Lectures on Dynamic Systems and Control

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Chapter 29

Observers, Model-based Controllers

29.1 Introduction

In here we deal with the general case where only a subset of the states, or linear combinations of them, are obtained from measurements and are available to our controller. Such a situation is referred to as the output feedback problem. The output is of the form

$$y = Cx + Du \ . \tag{29.1}$$

We shall examine a class of output feedback controllers constructed in two stages:

1. building an *observer* — a dynamic system that is driven by the inputs and the outputs of the plant, and produces an estimate of its state variables;
2. using the estimated state instead of the actual state in a state feedback scheme.

The resulting controller is termed an observer-based controller or (for reasons that will become clear) a *model-based controller*. A diagram of the structure of such a controller is given in Figure 29.1.

29.2 Observers

An observer comprises a *real-time simulation* of the system or plant, driven by the same input as the plant, and by a *correction term* derived from the difference between the actual output of the plant and the predicted output derived from the observer. Denoting the state vector of the observer by $\hat{x}$, we have the following state-space description of the observer:

$$\delta\hat{x} = A\hat{x} + Bu - L(y - \hat{y}) \ , \tag{29.2}$$

where L, the *observer gain*, is some matrix that will be specified later, and $\hat{y} = C\hat{x} + Du$ is an estimate of the plant output. The term "model-based" for controllers based on an observer refers to the fact that the observer uses a model of the plant as its core.

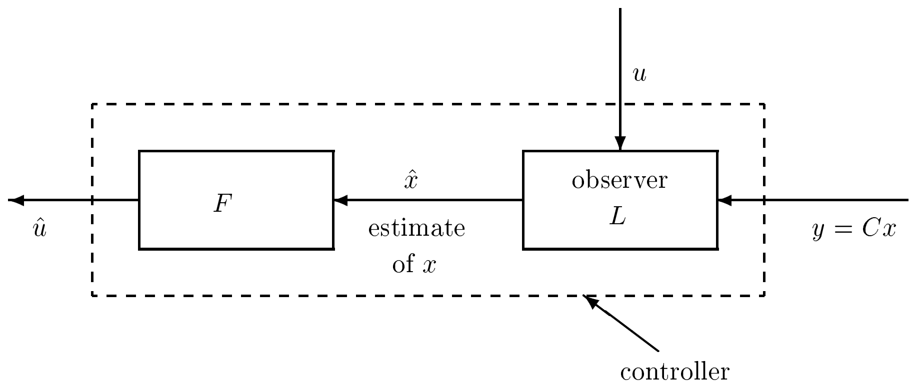


Figure 29.1: Structure of an observer-based, or model-based controller, where L denotes the observer gain and F the state feedback gain.

Define the error vector as $\tilde{x} = x - \hat{x}$. Given this definition, the dynamics of the error are determined by the following error model:

$$\begin{aligned}
\delta\tilde{x} &= \delta x - \delta\hat{x} \\
&= Ax + Bu - A\hat{x} - Bu + L(y - \hat{y}) \\
&= A(x - \hat{x}) + L(Cx - C\hat{x}) \\
&= (A + LC)\tilde{x}\ . \qquad (29.3)
\end{aligned}$$

In general, $\tilde{x}(0) \neq 0$, so we select an L which makes $\tilde{x}(t)$, the solution to (29.3), approach zero for large t. As we can see, $\tilde{x}(t) \to 0$ as $t \to \infty$ for any $\tilde{x}(0)$ if and only if $(A + LC)$ is stable. Note that if $\tilde{x}(t) \to 0$ as $t \to \infty$ then $\hat{x}(t) \to x(t)$ as $t \to \infty$. That is, the state estimates eventually converge to their actual values. A key point is that the estimation error does not depend on what the control inputs are.

