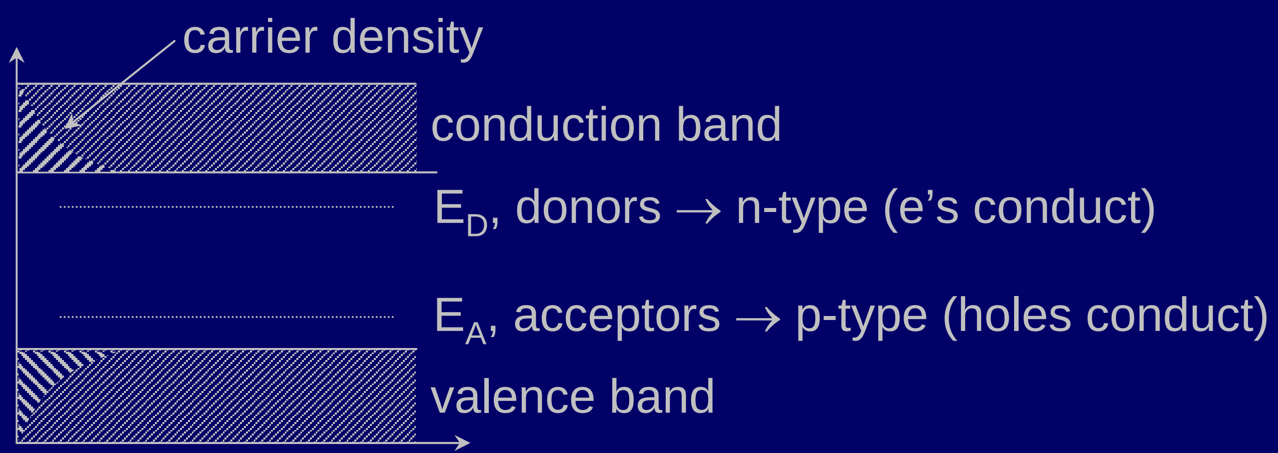
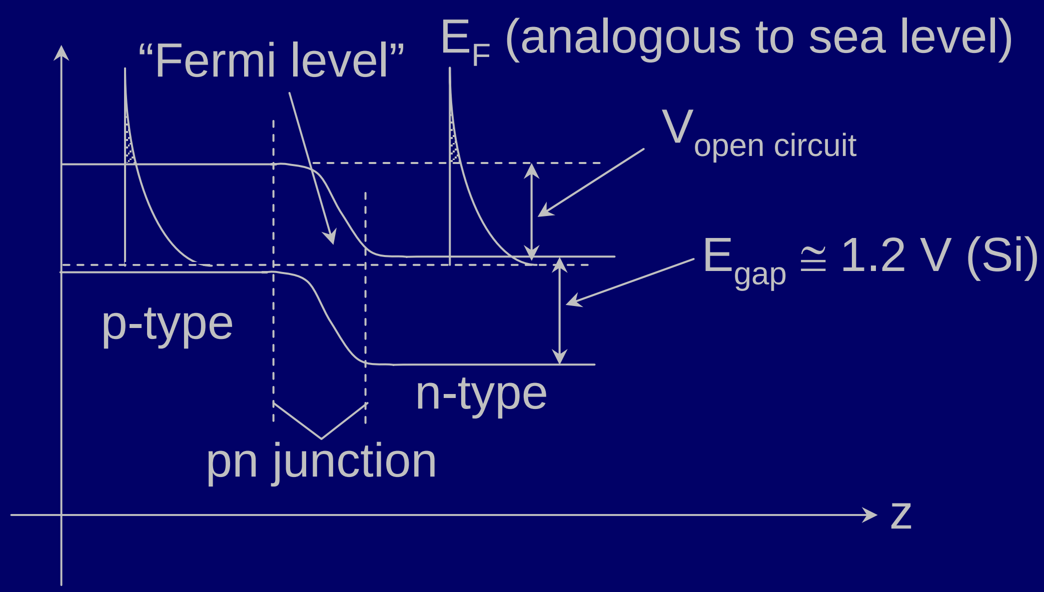
Semiconductor Diode Detectors

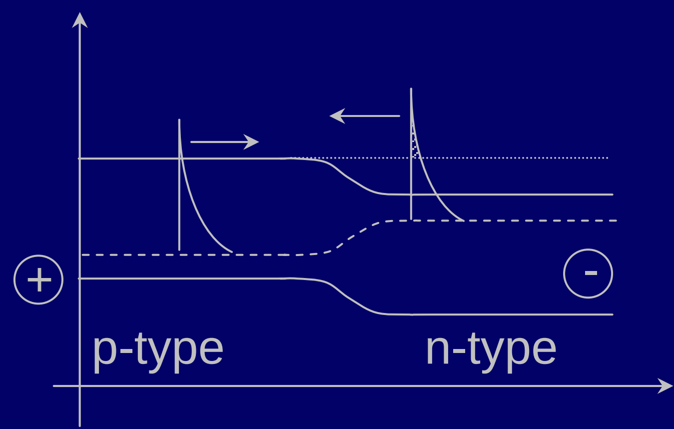
Electron energy diagram – (electrons fall downward)





