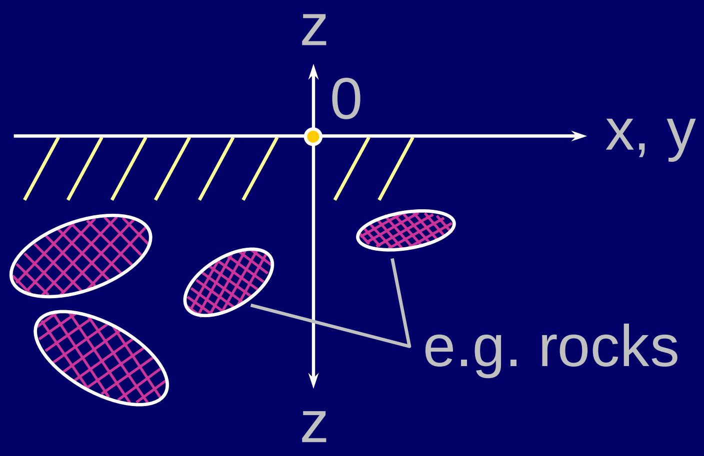
At Bragg angles $\Delta \mathrm{Path}_{1,2} = n \bullet 2\pi$ $\qquad n = 0, \ \pm 1, \ \pm 2, \ldots$