It should be clear that results on the stability of $(A + LC)$ can be obtained by taking the duals of the results on eigenvalue placement for $(A + BF)$. What we are exploiting here is the fact that the eigenvalues of $(A + LC)$ are the same as those of $(A' + C'L')$. Specifically we have the following result:

Theorem 29.1 *There exists a matrix L such that*

$$\det(\lambda I - [A + LC]) = \prod_{i=1}^{n} (\lambda - \mu_i) \qquad (29.4)$$

for any arbitrary self-conjugate set of complex numbers $\mu_1, \ldots, \mu_n \in \mathbb{C}$ *if and only if* (C, A) *is observable.*

In the case of a single-output system; i.e c is a row vector, one can obtain a formula that is dual to the feedback matrix formula for pole-assignment. Suppose we want to find the matrix L such that $A + Lc$ has the characteristic polynomial $\alpha^d(\lambda)$ then the following formula will give the desired result

$$L = -\alpha^d(A)\mathcal{O}_n^{-1} e_n$$

where $\mathcal{O}_n$ is the observability matrix defined as

$$\mathcal{O}_n = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}.$$

The above formula is the dual of Ackermann's formula which was obtained earlier.

Some remarks are in order:

1. If (C, A) is not observable, then the unobservable modes, and only these, are forced to remain as modes of the error model, no matter how L is chosen.

2. The pair (C, A) is said to be *detectable* if its unobservable modes are all stable, because in this case, and only in this case, L can be selected to change the location of all unstable modes of the error model to stable locations.

3. Despite what the theorem says we can do, there are good practical reasons for being cautious in applying the theorem. Trying to make the error dynamics very fast generally requires large L, but this can accentuate the effects of any noise in the measurement of y. If $y = Cx + \eta$, where η is a noise signal, then the error dynamics will be driven by a term $L\eta$, as you can easily verify. Furthermore, unmodeled dynamics are more likely to cause problems if we use excessively large gains.

   The *Kalman filter*, in the special form that applies to the problem we are considering here, is simply an optimal observer. The Kalman filter formulation models the *measurement noise* η as a white Gaussian process, and includes a white Gaussian *plant noise* term that drives the state equation of the plant. It then asks for the minimum error variance estimate of the state vector of the plant. The optimal solution is precisely an observer, with the gain L^* chosen in a particular way (usually through the solution of an algebraic Riccati equation). The measurement noise causes us to not try for very fast error dynamics, while the plant noise acts as our incentive for maintaining a good estimate (because the plant noise continually drives the state away from where we want it to be).

4. Since we are directly observing p linear combinations of the state vector via $y = Cx$, it might seem that we could attempt to estimate just $n - p$ other (independent) linear combinations of the state vector, in order to reconstruct the full state. One might think that this could be done with an observer of order $n - p$ rather than the n that our full-order observer takes. These expectations are indeed fulfilled in what is known as the *Luenberger/Gopinath reduced-order observer*. We leave exploration of associated details to some of the homework problems. With noisy measurements, the full-order observer (or Kalman filter) is to be preferred, as it provides some filtering action, whereas the reduced-order observer directly presents the unfiltered noise in certain directions of the $\hat{x}$ space.

29.3 Model-Based Controllers

Figure 29.2 shows the model-based controller in action, with the observer's state estimate being fed back through the (previously chosen) state feedback gain F.

Note that, for this model-based controller, the order of the plant and controller are the same. The number of state variables for the closed-loop system is thus double that of the open-loop plant,

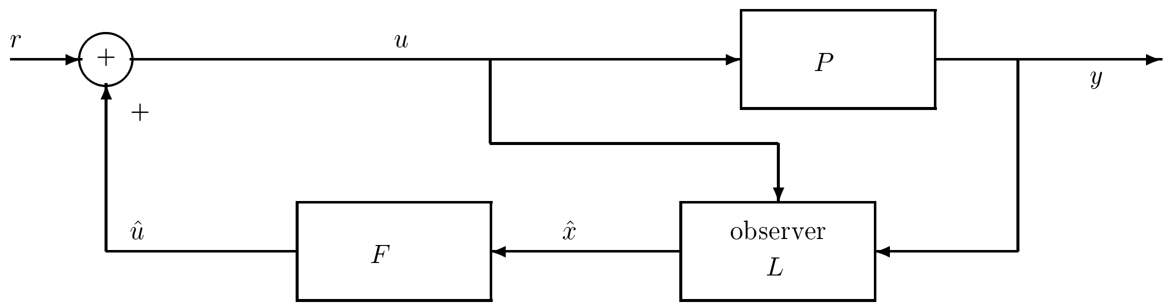


Figure 29.2: Closed-loop system using the model-based output feedback controller.