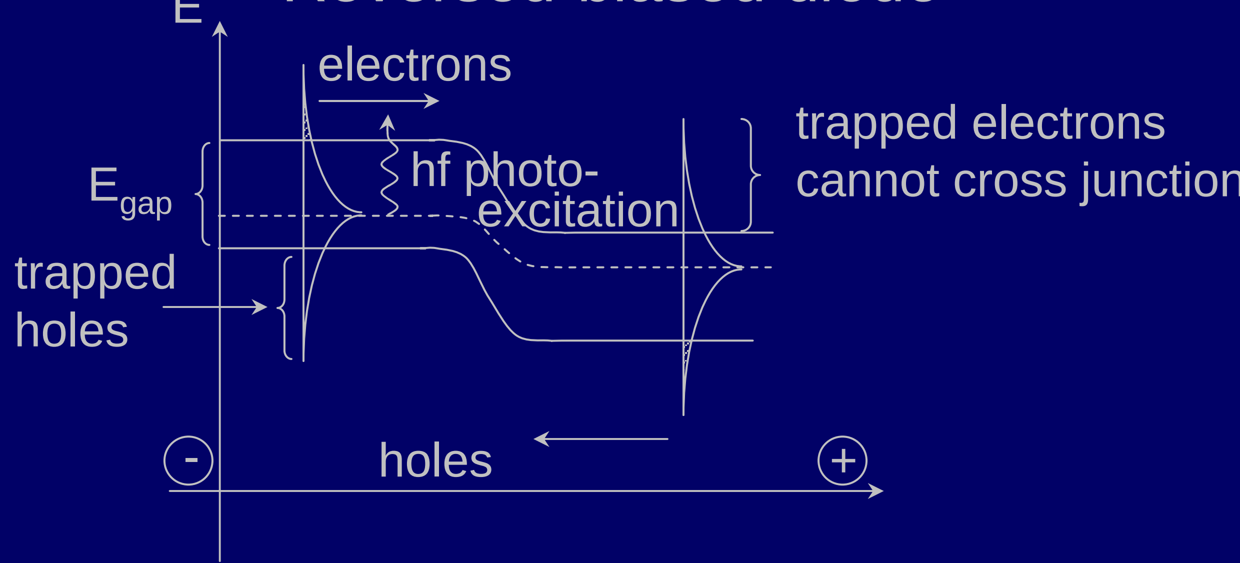
Conductivity of biased diodes

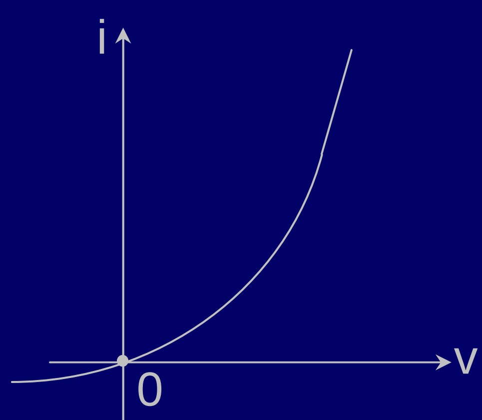
Forward-biased diode



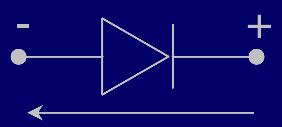
If T = 300K, kT ≅ 26 mv

Reversed-biased diode



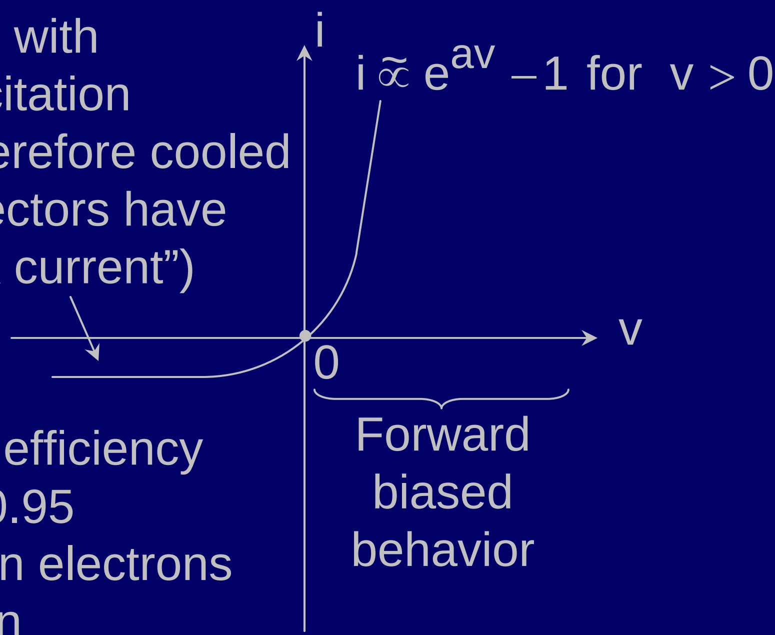


Photodiodes

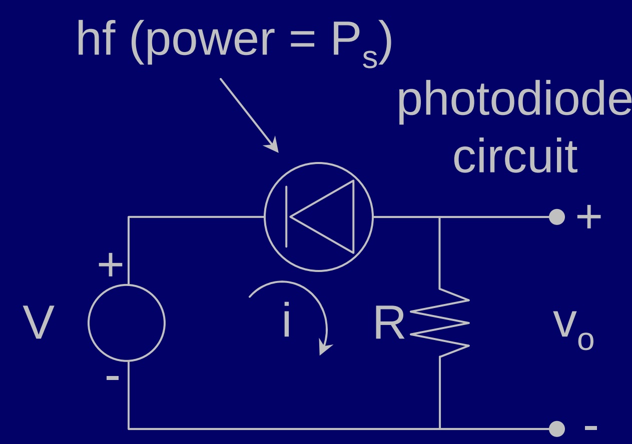


Reverse current

Increases with photo-excitation and T (therefore cooled photodetectors have less “dark current”)