6) Sub-Surface Inhomogeneities

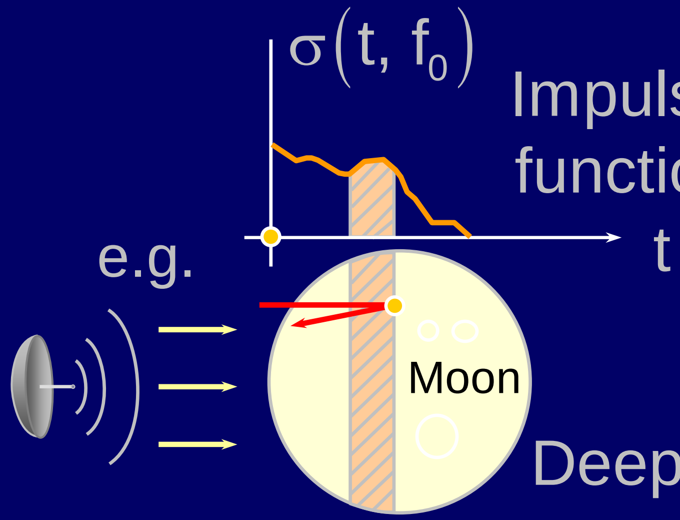


~ random Bragg scattering



Target Range-Doppler Response

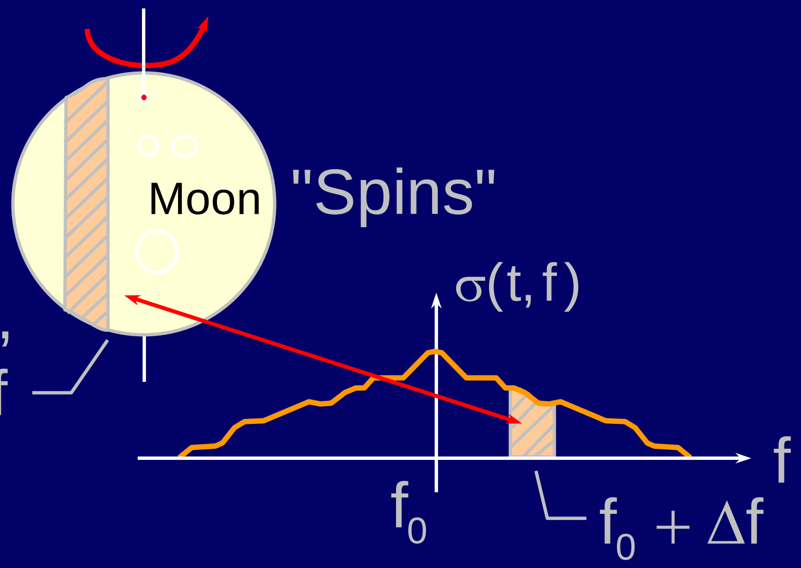
Narrowband Pulsed Radar



Impulse response function for a deep target

Deep Target

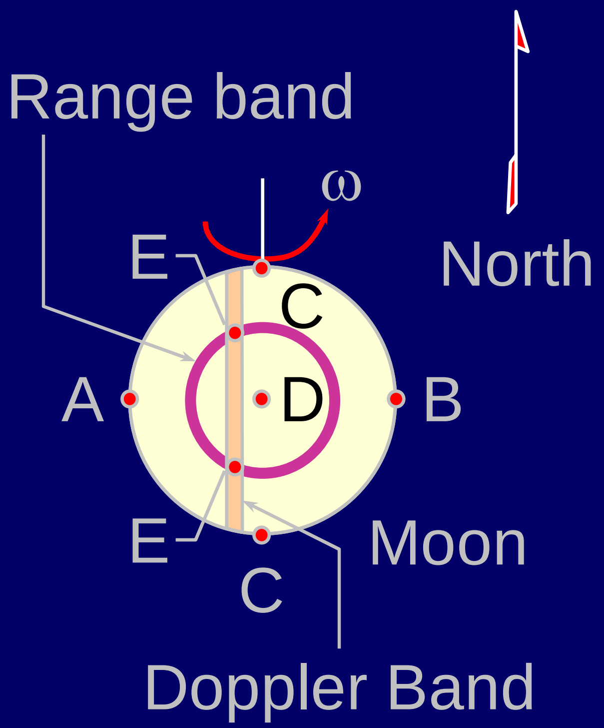
CW Radar

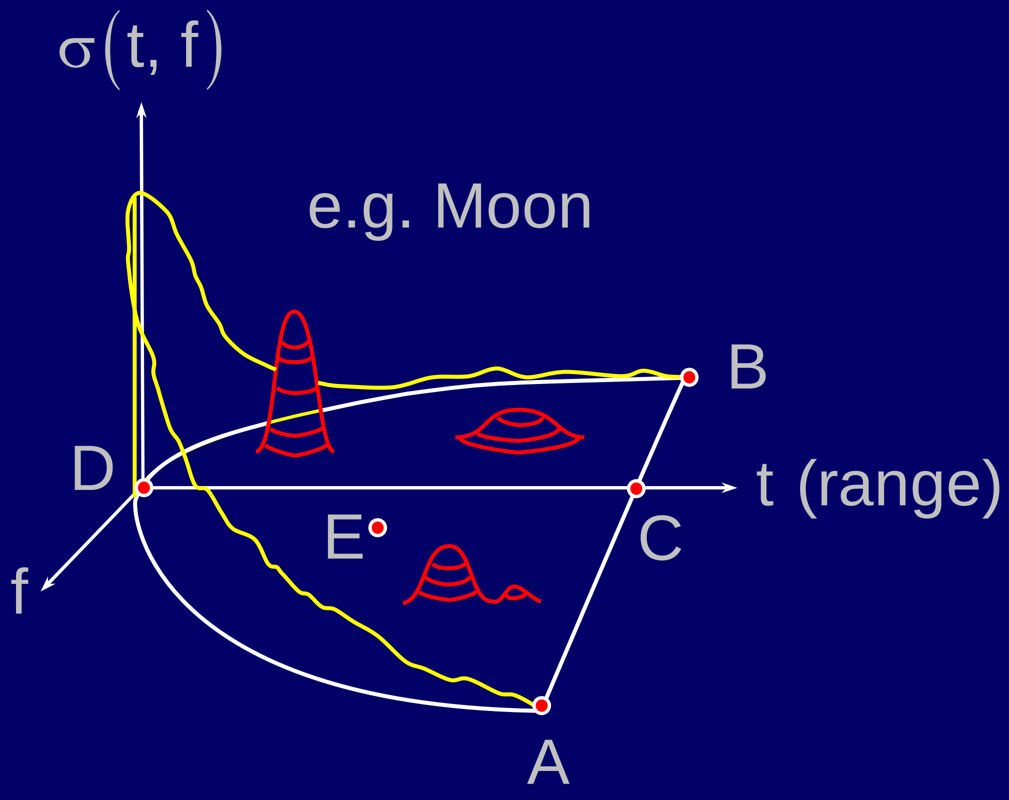


"Spins"

Return Doppler, shifted to $f_0 + \Delta f$

Range-Doppler Response for a CW Pulse





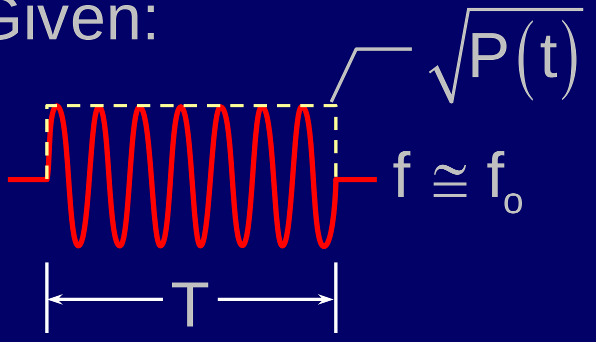
Note north-south ambiguity

Radar Range and Doppler Ambiguity

Professor David H. Staelin
Massachusetts Institute of Technology

Optimum CW Pulse Radar Receiver

Given:



CW transmitted pulse at f_o,

power = P(t)

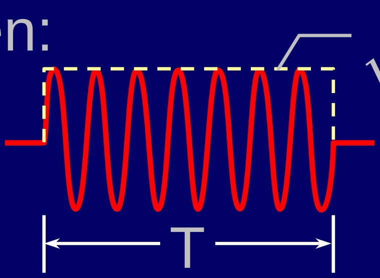
Receives for point source:

$$r(t)\ (\text{volts}) = k\underset{\substack{\uparrow \\ \propto\sqrt{G^2}\ \text{etc.}}}{\sqrt{\sigma}}\sqrt{P(t)}\cos\omega_o t + \underset{\substack{\uparrow \\ \text{Usually Gaussian} \\ \text{white noise}}}{m(t)}$$

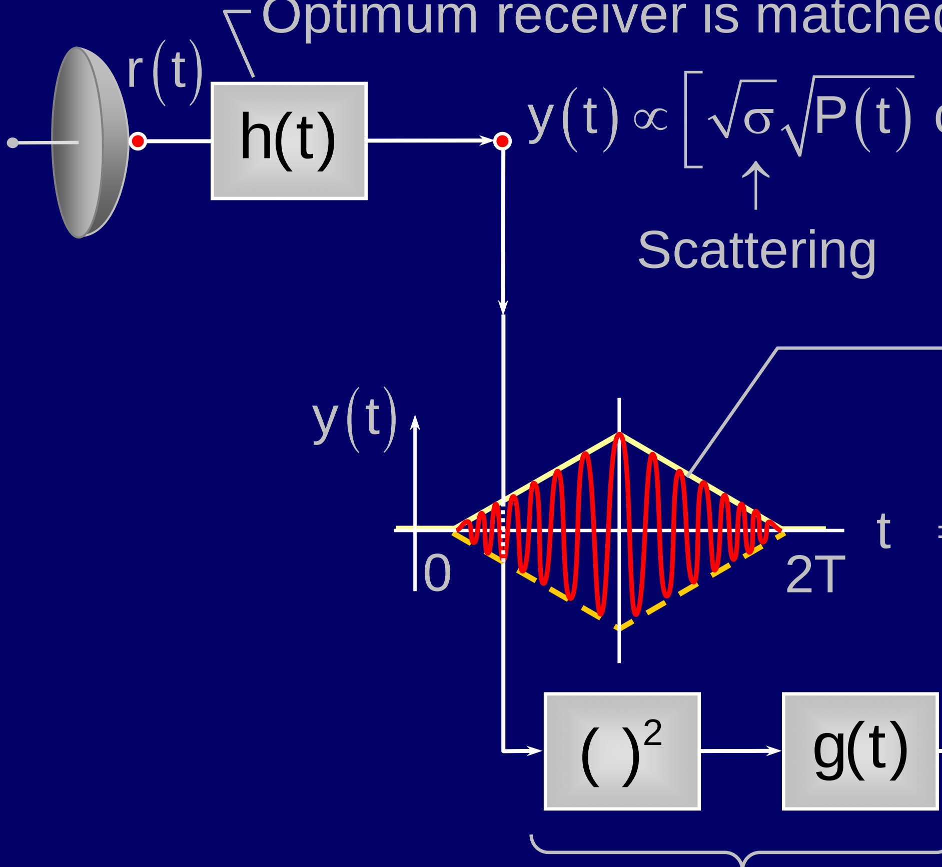
A bank of matched filters could test every possible delay, but the output envelope of a single filter matched to the transmitted waveform is equivalent.

Optimum CW Pulse Radar Receiver

Given:



$f \cong f_o$ CW transmitted pulse at f_o, power = $P(t)$

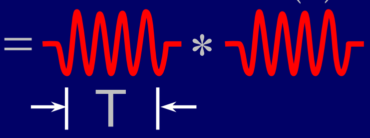


Optimum receiver is matched filter

$$y(t) \propto \left[\underset{\substack{\uparrow \\ \text{Scattering}}}{\sqrt{\sigma}}\sqrt{P(t)}\cos\omega_o t + m(t)\right] * \underbrace{\left[\frac{\sqrt{P(-t)}\cos\omega_o t}{\sqrt{P(0)}}\right]}_{h(t)\,\rightarrow}$$

$$R_{\sqrt{P}}(t) \triangleq \int_{-\infty}^{\infty} \sqrt{P(\tau)}\,\frac{\sqrt{P(t-\tau)}}{\sqrt{P(0)}}\,d\tau$$