since the state variables of both the plant, x, and of the estimator, $\hat{x}$ — or some equivalent set of variables — are required to describe the dynamics of the system. The state equation for the plant is

$$\delta x = Ax + Bu \ ,$$

which becomes

$$\begin{aligned} \delta x &= Ax + B\left(r + F\hat{x}\right) \\ &= Ax + BF\hat{x} + Br \end{aligned}$$

by substituting $F\hat{x}+r$ for the control u and expanding. To eliminate $\hat{x}$ so that this equation is solely in terms of the state variables x and $\tilde{x}$, we make the substitution $\hat{x} = x - \tilde{x}$ (since $\tilde{x} = x - \hat{x}$), producing the result

$$\begin{aligned} \delta x &= Ax + BF(x - \tilde{x}) + Br \\ &= (A + BF)x - BF\tilde{x} + Br \ . \end{aligned}$$

Coupling this with $\delta\tilde{x} = (A + LC)\tilde{x}$, which is the state equation for the estimator (derived in 29.3), we get the composite system's state description:

$$\begin{bmatrix} \delta x \\ \delta\tilde{x} \end{bmatrix} = \begin{bmatrix} A + BF & -BF \\ 0 & A + LC \end{bmatrix} \begin{bmatrix} x \\ \tilde{x} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} r \ . \qquad (29.5)$$

Since the composite system matrix is block upper triangular, the closed-loop eigenvalues are given by $\sigma(A+BF) \cup \sigma(A+LC)$, where, as indicated earlier, the notation $\sigma(A)$ represents the set of eigenvalues of A. This fact is referred to as the *separation theorem*, and indicates that the plant stabilization and estimator design can be tackled separately.

In the stochastic setting, with both plant noise and measurement noise, one can pose the so-called LQG problem (where the initials stand for *linear* system, *quadratic* criteria, *Gaussian* noise). The solution turns out to again be a model-based compensator, with a closed-loop system that is again governed by a separation result: the optimal F^* can be chosen according to an LQR formulation, ignoring noise, and the optimal L^* can be determined as a Kalman filter gain, ignoring the specifics of the control that will be applied. For a summary of the equations that govern a model-based compensator designed this way, see the article on "$\mathcal{H}_2$ (LQG) and $\mathcal{H}_\infty$ Control" by Lublin, Grocott and Athans in *The Control Handbook* referred to earlier (specifically look at Theorem 1 there).

A comment about the effect of modeling errors: If there are differences between the parameter matrices A, B, C of the plant and those assumed in the observer, these will cause the entries in the $2n \times 2n$ matrix above to deviate from the values shown there. However, for small enough deviations, the stability of the closed-loop system will not be destroyed, because eigenvalues are continuous functions of the entries of a matrix. The situation can be much worse, however, if (as is invariably the case) there are uncertainities in the *order* of the model. The field of *robust control* is driven by this issue, and we shall discuss it more later.

Example 29.1 Inverted Pendulum with Output Feedback

In the previous section we discussed the inverted pendulum problem. In that example a state feedback controller was given that stabilizes the pendulum around the equilibrium point of the vertical position. We will continue with this example by designing an observer-based stabilizing controller. Recall that the nonlinear system's equationsa are given by

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} &= \begin{bmatrix} x_2 \\ -\frac{mlg}{M_tL}\frac{1}{\alpha(x_3)}\sin(x_3)\cos(x_3) + \frac{ml}{M_t}\frac{1}{\alpha(x_3)}\sin(x_3)(x_4)^2 \\ x_4 \\ \frac{g}{L}\frac{1}{\alpha(x_3)}\sin(x_3) - \frac{ml}{M_tL}\frac{1}{\alpha(x_3)}\sin(x_3)\cos(x_3)(x_4)^2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M_t}\frac{1}{\alpha(x_3)} \\ 0 \\ -\frac{1}{M_tL}\frac{\cos(x_3)}{\alpha(x_3)} \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \end{aligned}$$

where $x_1 = s$, $x_2 = \dot{s}$, $x_3 = \theta$, and $x_4 = \dot{\theta}$. The function $\alpha(x_3)$ is given by

$$\alpha(x_3) = \left(1 - \frac{ml}{M_tL}\cos(x_3)^2\right).$$

The linearized system was also obtained in the previous example and was shown to have the following description

$$\begin{aligned} \frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} &= \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\alpha\frac{ml}{M_tL}g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \alpha\frac{g}{L} & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{\alpha}{M_t} \\ 0 \\ -\frac{\alpha}{LM_t} \end{pmatrix} u \\ y &= \begin{bmatrix} 1 & 0 & & 0 \end{bmatrix} x. \end{aligned}$$