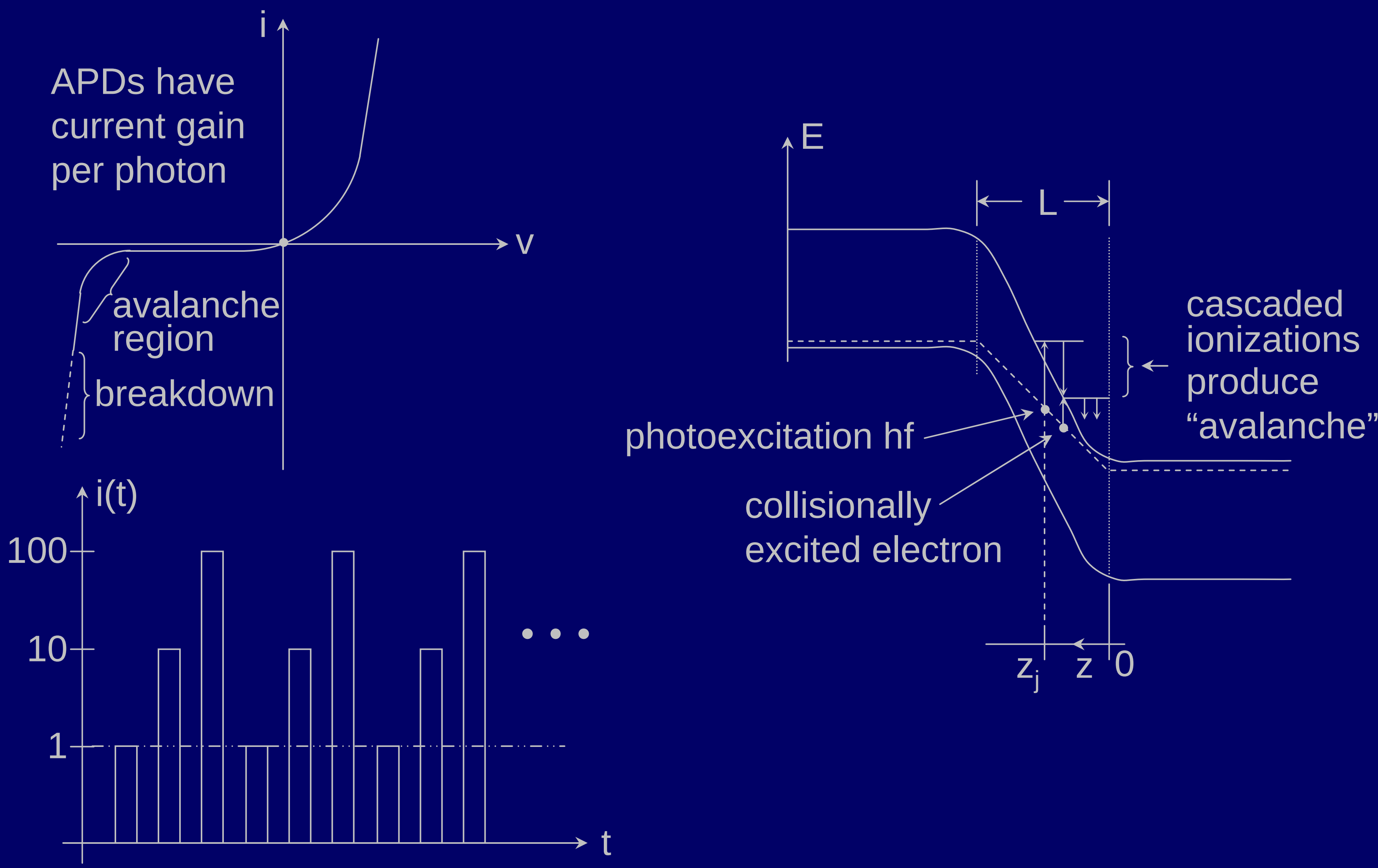


Quantum efficiency
$\eta \cong 0.8 - 0.95$ conduction electrons per photon if $hf > E_{gap}$

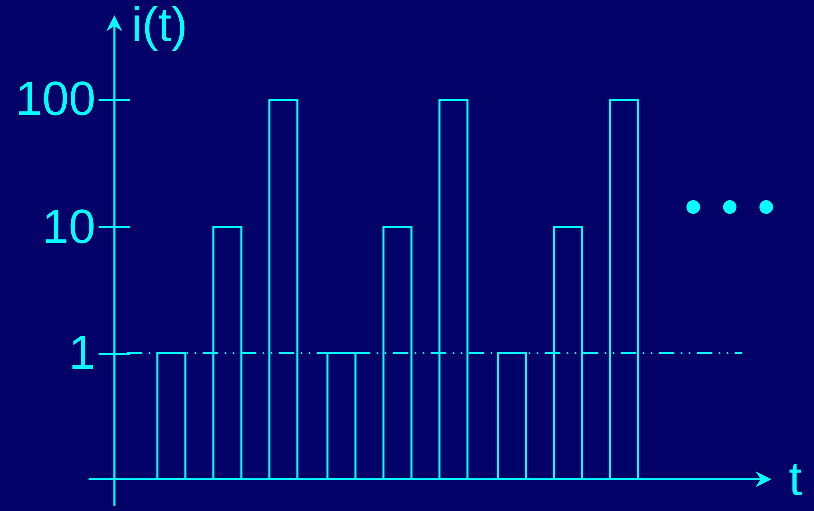


$$v_o = \left(i_{dark} + \frac{\eta P_s}{hf} e \right) R$$

Avalanche Photo Diodes, “APD”



Avalanche Photodiodes, "APD"



Simple model for avalanche current and noise:

i^{th} photon has gain $g_i \cong e^{g_o z_i}$

$$G = E\left[e^{g_o z_i}\right] \cong \frac{1}{L}\int_0^L e^{g_o z} dz = \frac{1}{g_o L}\left(e^{g_o L} - 1\right) \left[L \triangleq \text{junction thickness}\right]$$

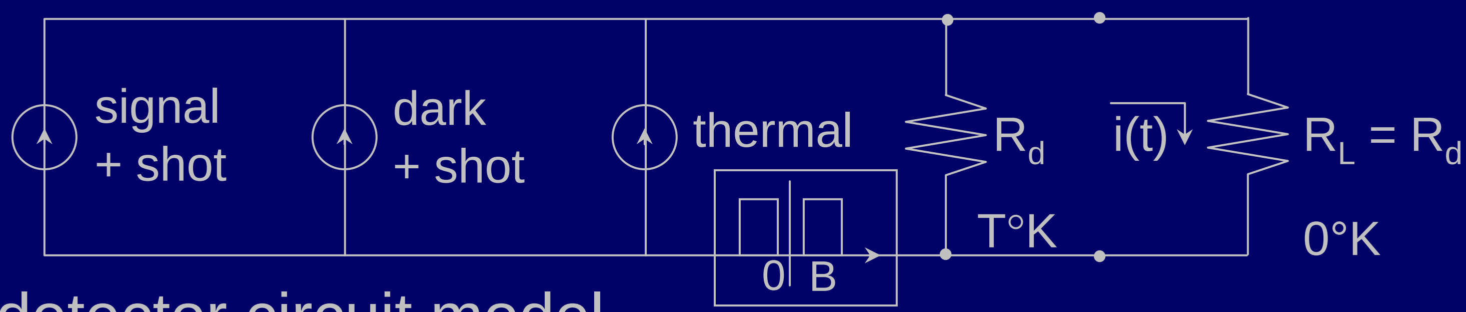
$$E\left[g^2\right] = E\left[e^{2 g_o z_i}\right] \cong \frac{1}{2 g_o L}\left[e^{2 g_o L} - 1\right]$$

