6.641 Electromagnetic Fields, Forces, and Motion
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Formula Sheet

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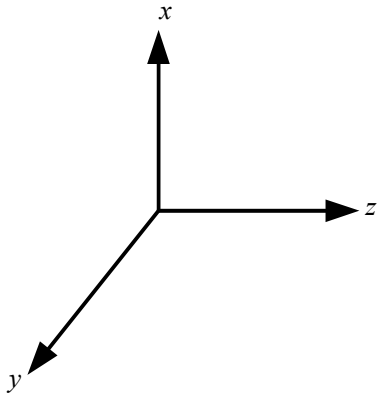
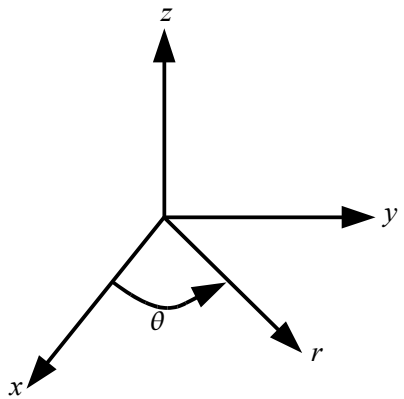
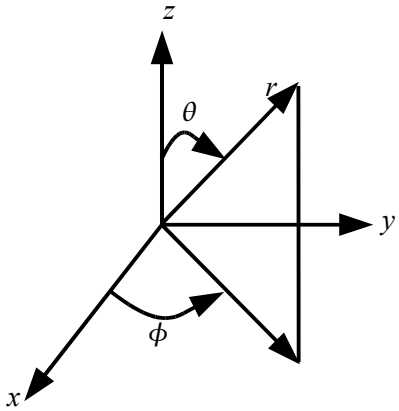
Differential Operators in Cartesian, Cylindrical, and Spherical Coordinates

Operator	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
			
$(\nabla \cdot \vec{A})$	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$
$\nabla\Phi$	$\frac{\partial \Phi}{\partial x}\vec{i}_x + \frac{\partial \Phi}{\partial y}\vec{i}_y + \frac{\partial \phi}{\partial z}\vec{i}_z$	$\frac{\partial \Phi}{\partial r}\vec{i}_r + \frac{1}{r}\frac{\partial \Phi}{\partial \theta}\vec{i}_\theta + \frac{\partial \Phi}{\partial z}\vec{i}_z$	$\frac{\partial \Phi}{\partial r}\vec{i}_r + \frac{1}{r}\frac{\partial \Phi}{\partial \theta}\vec{i}_\theta + \frac{1}{r\sin\theta}\frac{\partial \Phi}{\partial \phi}\vec{i}_\phi$
$(\nabla^2\Phi)$	$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2}$	$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial \Phi}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial \Phi}{\partial r} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\frac{\partial \Phi}{\partial \theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 \Phi}{\partial \phi^2}$
$(\nabla \times \vec{A})$	$\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\vec{i}_x$ $+\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\vec{i}_y$ $+\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\vec{i}_z$	$\left(\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right)\vec{i}_r$ $+\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\vec{i}_\theta$ $+\left(\frac{1}{r}\frac{\partial}{\partial r}(rA_\theta) - \frac{1}{r}\frac{\partial A_r}{\partial \theta}\right)\vec{i}_z$	$\left(\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{1}{r\sin\theta}\frac{\partial A_\theta}{\partial \phi}\right)\vec{i}_r$ $+\left(\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{1}{r}\frac{\partial}{\partial r}(rA_\phi)\right)\vec{i}_\theta$ $+\left(\frac{1}{r}\frac{\partial}{\partial r}(rA_\theta) - \frac{1}{r}\frac{\partial A_r}{\partial \theta}\right)\vec{i}_\phi$

$\nabla^2 \vec{A}$	$\left[\frac{\partial^2 A_x}{\partial x^2}+\frac{\partial^2 A_x}{\partial y^2}+\frac{\partial^2 A_x}{\partial z^2}\right]\vec{i}_x$ $+\left[\frac{\partial^2 A_y}{\partial x^2}+\frac{\partial^2 A_y}{\partial y^2}+\frac{\partial^2 A_y}{\partial z^2}\right]\vec{i}_y$ $+\left[\frac{\partial^2 A_z}{\partial x^2}+\frac{\partial^2 A_z}{\partial y^2}+\frac{\partial^2 A_z}{\partial z^2}\right]\vec{i}_z$	$\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rA_r)\right)+\frac{1}{r^2}\frac{\partial^2 A_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial A_\theta}{\partial \theta}+\frac{\partial^2 A_r}{\partial z^2}\right]\vec{i}_r$ $+\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rA_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 