# 6.441 Transmission of Information
# Problem Set 6

Spring 2010

Due date: April 6

**Problem 1** The following is an attempt to prove the converse of the channel coding theorem using conditionally typical set. Point out all the mistakes, if any, in the proof.

Suppose $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ are the codewords of a fixed-rate fixed-length $(2^{nR}, n)$ code for two distinct messages $w_1, w_2 \in \mathcal{W}$, respectively. If $A_\epsilon^{(n)}(Y|\boldsymbol{x}_1)$ overlaps with $A_\epsilon^{(n)}(Y|\boldsymbol{x}_2)$, then there exists $\boldsymbol{y}$ jointly typical with both $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$, leading to decoding error. To ensure reliable communication, $A_\epsilon^{(n)}(Y|\boldsymbol{x}(w))$ must be disjoint for different $w$, i.e.,

$$\lim_{n\to\infty} \left( \sum_{w \in \mathcal{W}} |A_\epsilon^{(n)}(Y|\boldsymbol{x}(w))| \right)^{\frac{1}{n}} \leq \lim_{n\to\infty} |A_\epsilon^{(n)}(Y)|^{\frac{1}{n}}.$$

Hence,

$$2^{(R+H(Y|X)-2\epsilon)} \leq 2^{H(Y)+\epsilon}.$$

Therefore, $R \leq I(X;Y) + 3\epsilon$. If we let $\epsilon \to 0$, we get $R \leq C$, which proves the converse.

**Problem 2**

Consider a bursty channel which has two states. In state $s_1$, it is a BSC with $\epsilon_1$ crossover probability; in state $s_2$, it is a BSC with $\epsilon_2$ crossover probability. One binary input is sent over the channel in each time period. If the channel is in state $i$ in time period $n$, in time period $n+1$ it remains in state $i$ with probability $q_i$ or switches to the opposite state with probability $1 - q_i$.

a) What is the capacity of this channel without feedback?

b) What effect, if any, would perfect feedback have on the capacity of the channel? Please explain.

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