In practice $E\left[g^2\right] / G^2 \cong G^x$; where $x \cong 0.25$; $(x = 0.2\text{-}0.5 \text{ is typical})$

$G \cong 200 \pm \sim 6$ dB, for typical applications

Carrier-to-Noise Ratio, "CNR"

for photon detectors



Photodetector circuit model

$\text{CNR} \triangleq \overline{i_s(t)}^2 / E[i - \bar{i}]^2$ where $i_s(t)$ is the signal current through R_L and i is the total current

signal power (W) ≅ constant for PMT

$\overline{i_s(t)} = \eta P_s eG / hf$ $\overline{i^2_{n\,\text{shot}}} = 2B(eG)\bar{i}_{s+D}$

Only $i_n/2$ flows in R_L from R_d noise (assume $T_L << T_{diode}$),

and $(i_n/2)^2 R_L = kTB$ since R_L matched to diode R_d

Therefore $\overline{i^2_{n\,\text{thermal}\,R_L}} = 4kTB/R_L$

Carrier-to-Noise Ratio, "CNR"

$\text{CNR} \triangleq \overline{i_s(t)}^2 / E[i - \bar{i}]^2$ where $i_s(t)$ is the signal current through R_L and i is the total current

signal power (W) ↓

$$\overline{i_s(t)} = \eta P_s eG/hf$$

≅ constant for PMT ↓

$$\overline{i^2_{n\,shot}} = 2B(eG)\bar{i}_{s+D}$$

$$\overline{i^2_{n\,thermal\,R_L}} = 4kTB/R_L$$

$$\text{CNR} = \frac{(\eta P_s eG/hf)^2}{\underbrace{(2BeG\eta(P_s + P_D))eG/hf}_{i^2_{n\,shot}} + \underbrace{(4kTB/R_L)}_{i^2_{n\,thermal}}}$$

for photo diodes or PMT with constant G

CNR for constant-G photodiodes, photomultipliers

$$\mathrm{CNR} = \frac{\left(\eta P_s eG/hf\right)^2}{\underbrace{\left(2BeG\eta\left(P_s + P_D\right)\right)eG/hf}_{i^2_{n_{shot}}} + \underbrace{\left(4kTB/R_L\right)}_{i^2_{n_{thermal}}}}$$

for photo diodes or PMT with constant G

$$\mathrm{CNR} = \frac{\eta P_s/hf\,2B}{1 + P_D/P_s + 2kThf/R_L\eta P_s(eG)^2}$$

For P_D = T =0, or $P_s \rightarrow \infty$; ideal quantum limit (denominator equals unity)

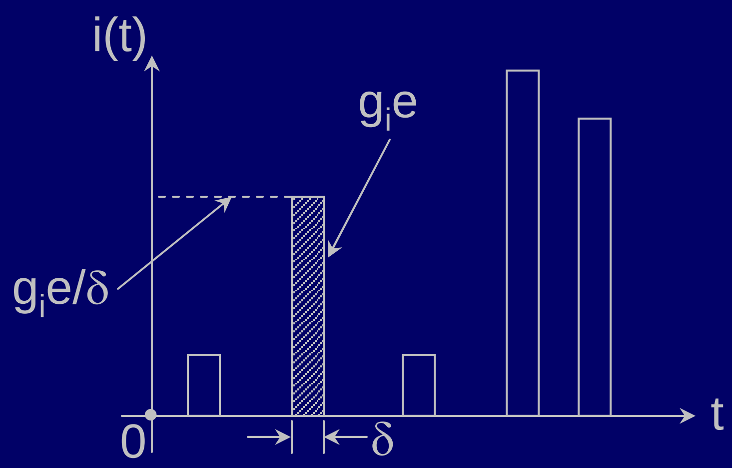
In the quantum limit, we want large η and P_s, and small B

Why not let $R_L \rightarrow \infty$?

Because $R_L C = \tau$ sec; C = detector capacitance $\cong 10^{-12}$ (say)
Then $R_L \cong 1000\ \Omega$ for $\tau = 10^{-9}$ (f $\cong$ 150 MHz)

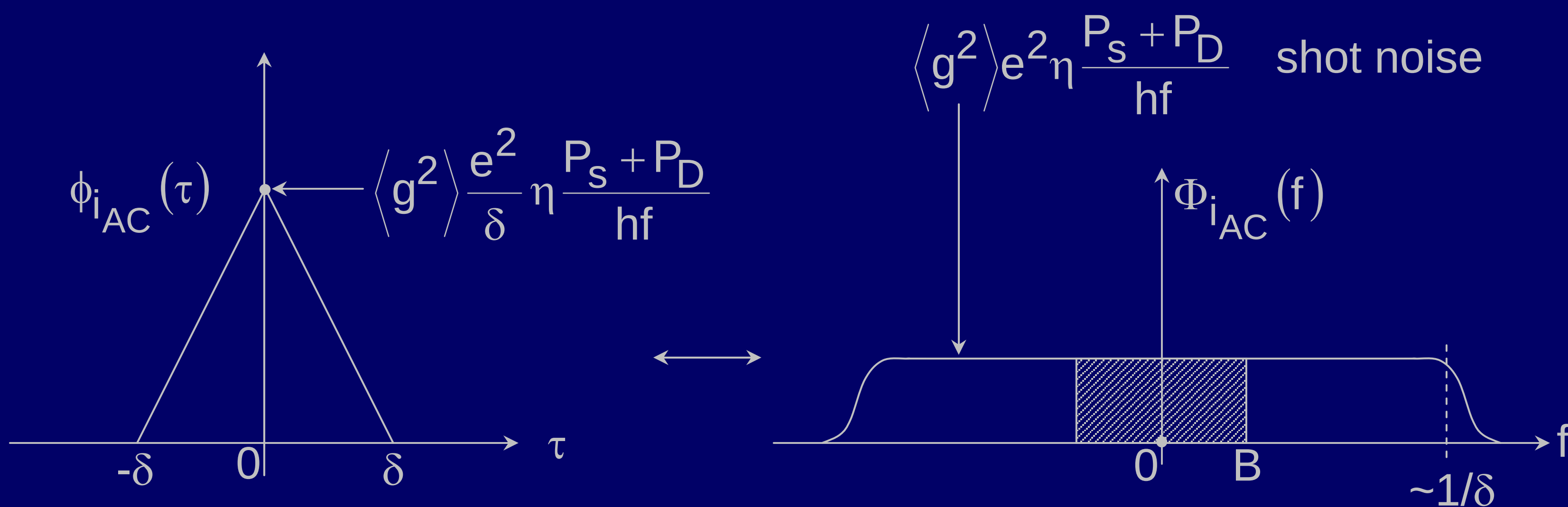
Also, generally: set T so $P_D < P_s$
set $R_L G^2$ large to contain thermal noise

