Slide 1

Mathematics for Computer Science
MIT 6.042J/18.062J

Structural Induction

Albert R Meyer, February 29, 2012

lec 5M.1

Slide 2

Structural Induction

To prove P(x) holds for all x in recursively defined set R, prove

- P(b) for each base case $b \in$ R
- P($c(x)$) for each constructor, c, assuming ind. hyp. P(x)

Albert R Meyer, February 29, 2012

7W.2

Slide 3

E ⊆ Even

by structural induction on $x \in$ E with ind. hyp. "x is even"

- 0 is even
- if n is even, then so is $n+2$, $-n$

Albert R Meyer, February 29, 2012

lec 5M.3

Slide 4

Matched Paren Strings M

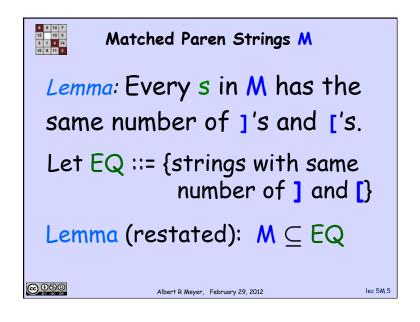*Lemma:* Every s in M has the same number of]'s and ['s.

Proof by structural induction on the definition of M

Albert R Meyer, February 29, 2012

lec 5M.4

Matched Paren Strings M

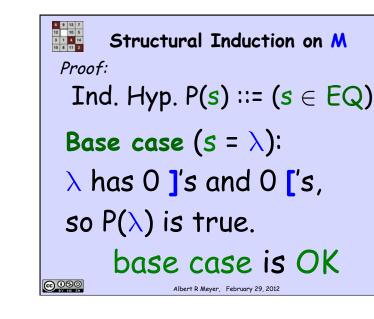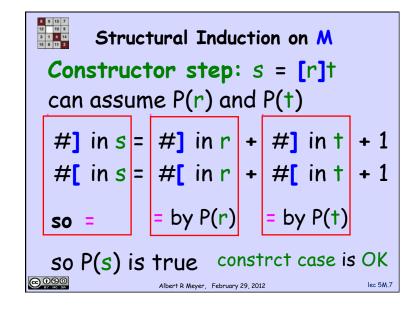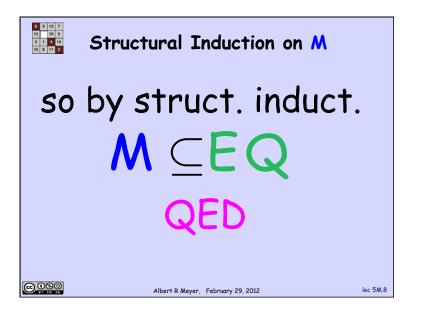*Lemma:* Every s in M has the same number of **]**'s and **[**'s.

Let EQ ::= {strings with same number of **]** and **[**}

Lemma (restated): M $\subseteq$ EQ

Structural Induction on M

Proof:

Ind. Hyp. P(s) ::= ($s \in$ EQ)

Base case ($s = \lambda$):

λ has 0 **]**'s and 0 **[**'s, so P(λ) is true.

base case is OK

Structural Induction on M

Constructor step: s = **[**r**]**t
can assume P(r) and P(t)

#**]** in s =	#**]** in r +	#**]** in t + 1
#**[** in s =	#**[** in r +	#**[** in t + 1
so =	= by P(r)	= by P(t)

so P(s) is true   constrct case is OK

Structural Induction on M

so by struct. induct.

M $\subseteq$ EQ

QED

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Lemma.

 F18 is closed under
taking derivatives:
if f ∈ **F18**, then f´ ∈ **F18**

Class Problem

lec 5M.11

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