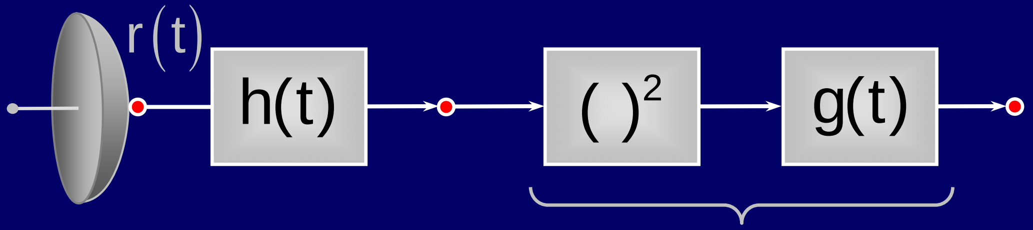
$=$ Signal $*$ $h(t)$



$$z(t) \cong k^2\sigma_o R^2_{\sqrt{P}}(t) + n(t)$$

Envelope Detector

Optimum CW Pulse Radar Receiver



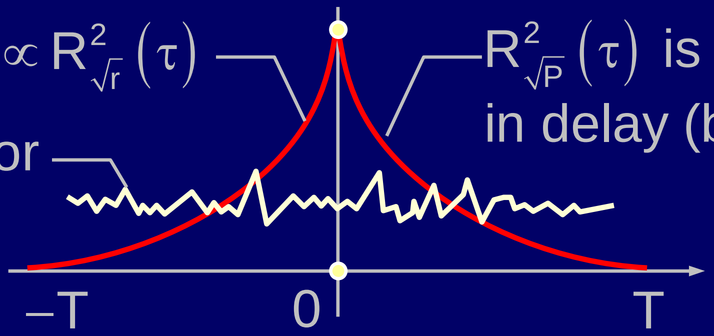
$z(t) \cong k^2\sigma_o R^{\,2}_{\sqrt{P}}(t) + n(t)$

Envelope Detector

$z(t) \propto R^2_{\sqrt{r}}(\tau)$

$R^2_{\sqrt{P}}(\tau)$ is "ambiguity function" in delay (blur function)

$n(t)$, noise floor



Ambiguity function

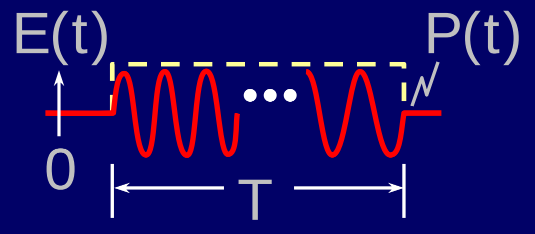
$$E\left[z(t)\right] = k^2\underset{\uparrow}{\sigma}(t) * \overset{\downarrow}{R^2_{\sqrt{P}}}(t) + n(t)$$