In order to design an observer-based controller we need to compute the observer gain L to place the eignevalues of $A + LC$ at stable locations. Suppose we choose to place the eigenvalues at $\{-4, -4, -4, -4\}$ then by Ackermann's formula the observer gain will be given by

$$\begin{aligned} L &= -\alpha^d(A)\mathcal{O}_4^{-1}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \\ &= \begin{bmatrix} -16.0 \\ -106.0 \\ 1743.0 \\ 5524.7 \end{bmatrix} \end{aligned}$$

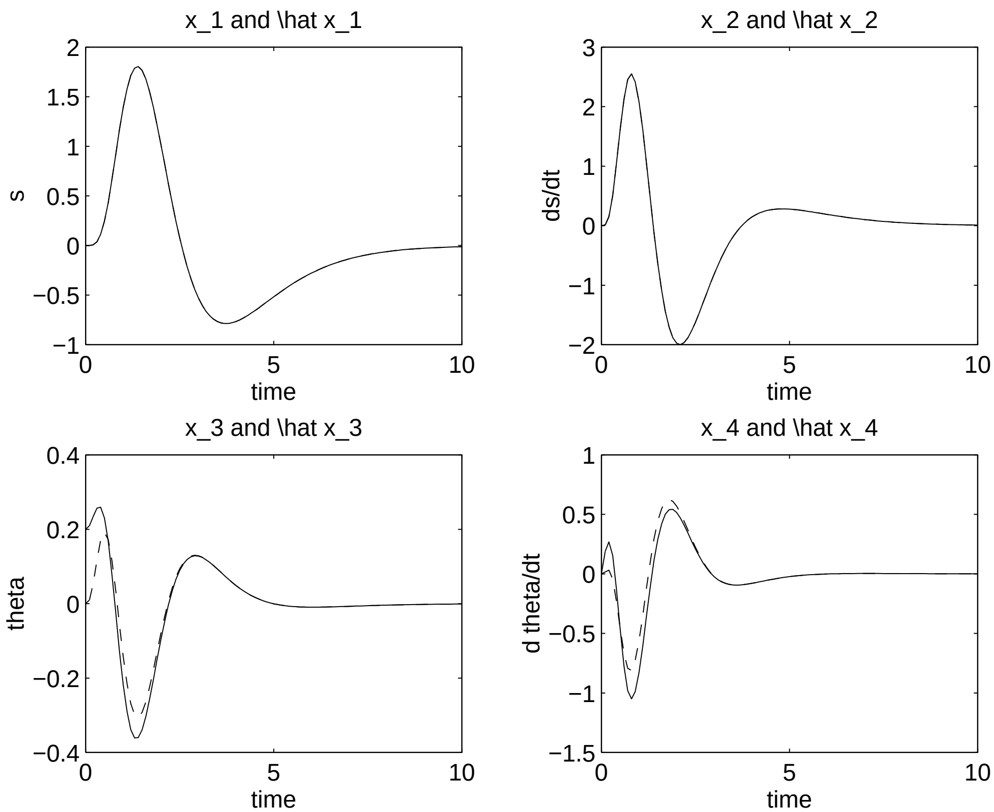


Figure 29.3: Plot of the state variables and the observer variables of the closed-Loop linearized system with $r = 0$ and the initial condition $s = 0$, $\dot{s} = 0$, $\theta = .2$, $\dot{\theta} = 0$, $\hat{x}_1 = 0$, $\hat{x}_2 = 0$, and $\hat{x}_3 = 0$, $\hat{x}_4 = 0$. The solid lines represent the state variables and the dashed lines represent the observer variables

where in the above expression we have $\alpha^d(\lambda) = (\lambda + 4)^4$. The closed loop system is simulated as shown in Figure 29.3. Note that the feedback matrix F is the same as was obtained in the first example in this chapter. It is clear that the estimates $\hat{x}_1$, $\hat{x}_2$, $\hat{x}_3$ and $\hat{x}_4$ converge to the state variables x_1, x_2, x_3 and x_4. The initial angle of the pendulum is chosen to be .2 radians and the initial condition for the observer variables as well as the other state variables are chosen to be zero, and .