CNR for variable-G avalanche photodiodes

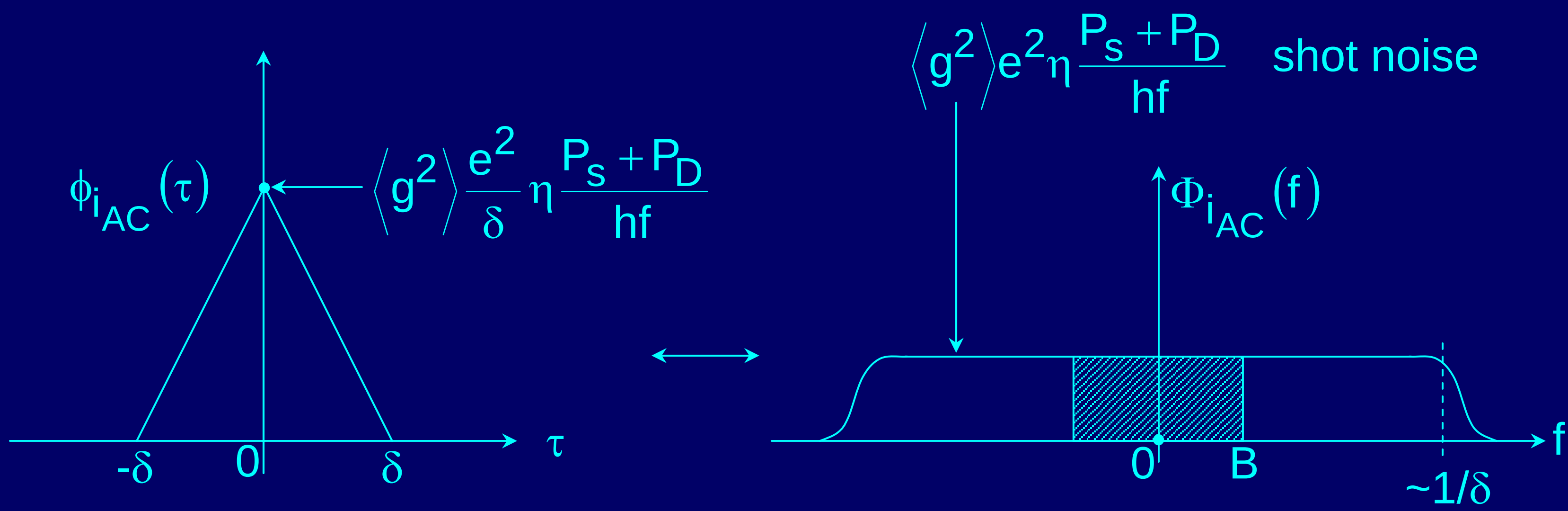


Assume rectangular pulses
to simplify analysis

$$\bar{n} = \frac{\eta(P_s + P_D)}{hf} \text{ events s}^{-1}$$



CNR for variable-G avalanche photodiodes



$$\sigma_i^2 = \left[\left(i - \bar{i}\right)^2 \right] = \int_{-B}^{B} \Phi_i(f)\,df = 2B\langle g^2 \rangle e^2 \eta \left(P_s + P_D\right) / hf$$

$$\text{Therefore CNR (APD)} = \frac{\left(\eta P_s e G / hf\right)^2}{\underbrace{2B\langle g^2 \rangle e^2 \eta \left(P_s + P_D\right) / hf}_{\sigma_i^2 \text{ shot}} + \underbrace{4kTB / R_L}_{i_n^2 \text{ thermal}}}$$

CNR for variable-G avalanche photodiodes

$$\text{Therefore CNR (APD)} = \frac{\left(\eta P_s eG/hf\right)^2}{\underbrace{2B\langle g^2\rangle e^2\eta\left(P_s + P_D\right)/hf}_{\sigma_i^2 \text{ shot}} + \underbrace{4kTB/R_L}_{i_n^2 \text{ thermal}}}$$

$$\text{CNR (APD)} = \frac{\eta P_s/hf\,2B}{\dfrac{\langle g^2\rangle}{G^2}\left(1+\dfrac{P_D}{P_s}\right) + \dfrac{2kThf}{R_L\eta P_s(eG)^2}}$$

Only change is $\langle g^2\rangle/G^2 \cong G^x\ ,\ x \cong 0.2 - 0.5$

In general, to maximize APD CNR we want G^2 large to make thermal noise negligible, but small so G^x is still modest; e.g. $G^x \cong 4$ is typical