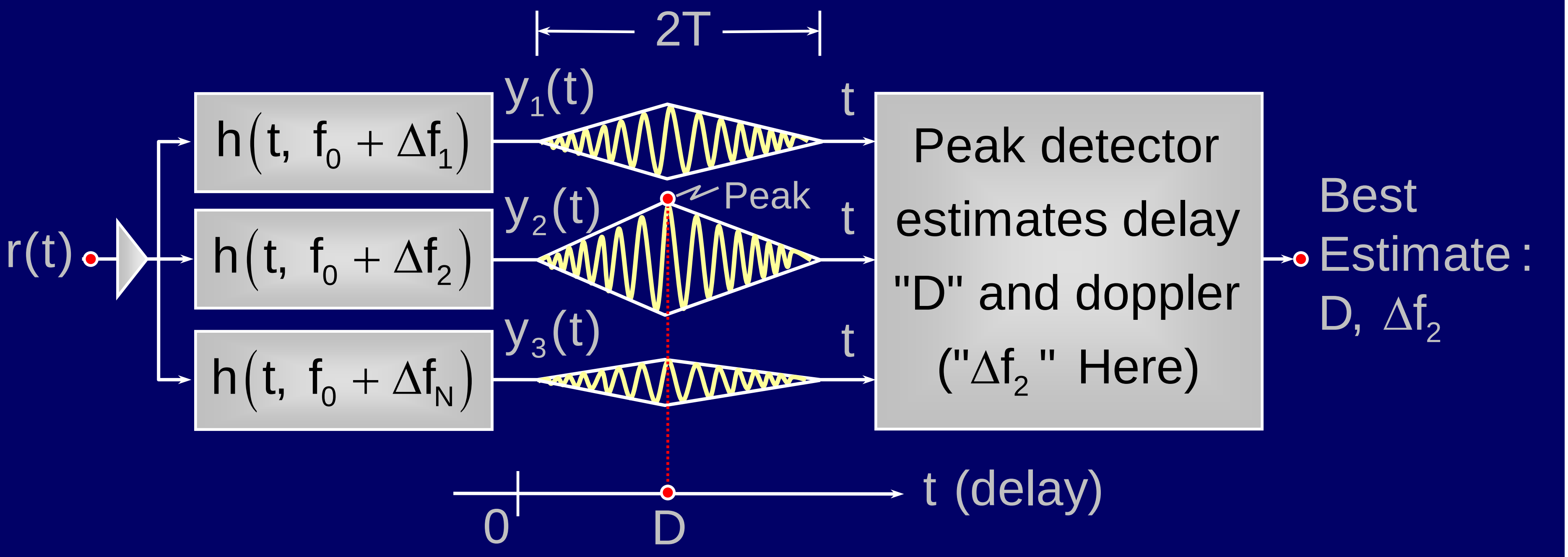
Scattering cross-section

Note: The matched filter can operate on the RF signal (as here) or on its detected envelope.

Range-Doppler Matched-Filter Receiver

Pulsed CW (Continuous Wave) transmitted signal (e.g.):





$h\left(t,\ f_0 + \Delta f_1\right)$

$h\left(t,\ f_0 + \Delta f_2\right)$

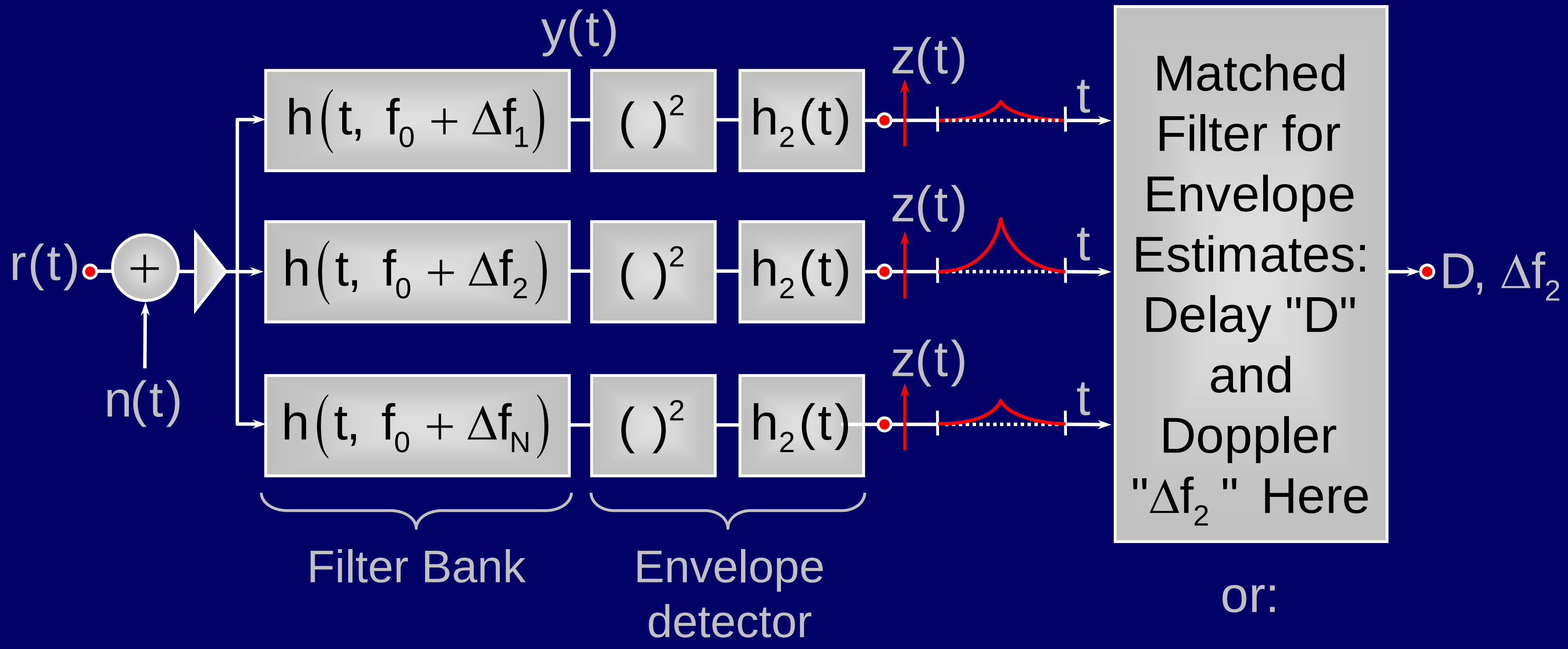
$h\left(t,\ f_0 + \Delta f_N\right)$

Peak detector estimates delay "D" and doppler ("Δf_2" Here)

