

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Binomial Theorem



6	9	13	7
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Polynomials Express Choices & Outcomes

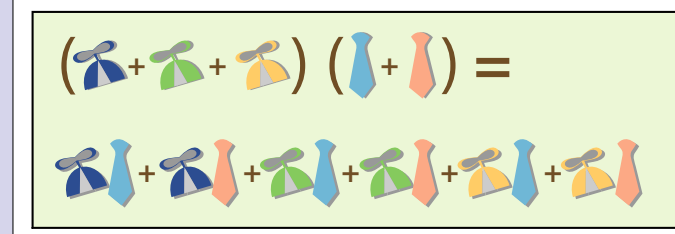


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Products of Sums = Sums of Products



6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

expression for c_k ?

$$(1+X)^n =$$

$$c_0 + c_1X + c_2X^2 + \dots + c_nX^n$$



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binomial expressions

$$(1+X)^0 = 1$$


$$(1+X)^1 = 1 + 1X$$

$$(1+X)^2 = 1 + 2X + 1X^2$$

$$(1+X)^3 = 1 + 3X + 3X^2 + 1X^3$$

$$(1+X)^4 = 1 + 4X + 6X^2 + 4X^3 + 1X^4$$





 expression for c_k ?

$$(1 + X)^n \quad n \text{ times}$$

$$= \overbrace{(1 + X)(1 + X)(1 + X)(1 + X) \dots (1 + X)}^{n \text{ times}}$$

multiplying gives 2^n product terms:
 $11 \dots 1 + X1X \dots X1 + 1XX \dots 1X1 + \dots + XX \dots X$
 a term corresponds to selecting 1 or X from each of the n factors

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 expression for c_k ?


$$(1 + X)^n \quad n \text{ times}$$

$$= \overbrace{(1 + X)(1 + X)(1 + X)(1 + X) \dots (1 + X)}^{n \text{ times}}$$

the X^k coeff, c_k , is # terms with exactly k X's selected

$$c_k = \binom{n}{k}$$


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 The Binomial Formula

$(1 + X)^n =$ binomial expression

$$\binom{n}{0} + \binom{n}{1}X + \binom{n}{2}X^2 + \dots + \binom{n}{k}X^k + \dots + \binom{n}{n}X^n$$

binomial coefficients


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 The Binomial Formula

$$(X + Y)^n =$$

$$\binom{n}{0}Y^n + \binom{n}{1}XY^{n-1} + \binom{n}{2}X^2Y^{n-2} +$$

$$\dots + \binom{n}{k}X^kY^{n-k} + \dots + \binom{n}{n}X^n$$

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6	9	13	7
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The Binomial Formula

$$(X + Y)^n = \sum_{k=0}^n \binom{n}{k} X^k Y^{n-k}$$



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