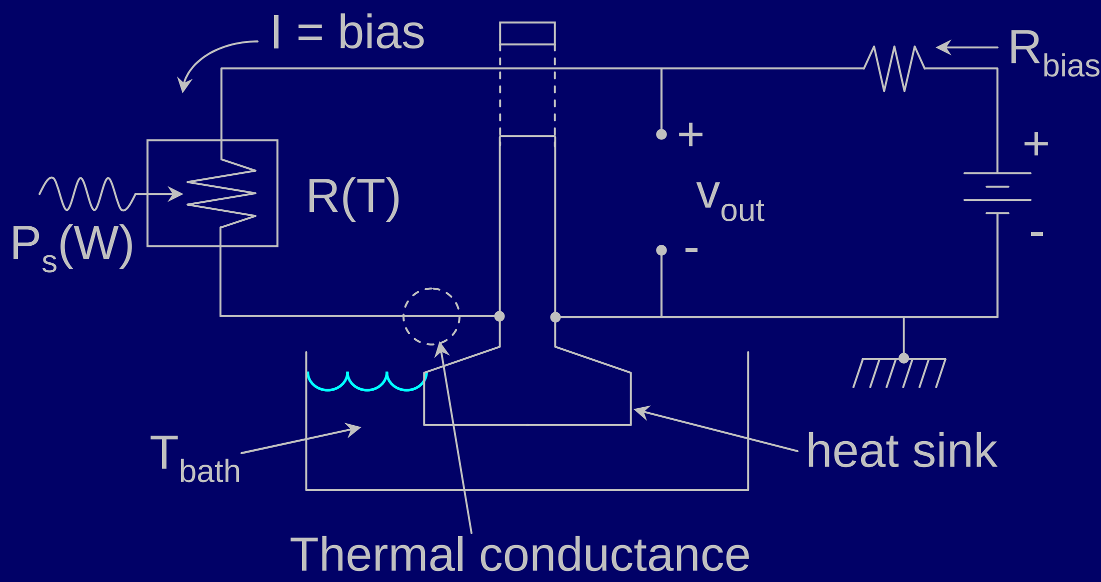
Infrared Detection

Types of electromagnetic detectors

hf << kT	"radio"
hf >> kT	"optical," "visible," etc.
hf = kT	"infrared"

Infrared Detection

Bolometers (measure heat)

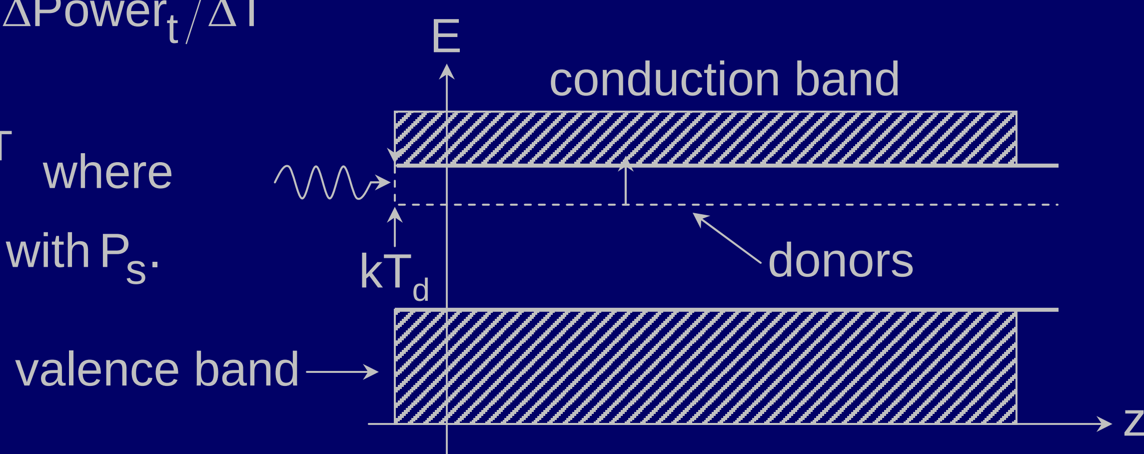


Signal power P_s raises T, changing R(T) and v_o. Heat flows to heat sink and to bath; $T_{bath} \lesssim 4 - 250$K.

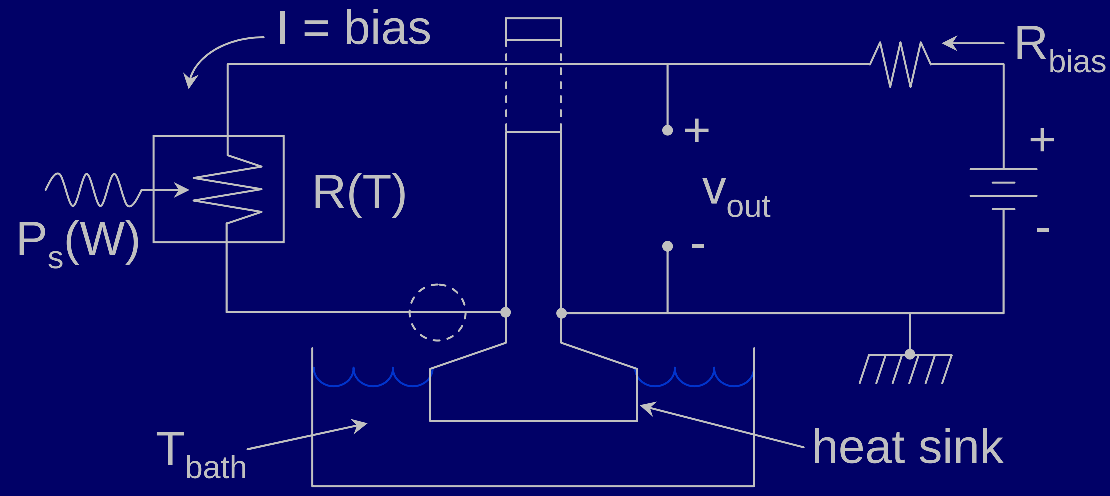
$$G_t\left(WK^{-1}\right) \triangleq \Delta Power_t / \Delta T$$

$R \cong R_o e^{T_d/T}$ where

T increases with P_s.