Best Estimate : D, Δf_2

Range-Doppler Matched-Filter Receiver

Alternative range-doppler matched filter receiver

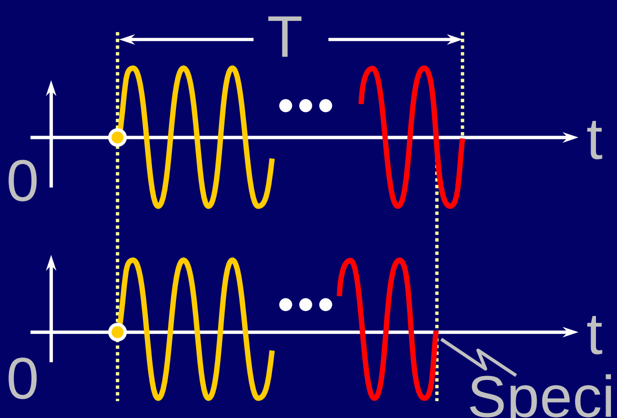


or:

$$h(t, f_0 + \Delta f_1) = p(-t)\cos\left(-(\omega + \Delta\omega_1)\,t\right)$$

Range-Doppler Ambiguity Function

CW Pulse:



Special case:

180° out of phase

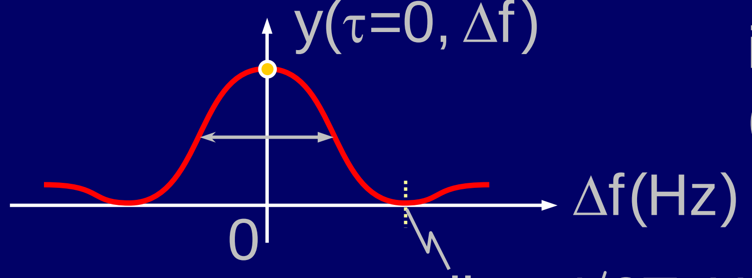
yields y(t) null response

First null in y(t, Δf):

Shifted $\frac{\Delta f}{f} = \frac{1/2 \text{ cycle}}{fT \text{ cycles}}$

Therefore $\Delta f = 1/2T$ Hz for first Doppler null

Doppler ambiguity function for y(t):

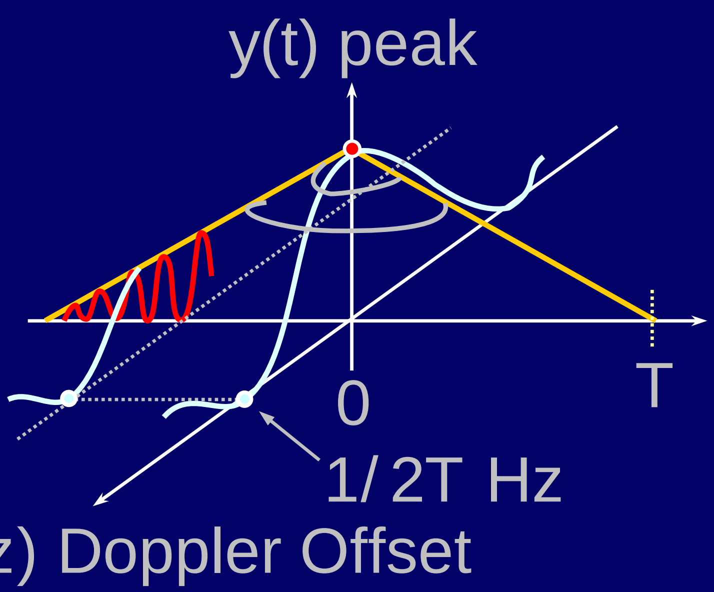


null at $1/2T$ Hz

Doppler resolution $\cong 1/2T$ Hz
in confusion limit
(better if point-source reflector)

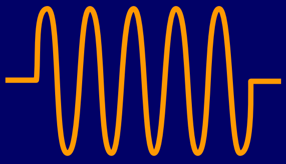
Range-Doppler Ambiguity Function

Range-doppler ambiguity function for y(t);
Represents point-source response:



Half-power width in
range $\cong$ T seconds;
width in Doppler $\cong$ 1/ 2T Hz

Heisenberg uncertainty principle:

$\Delta t \Delta f \cong 1$ for  or: $BT \cong 1 =$ time-bandwidth product,
where $B \cong 1/T$ Hz here

CW Pulse Radar Response $Z \cong \sigma(\tau,\Delta f) * R(\tau,\Delta f)$

Ambiguity function R(τ, Δf) = z(τ, Δf) for point target = "impulse response" for radar.