The observer-based controller is applied to the nonlinear model and the simulation is given in Figure 29.4.

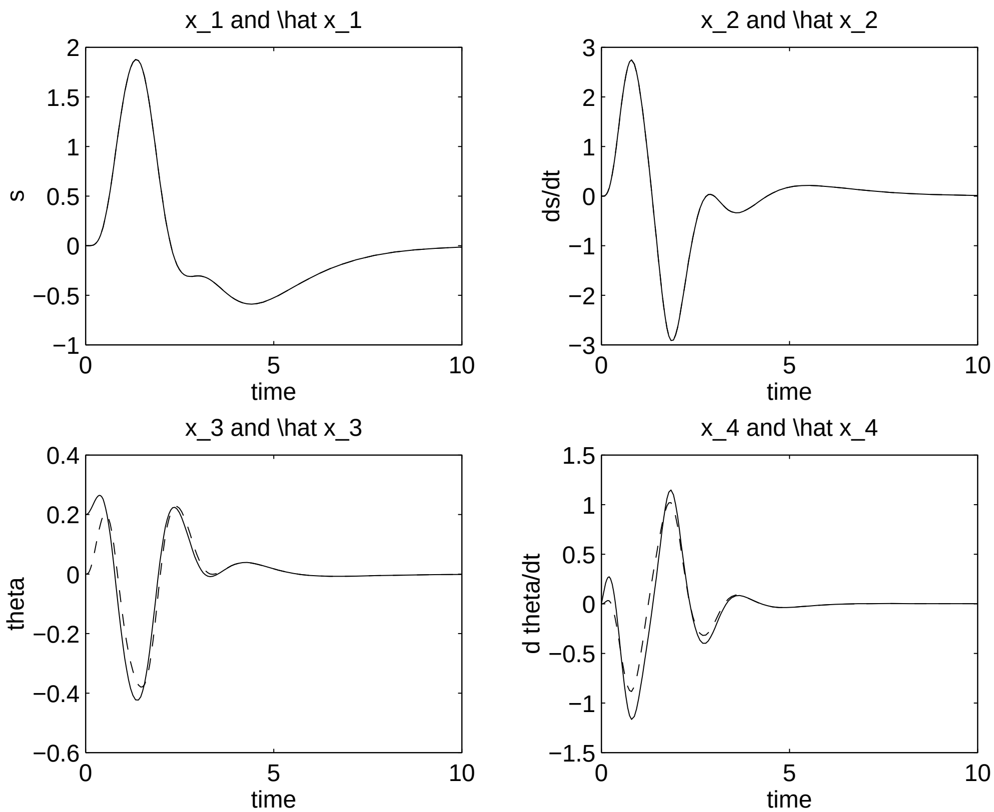
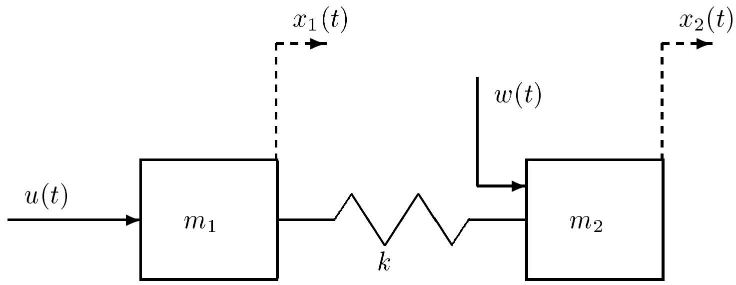


Figure 29.4: Plot of the state variables and the observer variables of the closed-Loop nonlinear system with $r = 0$ and the initial condition $s = 0$, $\dot{s} = 0$, $\theta = .2$, $\dot{\theta} = 0$, $\hat{x}_1 = 0$, $\hat{x}_2 = 0$, and $\hat{x}_3 = 0$, $\hat{x}_4 = 0$. The solid lines represent the state variables and the dashed lines represent the observer variables

Exercises

Exercise 29.1 Consider the mass-spring system shown in the figure below.



