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6.642 Continuum Electromechanics
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Problem 1

Prob.7.6.3 (Melcher)

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Problem 4

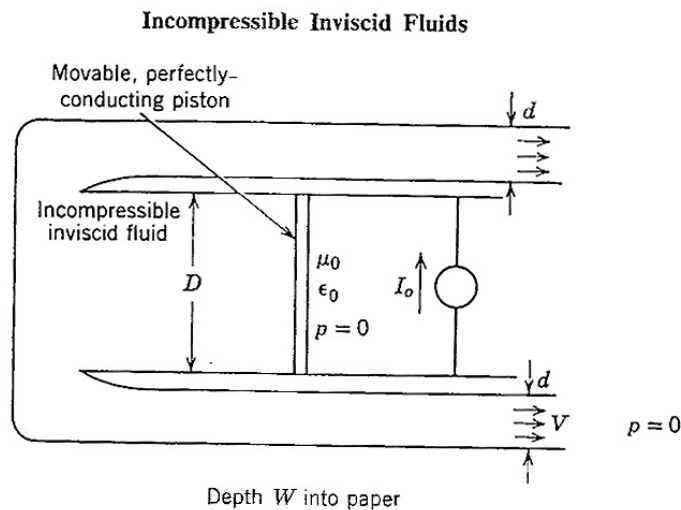


Fig. 12P.3

12.3. A magnetic rocket is shown in Fig. 12P.3. A current source I_o (distributed over the depth W) drives a circuit composed in part of a movable piston. This piston drives an incompressible, inviscid, nonconducting fluid through two orifices, each of height d and depth W . Because $D \gg d$, the flow is essentially steady.

- (a) Find the exit velocity V as a function of I_o .
- (b) What is the thrust developed by the rocket? (You may assume that it is under static test.)

Problems for Chapter 7

For Section 7.2:

Prob. 7.2.1 In Sec. 3.7, α_i is defined such that in the conservative subsystem, Eq. 3.7.3 holds. Show that α_i satisfies Eq. 7.2.3 with $\rho \rightarrow \alpha_i$. Further, show that if a "specific" property β_i is defined such that $\beta_i \equiv \rho \alpha_i$, then by virtue of conservation of mass, the convective derivative of β_i is zero.

For Section 7.6:

Prob. 7.6.1 Show that Eq. (b) of Table 7.6.2 is correct.

Prob. 7.6.2 Show that Eqs. (j) and (l) from Table 7.6.2 are correct.

Prob. 7.6.3 A pair of bubbles are formed with the tube-valve system shown in the figure. Bubble 1 is blown by closing valve V_2 and opening V_1 . Then, V_1 is closed and V_2 opened so that the second bubble is filled. Each bubble can be regarded as having a constant surface tension γ . With the bubbles having the same initial radius ξ_0 , when $t = 0$, both valves are opened (with the upper inlet closed off). The object of the following steps is to describe the resulting dynamics.

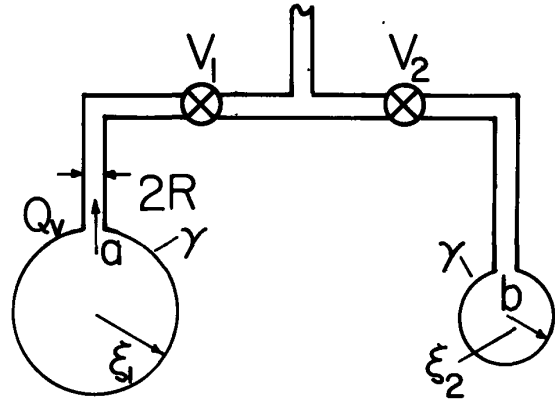


Fig. P7.6.3

- (a) Flow through the tube that connects the bubbles is modeled as being fully developed and viscous dominated. Hence, for a length of tube ℓ having inner radius R and with a viscosity of the gas η , the volume rate of flow is related to the pressure difference by

$$Q_v = \frac{\pi R^4}{8\eta} \frac{(p_a - p_b)}{\ell} \quad \text{m}^3/\text{sec}$$

The inertia of the gas and bubble is ignored, as is that of the surrounding air. Find an equation of motion for the bubble radius ξ_1 .

- (b) With the bubbles initially of equal radius ξ_0 , there is a slight departure of the radius of one of the bubbles from equilibrium. What happens?
 (c) In physical terms, explain the result of (b).

For Section 7.8:

Prob. 7.8.1 A conduit forming a closed loop consists of a pair of tubes having cross-sections with areas A_r and A_ℓ . These are arranged as shown with a fluid having density ρ_b filling the lower half and a second fluid having density ρ_a filling the upper half. The object of the following steps is to determine the dynamics of the fluid, specifically the time dependence of the interfacial positions ξ_r and ξ_ℓ .