Therefore $\boxed{z(\tau,\Delta f) = R(\tau,\Delta f) * \sigma_s(\tau,\Delta f)}$

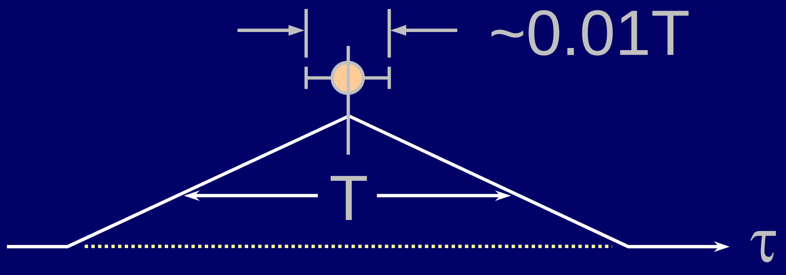
where $\sigma_s(\tau,\Delta f) =$ target response function

That is: Radar response = (ambiguity function) $*$ (target response)

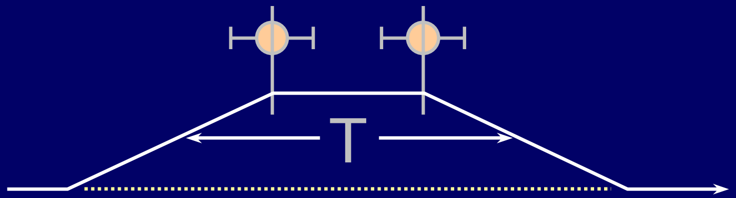
Note: For good image reconstruction of complex images we are limited largely by T and 1/2T resolution in delay, Doppler

CW Pulse Radar Response – Simple Targets

For a point target, delay and Doppler resolution can achieve ~0.01T and 1/200T, or better, if the SNR is sufficiently high; almost the same is possible for 2 point targets



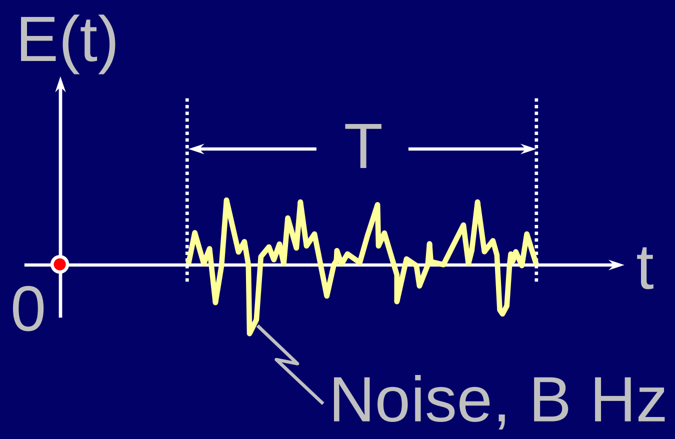
Single point target
range delay



Pair of point targets

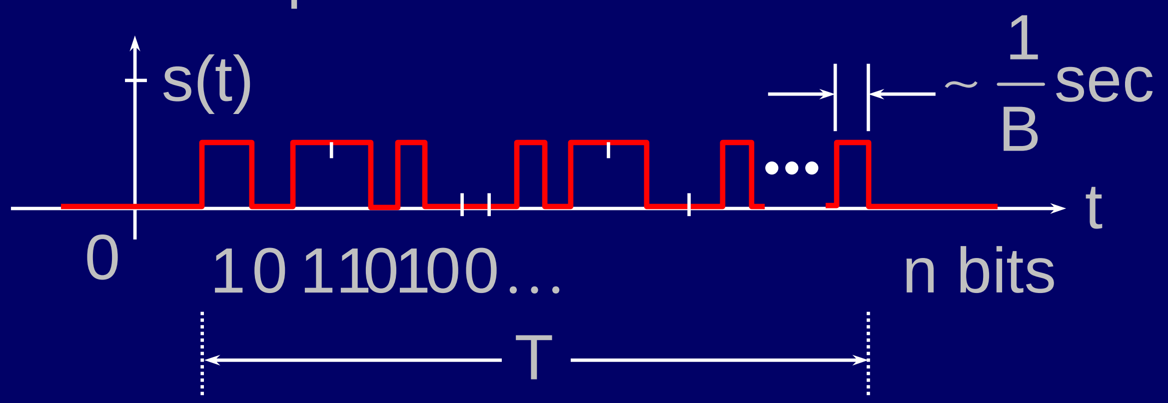
Improved Resolution for Signals with BT>>1