Let $x_1(t)$ denote the position of mass m_1, $x_2(t)$ the position of mass m_2, $x_3(t)$ the velocity of mass m_1, $x_4(t)$ the velocity of mass m_2, $u(t)$ the applied force acting on mass m_1, and $w(t)$ a disturbance force acting on mass m_2, k is the spring constant. There is no damping in the system.

The equations of motion are as follows:

$$
\begin{aligned}
\dot{x}_1(t) &= x_3(t) \\
\dot{x}_2(t) &= x_4(t) \\
\dot{x}_3(t) &= -(k/m_1)x_1(t) + (k/m_1)x_2(t) + (1/m_1)u(t) \\
\dot{x}_4(t) &= (k/m_2)x_1(t) - (k/m_2)x_2(t) + (1/m_2)w(t)
\end{aligned}
$$

The output is simply the position of mass m_2, so

$$y(t) = x_2(t)$$

Assume the following values for the parameters:

$$m_1 = m_2 = 1; \quad k = 1$$

(a) Determine the natural frequencies of the system, the zeros of the transfer function from u to y, and the zeros of the transfer function from w to y.

(b) Design an observed-based compensator that uses a feedback control of the form $u(t) = F\hat{x}(t)+r(t)$, where $\hat{x}(t)$ is the state-estimate provided by an observer. Choose F such that the poles of the transfer function from r to y are all at -1. Design your observer such that the natural frequencies governing observer error decay are all at -5.

(c) Determine the closed-loop transfer function from the disturbance w to the output y and obtain its Bode magnitude plot. Comment on the disturbance rejection properties of your design.

(d) Plot the transient response of the two position variables and of the control when $x_2(0) = 1$ and all the other state variables, including the compensator state variables, are initially zero.

(e) Plot the transient response of the two position variables and of the control when the system is initially at rest and the disturbance $w(t)$ is a unit step at time $t = 0$.

Exercise 29.2 Reduced Order Observer

The model-based observer that we discussed in class always has dimension equal to the dimension of the plant. Since the output measures part of the states (or linear combinations), it seems natural that only a subset of the states need to be estimated through the observer. This problem shows how one can derive a reduced order observer.

Consider the following dynamic system with states $x_1 \in \mathbb{R}^r$, $x_2 \in \mathbb{R}^p$:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} u,$$

and

$$y = x_2.$$

Since x_2 is completely available, the reduced order observer should provide estimates only for x_1, and its dimension is equal to r, the dimension of x_1. Thus

$$\hat{x} = \begin{pmatrix} \hat{x}_1 \\ x_2 \end{pmatrix}.$$

One may start with the following potential observer:

$$\dot{\hat{x}}_1 = A_{11}\hat{x}_1 + A_{12}y + B_1 u + L(y - \hat{y})$$

Since $\hat{y} = C\hat{x} = x_2$ (since x_2 is known exactly), the correction term in the above equation is equal to zero ($L(y - \hat{y}) = 0$). This indicates that this proceedure may not work.

Suppose instead, that we define a new variable $z = x_1 - Lx_2$, where L is an $r \times p$ matrix that we will choose later. Then if we can derive an estimate for z, denoted by $\hat{z}$, we immediately have an estimate for x_1, namely, $\hat{x}_1 = \hat{z} + Lx_2$.

(a) Express $\dot{z}$ in terms of z, y, and u. Show that the state matrix (matrix multiplying z) is given by $A_{11} - LA_{21}$.

(b) To be able to place the poles of $A_{11} - LA_{21}$ in the left half plane, the pair (A_{11}, A_{21}) should be observable (i.e., a system with dynamic matrix A_{11} and output matrix A_{21} should be observable). Show that this is the case if and only if the original system is observable.

(c) Suggest an observer for z. Verify that your choice is good.

(d) Suppose a constant state feedback matrix F has been found such that $A + BF$ is stable. Since not all the states are availabe, the control law can be implemented as:

$$u = F\hat{x} = F_1\hat{x}_1 + F_2 x_2$$

where $F = \begin{pmatrix} F_1 & F_2 \end{pmatrix}$ is decomposed conformally with x_1 and x_2. Where do the closed loop poles lie? Justify your answer.