A_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial A_r}{\partial \theta}+\frac{\partial^2 A_\theta}{\partial z^2}\right]\vec{i}_\theta$ $+\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 A_z}{\partial \theta^2}+\frac{\partial^2 A_z}{\partial z^2}\right]\vec{i}_z$	$\left[\nabla^2 A_r-\frac{2A_r}{r^2}-\frac{2}{r^2}\frac{\partial A_\theta}{\partial \theta}-\frac{2A_\theta\cot\theta}{r^2}-\frac{2}{r^2\sin\theta}\frac{\partial A_\phi}{\partial \phi}\right]\vec{i}_r$ $+\left[\nabla^2 A_\theta+\frac{2}{r^2}\frac{\partial A_r}{\partial \theta}-\frac{A_\theta}{r^2\sin^2\theta}-\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\phi}{\partial \phi}\right]\vec{i}_\theta$ $+\left[\nabla^2 A_\phi-\frac{A_\phi}{r^2\sin^2\theta}+\frac{2}{r^2\sin\theta}\frac{\partial A_r}{\partial \phi}+\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\theta}{\partial \phi}\right]\vec{i}_\phi$
$\vec{C}\cdot\nabla\vec{A}$	$\left(C_x\frac{\partial A_x}{\partial x}+C_y\frac{\partial A_x}{\partial y}+C_z\frac{\partial A_x}{\partial z}\right)\vec{i}_x$ $+\left(C_x\frac{\partial A_y}{\partial x}+C_y\frac{\partial A_y}{\partial y}+C_z\frac{\partial A_y}{\partial z}\right)\vec{i}_y$ $+\left(C_x\frac{\partial A_z}{\partial x}+C_y\frac{\partial A_z}{\partial y}+C_z\frac{\partial A_z}{\partial z}\right)\vec{i}_z$	$\left(C_r\frac{\partial A_r}{\partial r}+\frac{C_\theta}{r}\frac{\partial A_r}{\partial \theta}+C_z\frac{\partial A_r}{\partial z}-\frac{C_\theta A_\theta}{r}\right)\vec{i}_r$ $+\left(C_r\frac{\partial A_\theta}{\partial r}+\frac{C_\theta}{r}\frac{\partial A_\theta}{\partial \theta}+C_z\frac{\partial A_\theta}{\partial z}+\frac{C_\theta A_r}{r}\right)\vec{i}_\theta$ $+\left(C_r\frac{\partial A_z}{\partial r}+\frac{C_\theta}{r}\frac{\partial A_z}{\partial \theta}+C_z\frac{\partial A_z}{\partial z}\right)\vec{i}_z$	$\left(C_r\frac{\partial A_r}{\partial r}+\frac{C_\theta}{r}\frac{\partial A_r}{\partial \theta}+\frac{C_\phi}{r\sin\theta}\frac{\partial A_r}{\partial \phi}-\frac{C_\theta A_\theta}{r}-\frac{C_\phi A_\phi}{r}\right)\vec{i}_r$ $+\left(C_r\frac{\partial A_\theta}{\partial r}+\frac{C_\theta}{r}\frac{\partial A_\theta}{\partial \theta}+\frac{C_\phi}{r\sin\theta}\frac{\partial A_\theta}{\partial \phi}+\frac{C_\theta A_r}{r}-\frac{C_\phi A_\phi\cot\theta}{r}\right)\vec{i}_\theta$ $+\left(C_r\frac{\partial A_\phi}{\partial r}+\frac{C_\theta}{r}\frac{\partial A_\phi}{\partial \theta}+\frac{C_\phi}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}+\frac{C_\phi A_r}{r}+\frac{C_\phi A_\theta\cot\theta}{r}\right)\vec{i}_\theta$