Responsivity S of a bolometer



$$\frac{\partial v_o}{\partial P_s} \triangleq S \ \text{"responsivity"} = I\frac{\partial R}{\partial P} \bullet \frac{\partial P}{\partial P_s} \quad \text{where } P = I^2 R + P_s$$

$$\text{Thus } \frac{\partial P}{\partial P_s} = I^2 \frac{\partial R}{\partial P}\frac{\partial P}{\partial P_s} + 1 = \left(1 - I^2\frac{\partial R}{\partial P}\right)^{-1} \qquad (R_{bias} >> R)$$

$$\text{Thus } S = I\frac{\partial R}{\partial P} \Big/ \left(1 - I^2 \frac{\partial R}{\partial P}\right) \quad \text{where } \frac{\partial R}{\partial P} = \frac{\partial R}{\partial T} \bullet \left.\frac{\partial T}{\partial P}\right\} = 1/G_t$$

$$\frac{\partial R}{\partial T} = \frac{\partial}{\partial T}\left(R_o e^{T_d/T}\right) \cong \frac{-R_o T_d}{T^2} \qquad \left(\frac{T_d}{T} << 1\right)$$

$$S = \frac{-IT_d R}{G_t T^2} \Big/ \left(1 + \frac{I^2 T_d R}{G_t T^2}\right)$$

$T_d << T$ may not apply

Responsivity S of a bolometer

$$S = \frac{-IT_dR}{G_tT^2} \Bigg/ \left(1 + \frac{I^2T_dR}{G_tT^2}\right) \quad \text{If } T_d << T$$

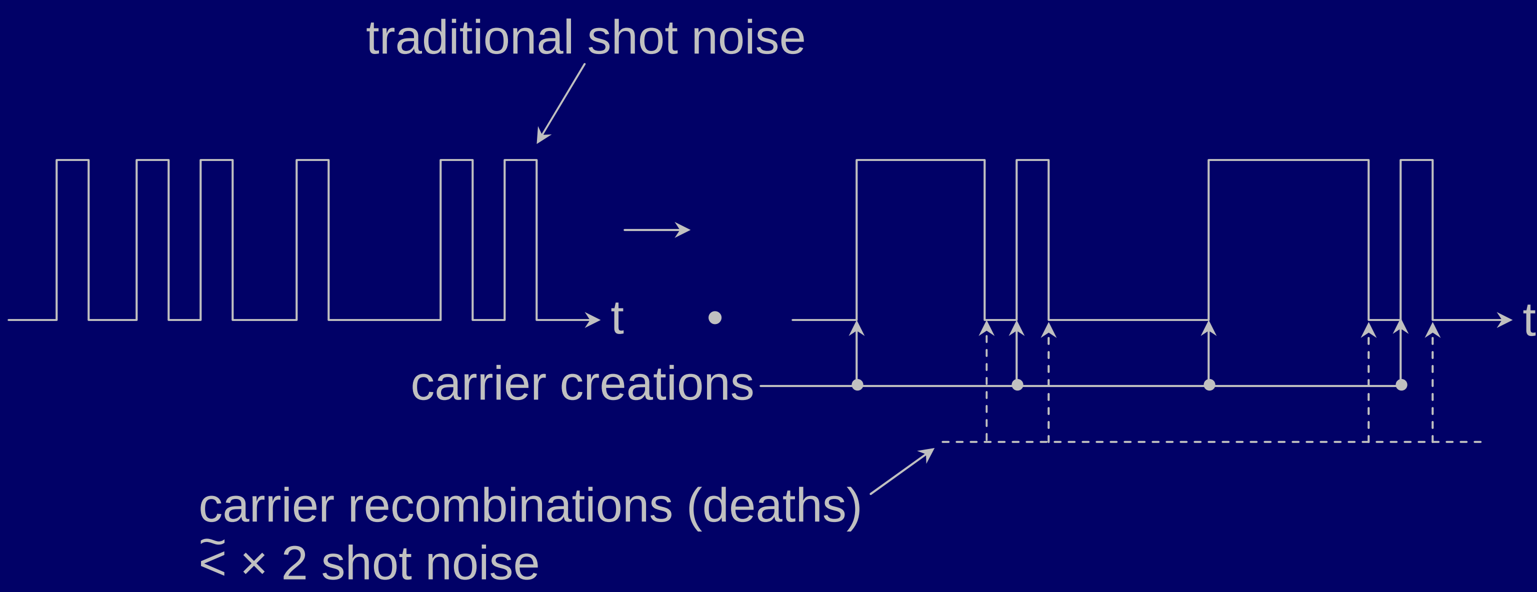
Thus $S \rightarrow 0$ and thermal noise in R_{bias} dominates as $I_{bias} \begin{smallmatrix}\rightarrow 0 \\ \rightarrow \infty\end{smallmatrix}$

Thus there is optimum $I_{bias} = f\left[T_dR \big/ G_tT^2\right]$ (near maximum S)

Bolometer noise sources

1) Can be recombination noise
2) Johnson noise in R, R_{bias} (thermal)
3) Phonon noise via G_t
4) Photon noise ("radiation noise")

Recombination Noise



Johnson noise

$$V_{J\,rms} = \sqrt{4kTR}\left(v\ Hz^{-1/2}\right) \quad \left[\text{Recall } \overline{v^2} = 4kTRB\right]$$

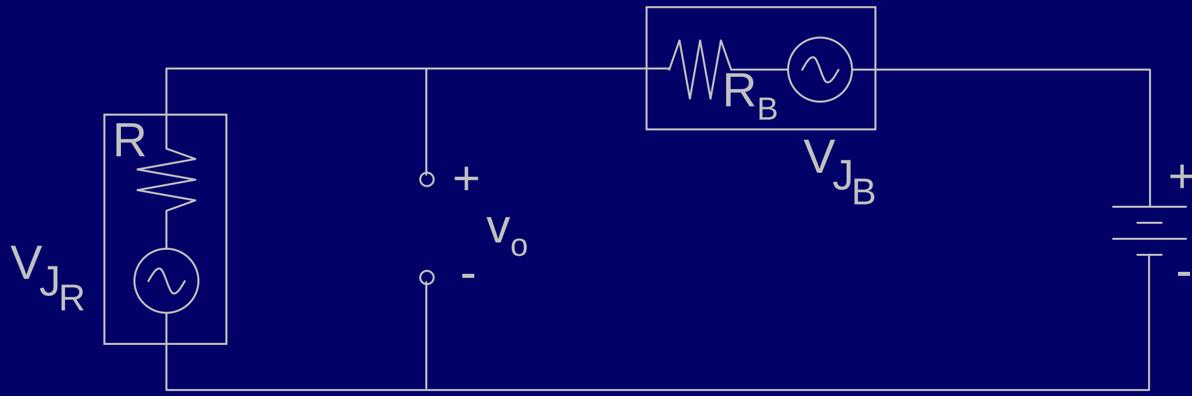
$$\text{"Noise - Equivalent Power"} \equiv \text{"NEP"} \left(W\ Hz^{-1/2}\right)$$

$$NEP_J = V_J / S; \left(S \triangleq \frac{\partial V}{\partial P_s} \right)$$

$$NEP_J \text{ is analogous to: } \Delta T_{rms} = \frac{v_{o_{rms}}}{\partial v_o / \partial T_A}$$

