MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.632 Electromagnetic Wave Theory
Quiz No. 1

Problem 1(40%)

Consider a plane wave incident on a planar boundary at $x = 0$ from a dielectric medium with $\epsilon = 9\epsilon_o$ as shown in the Fig. 1. The right-hand circularly polarized incident electric field is

$$\overline{E}_i = E_0 \left[2\hat{y} \sin(k_x x + k_z z - \omega t) + (\sqrt{3}\hat{z} - \hat{x}) \cos(k_x x + k_z z - \omega t) \right]$$

where E_0 is a real constant. The reflected field is

$$\overline{E}_r = E_0 [R^{TE} 2\hat{y} \sin(-k_x x + k_z z - \omega t) + R^{TM} (-\sqrt{3}\hat{z} - \hat{x}) \cos(-k_x x + k_z z - \omega t)]$$

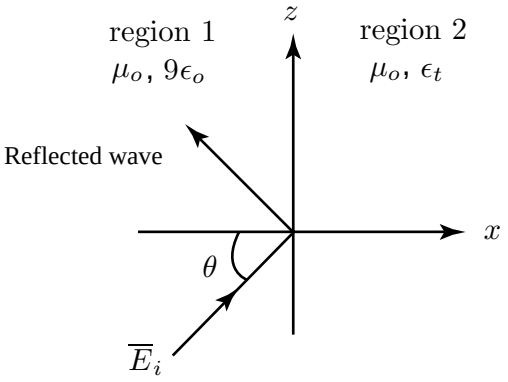

Fig. 1

(a) Show that the incident angle is $30°$.
(b) For $k_z = 1$ m^{-1}, find the frequency (in Hz) and the wavelength (in m) in region 1.
(c) Find the value of ϵ_t $(0 < \epsilon_t/\epsilon_o < \infty)$ for which the incident angle is equal to the critical angle.
(d) In the case of $\epsilon_t = 3\epsilon_o$, find the polarization of the reflected field.

Problem 2(40%)

Consider a circularly polarized electromagnetic wave normally incident upon a slab as shown in Fig. 2. The incident electric field is expressed by

$$\overline{E}_i = E_o \hat{x} \cos(k_y y - \omega t) + \alpha E_o \hat{z} \cos(k_y y - \omega t + \beta).$$

In this problem, neglect the reflection of the slab.

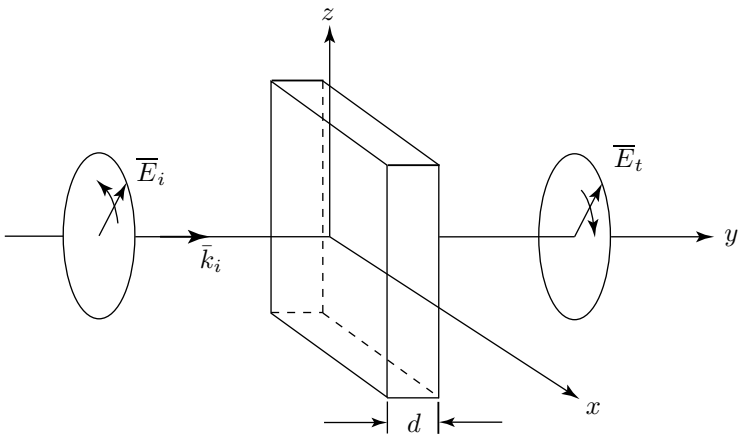

Fig. 2

(a) Let the incident wave be left-hand circularly polarized and assume that both α and β are positive, what is α and what is β?

(b) Let the slab be a uniaxial medium with the permittivity tensor

$$\overline{\overline{\epsilon}} = \begin{bmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}$$

where $\epsilon_x = \epsilon_y = 4\epsilon_o$, $\epsilon_z = 9\epsilon_o$ and the permeability $\mu = \mu_o$. Inside the uniaxial slab, what is the wave number k_y for the $\hat{x}$-polarized electric wave in terms of the wave number in free space k_o, where $k_o = \omega\sqrt{\mu_o\epsilon_o}$?

(c) For the uniaxial slab as in Part (b), let the incident wave be left-hand circularly polarized, what is the minimum thickness d in terms of the wavelength in free space λ_o, where $\lambda_o = 2\pi/k_o = 2\pi/\omega\sqrt{\mu_o\epsilon_o}$, such that the output electric field is right-hand circularly polarized?

Problem 3(20%)

Consider a rectangular waveguide with dimensions $1\,\text{cm} \times 0.5\,\text{cm}$.

(a) What are the cutoff frequencies for the first five modes?

(b) If the waveguide is excited at 20 GHz, what are the propagation modes? What are the corresponding propagation constants k_z?