$(\vec{\vec{T}}:\nabla\vec{A})$	$T_{xx}\left(\frac{\partial A_x}{\partial x}\right)+T_{yy}\left(\frac{\partial A_y}{\partial y}\right)+T_{zz}\left(\frac{\partial A_z}{\partial z}\right)$ $+T_{xy}\left(\frac{\partial A_x}{\partial y}+\frac{\partial A_y}{\partial x}\right)$ $+T_{yz}\left(\frac{\partial A_y}{\partial z}+\frac{\partial A_z}{\partial y}\right)+T_{zx}\left(\frac{\partial A_z}{\partial x}+\frac{\partial A_x}{\partial z}\right)$	$T_{rr}\left(\frac{\partial A_r}{\partial r}\right)+T_{\theta\theta}\left(\frac{1}{r}\frac{\partial A_\theta}{\partial \theta}+\frac{A_r}{r}\right)+T_{zz}\left(\frac{\partial A_z}{\partial z}\right)$ $+T_{r\theta}\left(r\frac{\partial}{\partial r}\left(\frac{A_\theta}{r}\right)+\frac{1}{r}\frac{\partial A_r}{\partial \theta}\right)+T_{\theta z}\left(\frac{1}{r}\frac{\partial A_z}{\partial \theta}+\frac{\partial A_\theta}{\partial z}\right)$ $+T_{rz}\left(\frac{\partial A_z}{\partial r}+\frac{\partial A_r}{\partial z}\right)$	$T_{rr}\left(\frac{\partial A_r}{\partial r}\right)+T_{\theta\theta}\left(\frac{1}{r}\frac{\partial A_\theta}{\partial \theta}+\frac{A_r}{r}\right)+T_{\phi\phi}\left(\frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}+\frac{A_r}{r}+\frac{A_\theta\cot\theta}{r}\right)$ $+T_{r\theta}\left(\frac{\partial A_\theta}{\partial r}+\frac{1}{r}\frac{\partial A_r}{\partial \theta}-\frac{A_\theta}{r}\right)+T_{r\phi}\left(\frac{\partial A_\phi}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial \phi}-\frac{A_\phi}{r}\right)$ $+T_{\theta\phi}\left(\frac{1}{r}\frac{\partial A_\phi}{\partial \theta}+\frac{1}{r\sin\theta}\frac{\partial A_\theta}{\partial \phi}-\frac{\cot\theta}{r}A_\phi\right)$
$\nabla\cdot\vec{\vec{T}}$	$\left(\frac{\partial T_{xx}}{\partial x}+\frac{\partial T_{xy}}{\partial y}+\frac{\partial T_{xz}}{\partial z}\right)\vec{i}_x$ $+\left(\frac{\partial T_{yx}}{\partial x}+\frac{\partial T_{yy}}{\partial y}+\frac{\partial T_{yz}}{\partial z}\right)\vec{i}_y$ $+\left(\frac{\partial T_{zx}}{\partial x}+\frac{\partial T_{zy}}{\partial y}+\frac{\partial T_{zz}}{\partial z}\right)\vec{i}_z$	$\left(\frac{1}{r}\frac{\partial}{\partial r}(rT_{rr})+\frac{1}{r}\frac{\partial}{\partial \theta}T_{r\theta}-\frac{1}{r}T_{\theta\theta}+\frac{\partial T_{rz}}{\partial z}\right)\vec{i}_r$ $+\left(\frac{1}{r}\frac{\partial T_{\theta\theta}}{\partial \theta}+\frac{\partial T_{r\theta}}{\partial r}+\frac{2}{r}T_{r\theta}+\frac{\partial T_{\theta z}}{\partial z}\right)\vec{i}_\theta$ $+\left(\frac{1}{r}\frac{\partial}{\partial r}(rT_{zr})+\frac{1}{r}\frac{\partial T_{z\theta}}{\partial \theta}+\frac{\partial T_{zz}}{\partial z}\right)\vec{i}_z$	$\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2T_{rr})+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(T_{r\theta}\sin\theta)+\frac{1}{r\sin\theta}\frac{\partial T_{r\phi}}{\partial \phi}-\frac{T_{\theta\theta}+T_{\phi\phi}}{r}\right)\vec{i}_r$ $+\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2T_{r\theta})+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(T_{\theta\theta}\sin\theta)+\frac{1}{r\sin\theta}\frac{\partial T_{\theta\phi}}{\partial \phi}+\frac{T_{r\theta}}{r}-\frac{\cot\theta}{r}T_{\phi\phi}\right)\vec{i}_\theta$ $+\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2T_{r\phi})+\frac{1}{r}\frac{\partial T_{\theta\phi}}{\partial \theta}+\frac{1}{r\sin\theta}\frac{\partial T_{\phi\phi}}{\partial \phi}+\frac{T_{r\phi}}{r}+\frac{2\cot\theta}{r}T_{\theta\phi}\right)\vec{i}_\phi$