Exercise 29.3 (Observers and Observer-Based Compensators) The optimal control in Problem 28.3 cannot be implemented when x is not available to us. We now examine, in the context of the (magnetic suspension) example in that problem, the design of an observer to produce an estimate $\hat{x}$, and the design of an observer-based compensator that uses this estimate instead of x. Assume for this problem that the output measurement available to the observer is the same variable y that is penalized in the quadratic criterion. [In general, the penalized "output" in the quadratic criterion *need not be the same* as the measured output used for the observer.]

(a) Design a full-order observer for the original open-loop system, to obtain an estimate $\hat{x}(t)$ of $x(t)$, knowing only u and y. The eigenvalues that govern error decay are both to be placed at -6.

(b) Suppose we now use the control $u(t) = F^*\hat{x}(t) + v(t)$, where F^* is the same as in (d), (e) of Problem 2, and $v(t)$ is some new external control. Show that the transfer matrix of the *compensator*, whose input vector is $(u \quad y)'$ and whose output is the scalar $f = F^*\hat{x}$, is given by

$$-\frac{1}{(s+6)^2}\,[\,6(s+15) \qquad 486(s+3)\,]$$

Also determine the transfer function from v to y.

(c) As an alternative to the compensator based on the full-order observer, design a *reduced-order* observer — see Problem 1(c) — and place the eigenvalue that governs error decay at -6. Show that the transfer matrix in (b) is now replaced by

$$-\frac{1}{(s+6)}\,[6 \qquad 54(s+3)]$$

and determine the transfer function from v to y.

Exercise 29.4 Motivated by what we have done with observer-based compensators designed via state-space methods, we now look for a direct transform-domain approach. Our starting point will be a given open-loop transfer function for the plant, $p(s)/a(s)$, with $a(s)$ being a polynomial of degree n that has no factors in common with $p(s)$. Let us look for a compensator with the structure of the one in Problem 3(b), with input vector $(u \quad y)'$, output f that constitutes the feedback signal, and transfer matrix

$$-\frac{1}{w(s)}\,[\,q(s) \qquad r(s)\,]$$

where $w(s)$ is a monic polynomial (i.e. the coefficient of the highest power of s equals 1) of degree n, while $q(s)$ and $r(s)$ have degrees $n-1$ or less. With this compensator, the input to the plant is given by $u = f + v$, where v is some new external control signal.

(a) Find an n-th order realization of the above compensator. (Hint: Use the familiar SISO observer canonical form, modified for 2 inputs.) (You will *not* need to use this realization for any of the remaining parts of this problem — the intent of this part is just to convince you that an n-th order realization of the compensator exists.)

(b) Show that the transfer function from v to y is

$$g(s) = \frac{p(s)w(s)}{[w(s) + q(s)]a(s) + r(s)p(s)}$$

and argue that the characteristic polynomial of the system must be

$$[w(s) + q(s)]a(s) + r(s)p(s)$$

It turns out that, since $a(s)$ and $p(s)$ are coprime, we can choose $[w(s) + q(s)]$ and $r(s)$ to make the characteristic polynomial equal to *any* monic polynomial of degree $2n$. The following strategy for picking this polynomial mimics what is done in the design of an observer-based compensator using state-space methods: pick $w(s)$ to have roots at desirable locations in the left-half-plane (these will correspond to observer error decay modes); then pick $q(s)$ and $r(s)$ so that the characteristic polynomial above equals $\alpha(s)w(s)$, where $\alpha(s)$ is a polynomial of degree n that also has roots at desirable positions in the left-half- plane. With these choices, we see that

$$g(s) = [p(s)w(s)]/[\alpha(s)w(s)] = p(s)/\alpha(s)$$

This compensator has thus shifted the poles of the closed-loop system from the roots of $a(s)$ to those of $\alpha(s)$, and the roots of $w(s)$ correspond to hidden modes.

(c) Design a compensator via the above route for a plant of transfer function $1/(s^2 - 9)$, to obtain an overall transfer function of $1/(s + 3)^2$, with two hidden modes at -6. Compare with the result in Problem 3(b).

(d) The above development corresponds to designing a compensator based on a full-order observer. A compensator based on a *reduced-order* observer — see Problem 1(c) — is easily obtained as well, by simply making $w(s)$ a monic polynomial of degree $n - 1$ rather than n and making any other changes that follow from this. After noting what the requisite changes would be, design a compensator for a plant of transfer function $1/(s^2 - 9)$, to obtain an overall transfer function of $1/(s + 3)^2$, with *one* hidden mode at -6. Compare with the result in Problem 3(c).