Johnson noise minimization

Bolometer equivalent circuit for V_{J_B}, V_{J_R} from bias and detector resistors:



$$V_{J_{B_{out}}} = \frac{R}{R+R_B} \underbrace{V_{J_B}}_{\propto \sqrt{R_B}} \rightarrow 0 \text{ as } \frac{R_B}{R} \rightarrow \infty$$

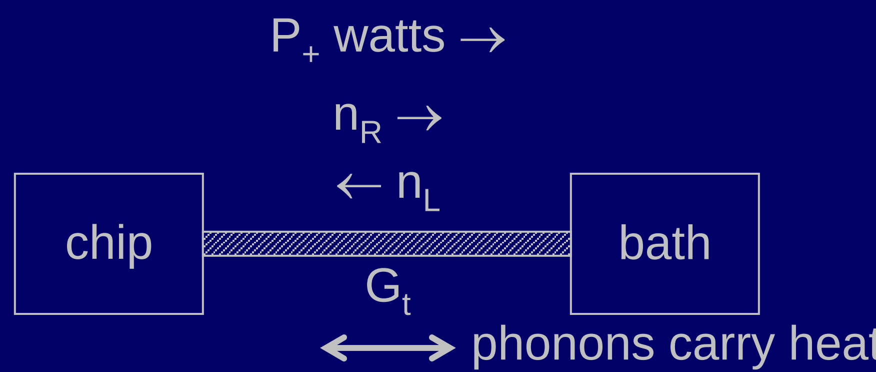
Therefore set $R_B >> R$ or $T_{bias} \cong 0°K$ and then $V_{J_{out}}$ mostly from R

$$V_{J_{R_{out}}} \cong \frac{R_B}{R+R_B} \bullet V_{J_R} \cong V_{J_R} = \sqrt{4kT_{bath}BR} \text{ volts rms}$$

Phonon noise

Approximate intuitive analysis:

For thermal phonons $hf \cong kT$



Say n_R phonons s^{-1}
moving right, n_L moving left

$$\text{Heat flux } P_+ \cong k\left(n_R T_{chip} - n_L T_{bath}\right) \cong \underbrace{kn}_{G_t}\Delta T\left(n = n \cong n_L\right) = G_t \Delta T$$

$$\text{Phonon shot noise: } NEP_{G_t} \cong \sigma_{P_+} \cong kT\sqrt{2n} \cong kT\sqrt{2G_t/k} = \sqrt{2kG_tT^2}$$

Actually,

$$NEP_{G_t} \cong \sqrt{4kG_tT^2} \;\; WHz^{-1/2}$$

If $G_t \rightarrow 0, NEP_{G_t} \rightarrow 0$ but then T rises, so there is an optimum G_t which can be computed

Photon shot noise

Thermal radiation contributes photon shot noise
Assume radiation comes from cavity attached to the detector
Cavity has n_i photons in mode i at energy $E_i = hf_i$

We seek σ_n^2 for a single mode, i.e. the variance of n

$p(n) = D\, e^{-n[hf/kT]}$ where p(n) = probability of n photons in each mode with frequency f;
D is to be determined
This is "Maxwell-Boltzmann" distribution for thermal equilibrium

Photon shot noise

$p(n) = D\,e^{-n[hf/kT]}$ where p(n) = probability of n photons in each mode with frequency f, is to be determined

This is "Maxwell-Boltzmann" distribution for thermal equilibrium

To solve for D let $x \triangleq hf/kT$ and define $S_k(x) \equiv \sum_{n=0}^{\infty} n^k e^{-nx}$

then $S_{k+1}(x) = \frac{-d}{dx} S_k(x)$

so $S_o(x) = 1/\left(1 - e^{-x}\right)$

Since $\sum_{n=0}^{\infty} p(n) = 1 = \sum_{n=0}^{\infty} D\,e^{-nx} = D \bullet S_o$

If follows that $D = 1/S_o = 1 - e^{-hf/kT}$

Photon shot noise

If follows that $D = 1/S_o = 1 - e^{-hf/kT}$

then $S_k(x) \equiv \sum_{n=0}^{\infty} n^k e^{-nx} \Rightarrow S_{k+1}(x) = \frac{-d}{dx} S_k(x)$

Therefore $S_1(x) = e^{-x} / \left(1 - e^{-x}\right)^2$

$$S_2(x) = S_1(x) + 2e^{-2x} / \left(1 - e^{-x}\right)^3$$

Here we seek $\sigma_n^2 \equiv E\left[\left(n - \bar{n}\right)^2\right] = E\left[n^2\right] - (E[n])^2 \equiv \overline{n^2} - \bar{n}^2$

$$\bar{n} = \sum_{n=0}^{\infty} n\, p(n) = \sum_{n=0}^{\infty} n \frac{1}{S_o} e^{-nx} = S_1 / S_o = e^{-x} / \left(1 - e^{-x}\right)$$

$$\overline{n^2} = \sum_{n=0}^{\infty} n^2 p(n) = S_2 / S_o = \frac{S_1}{S_o} + \frac{2e^{-2x}}{\left(1 - e^{-x}\right)^2} = \bar{n} + 2\bar{n}^2$$

Photon shot noise

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$$\sigma_n^2 = \overline{n^2} - \bar{n}^2 = \boxed{\bar{n} + \bar{n}^2 = \sigma_n^2}$$

where $\bar{n} = 1/\left(e^x - 1\right) = 1/\left(e^{hf/kT} - 1\right)$ photons/mode

Optical limit: $hf >> kT \Rightarrow \bar{n} << 1, \ \sigma_n \cong \sqrt{\bar{n}}$

Radio limit: $hf << kT \Rightarrow \bar{n} >> 1, \ \sigma_n \cong \bar{n}$

IR regime: $hf \cong kT \ \Rightarrow \bar{n} \cong 1, \ \sigma_n \cong \sqrt{\bar{n} + \bar{n}^2}$