Example:



BT >> 1 for white noise, PRN (pseudo-random noise), binary data with ~BT bits per block

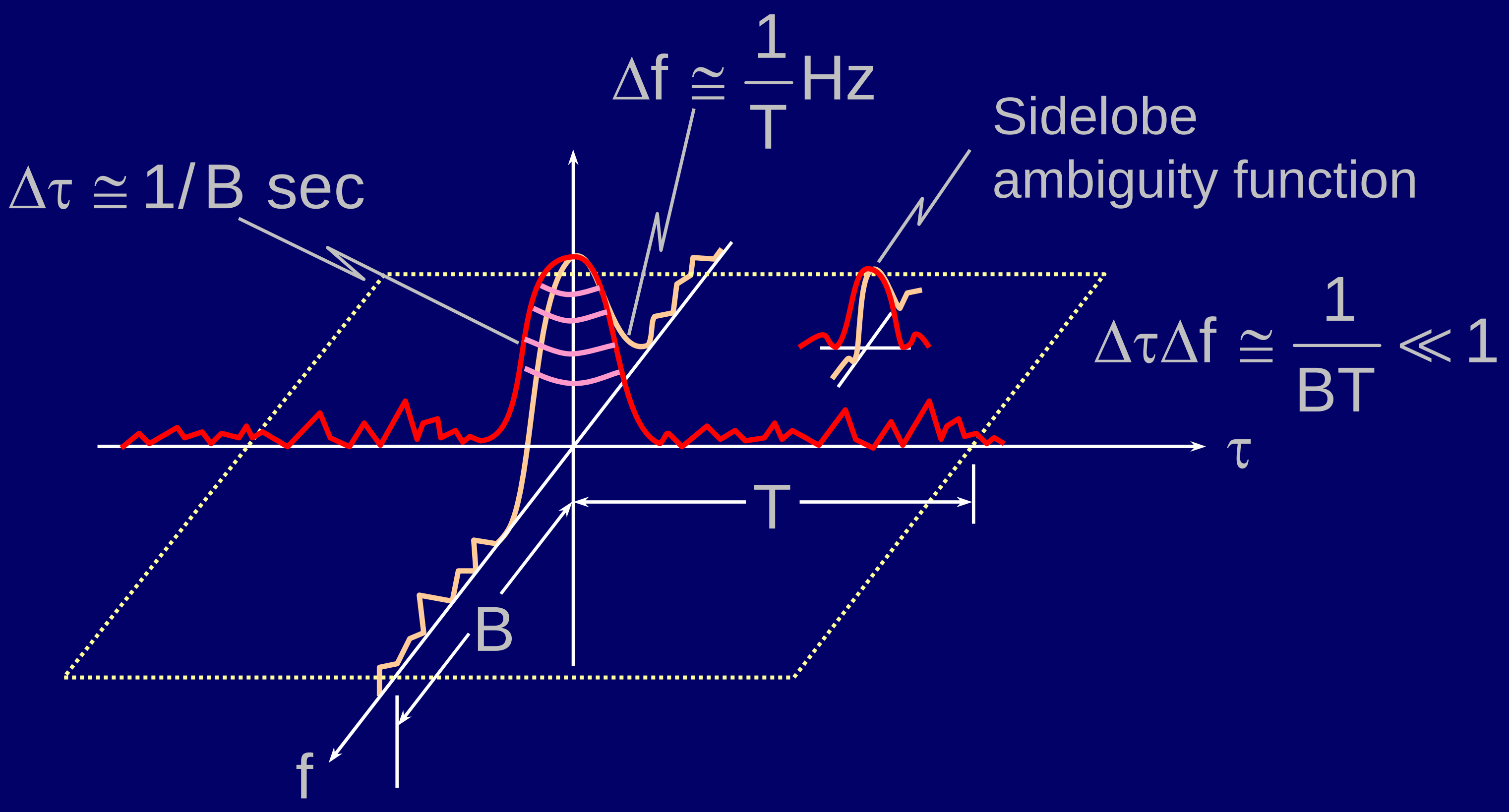
Binary Code Example:



$$T = n(1/B), \text{ so } BT = n >> 1$$

Improved Resolution for BT>>1 Signals

Ambiguity function $z(t, \Delta f)$ for $BT \gg 1$:



Design of s(t) cos ωt to yield minimum ambiguity sidelobes is difficult; trial and error is common design technique.