Exercise 29.5 Consider a plant described by the transfer function matrix

$$P(s) = \begin{pmatrix} \frac{1}{s-1} & \frac{1}{s-1} \\ \frac{2s-1}{s(s-1)} & \frac{1}{s-1} \end{pmatrix}$$

(a) Design a model-based (i.e. observer-based) controller such that the closed loop system has all eigenvalues at $s = -1$.

(b) Suppose that $P_{11}(s)$ is perturbed to $\frac{1+\epsilon}{s-1}$. For the controller you designed, give the range of ϵ for which the system remains stable. Discuss your answer.

Exercise 29.6 Assume we are given the controllable and observable system $\dot{x}(t) = Ax(t) + Bu(t)$, $z(t) = Cx(t)$, with transfer matrix $P(s)$. The available measurement is $y(t) = z(t) + d(t)$, where $d(t)$ is a disturbance signal. An observer for the system comprises a duplicate of the plant model, driven by the same input $u(t)$, but also by a correction term $e(t) = y(t) - C\hat{x}(t)$ acting through an observer gain L, which is chosen to obtain stable error dynamics.

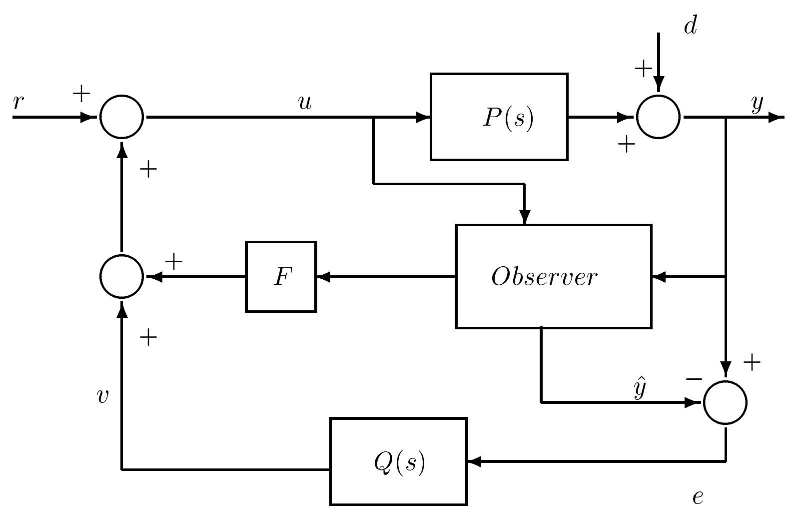
For an observer-based stabilizing compensator, suppose we pick $u(t) = F\hat{x}(t) + r(t) + v(t)$, where $\hat{x}(t)$ is the estimate produced by an observer, F is the gain we would have used to stabilize the system under perfect state feedback, $r(t)$ is some external input, and $v(t)$ is the output of a *stable* finite dimensional LTI system whose input is $e(t)$ and whose (proper, rational) transfer function matrix is $Q(s)$. (The case of $Q(s) = 0$ constitutes the "core" observer-based stabilizing compensator that we have discussed in detail in class.) A block diagram for the resulting system is given below.

(a) Show that this system is stable for any stable finite-dimensional system Q. [Hint: The transfer function from v to e is equal to zero regardless of what r and d are!]

(b) Obtain a state-space description of the overall system, and show that its eigenvalues are the union of the eigenvalues of $A + BF$, the eigenvalues of $A + LC$, and the poles of $Q(s)$.)

(c) What is the transfer function matrix $K(s)$ of the overall feedback compensator connecting y to u? Express it in the form $K(s) = [W(s) - Q(s)M(s)]^{-1}[J(s) - Q(s)N(s)]$, where the matrices W, M, J, N are also *stable*, proper rationals.

It turns out that, as we let $Q(s)$ vary over all proper, stable, rational matrices, the matrix $K(s)$ ranges *precisely* over the set of proper rational transfer matrices of feedback compensators that stabilize the closed-loop system. This is therefore referred to as the "Q-parametrization" of stabilizing feedback compensators.



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