- (a) Use mass conservation to relate the displacements (ξ_r , ξ_ℓ) to each other and to the fluid velocities (v_r , v_ℓ) on the right and left respectively. Assume that the fluid is inviscid and has a uniform profile over the cross-section of a tube.
 (b) Use Bernoulli's equation, Eq. 7.8.5 to relate quantities evaluated at the interfaces in the lower fluid, and in the upper fluid.
 (c) Write the boundary conditions that relate quantities across the interfaces.
 (d) Show that these laws combine to give an equation of motion for the right interface having the form

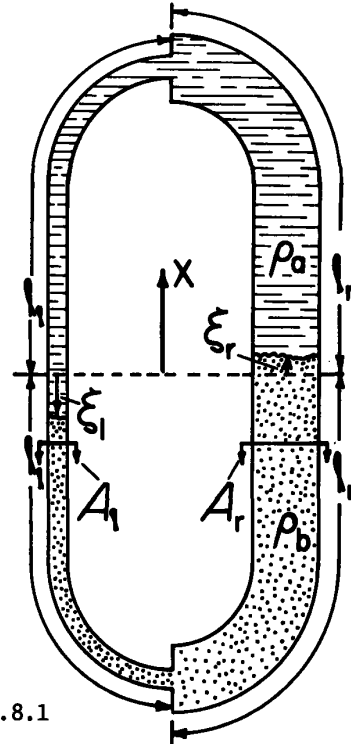


Fig. P7.8.1

$$m \frac{d^2 \xi_r}{dt^2} + \frac{1}{2} (\rho_b - \rho_a) \left[\left(\frac{d\xi_r}{dt} \right)^2 - \left(\frac{d\xi_\ell}{dt} \right)^2 \right] + K \xi_r = 0$$

Courtesy of MIT Press. Used with permission. Melcher, James R. *Continuum Electromechanics*. Cambridge, MA: MIT Press, 1981. ISBN: 9780262131650.

Prob. 3.10.3 Fig. P3.10.3 shows a circular cylindrical tube of inner radius a into which a second tube of outer radius b projects half way. On top of this inner tube is a "blob" of liquid metal (shown inside the broken-line box) having an arbitrary shape, but having a base radius equal to that of the inner tube. The outer and inner tubes, as well as the blob, are all essentially perfectly conducting on the time scale of interest. When $t=0^-$, there are no magnetic fields. When $t=0^+$, the outer tube is used to produce a magnetic flux which has density $B_0 \hat{i}_z$ a distance $l \gg a$ above the end of the inner tube. What is the magnetic flux density over the cross section of the annulus between tubes a distance l ($l \gg a$) below the end of the inner tube? Sketch the distribution of surface current on the perfect conductors (outer and inner tubes and blob), indicating the relative densities. Use qualitative arguments to state whether the vertical magnetic force on the blob acts upward or downward. Use the stress tensor to find the magnetic force acting on the blob in the z direction. This expression should be exact if $l \gg a$, and be written in terms of a , b , B_0 and the permeability of free space μ_0 .

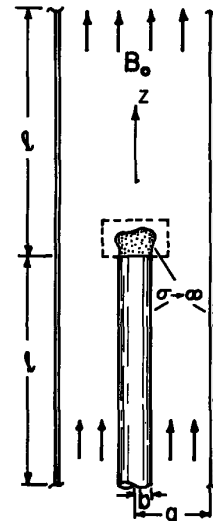


Fig. P3.10.3

Prob. 3.10.4 The mechanical configuration is as in Prob. 3.10.3. But, instead of the magnetic field, an electric field is produced by making the outer cylinder have the potential V_0 relative to the inner one. Sketch the distribution of the electric field, and give qualitative arguments as to whether the electrical force on the blob is upward or downward. What is the electric field in the annulus at points well removed from the tip of the inner cylinder? Use the electric stress tensor to determine the z -directed electric force on the blob.

Prob. 3.10.5 In an EQS system with polarization, the force density is not $\vec{F} = \rho_p \vec{E} + \rho_f \vec{E}$, where ρ_p is the polarization charge. Nevertheless, this force density can be used to correctly determine the total force on an object isolated in free space. The proof follows from the argument given in the paragraph following Eq. 3.10.4. Show that the stress tensor associated with this force density is

$$T_{ij} = \epsilon_0 E_i E_j - \frac{1}{2} \delta_{ij} \epsilon_0 E_k E_k$$

Show that the predicted total force will agree with that found by any of the force densities in Table 3.10.1.

Prob. 3.10.6 Given the force density of Eq. 3.8.13, show that the stress tensor given for this force density in Table 3.10.1 is correct. It proves helpful to first show that

$$[(\nabla \times \mathbf{H}) \times \mathbf{B}]_i = \left(\frac{\partial H_i}{\partial x_j} - \frac{\partial H_j}{\partial x_i} \right) B_j$$

Prob. 3.10.7 Given the Kelvin force density, Eq. 3.5.12, derive the consistent stress tensor of Table 3.10.1. Note the vector identity given in Prob. 3.10.6.

Prob. 3.10.8 Total forces on objects can sometimes be found by the energy method "ignoring" fringing fields and yet obtaining results that are "exact." This is because the change in total energy caused by a virtual displacement leaves the fringing field unaltered. There is a "theorem" than any configuration that can be described in this way by an energy method can also be described by integrating the stress tensor over an appropriately defined surface. Use Eqs. 3.7.22 of Table 3.10.1 to find the force derived in Prob. 2.13.2.

For Section 3.11:

Prob. 3.11.1 An alternative to the derivation represented by Eq. 3.11.7 comes from exploiting an integral theorem that is analogous to Stokes's theorem.¹

1. C. E. Weatherburn, Advanced Vector Analysis, G. Bell and Sons, Ltd., London, 1966, p. 126.