

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Bipartite Matching

Albert R Meyer. April 3, 2013 bipartite.1

6	9	13	7
12	10	5	
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15	8	11	2

Compatible Boys & Girls

compatible

Albert R Meyer. April 3, 2013 bipartite.2

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Compatible Boys & Girls

match each girl to a unique compatible boy

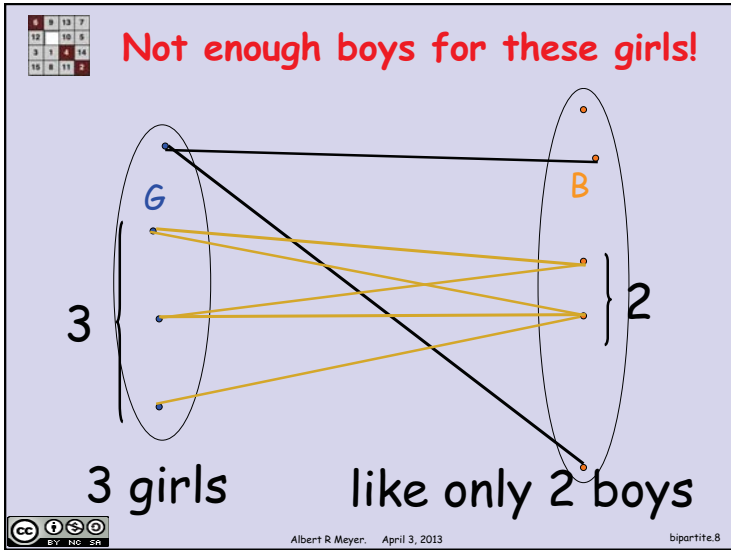
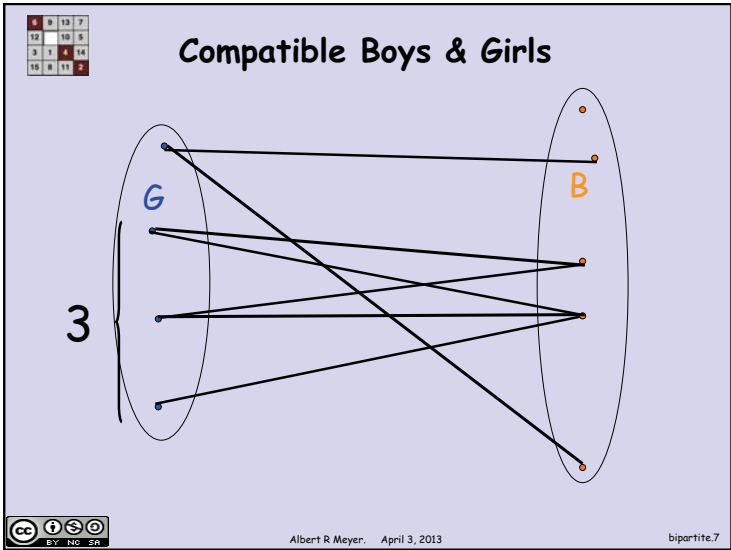
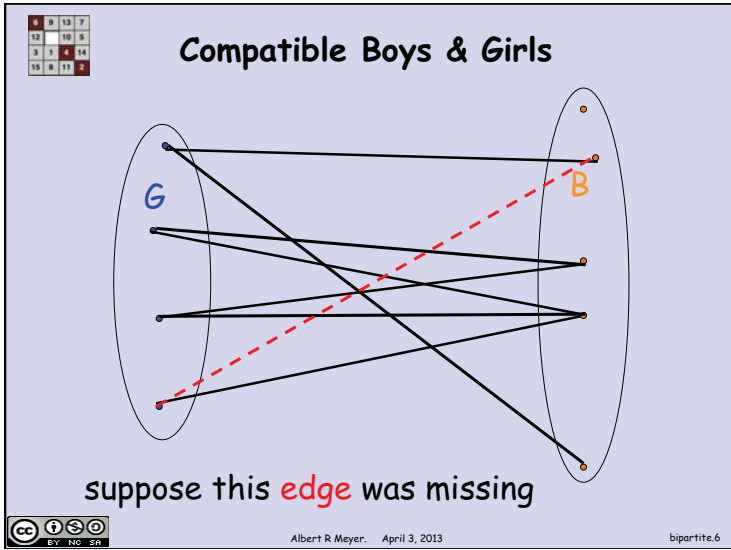
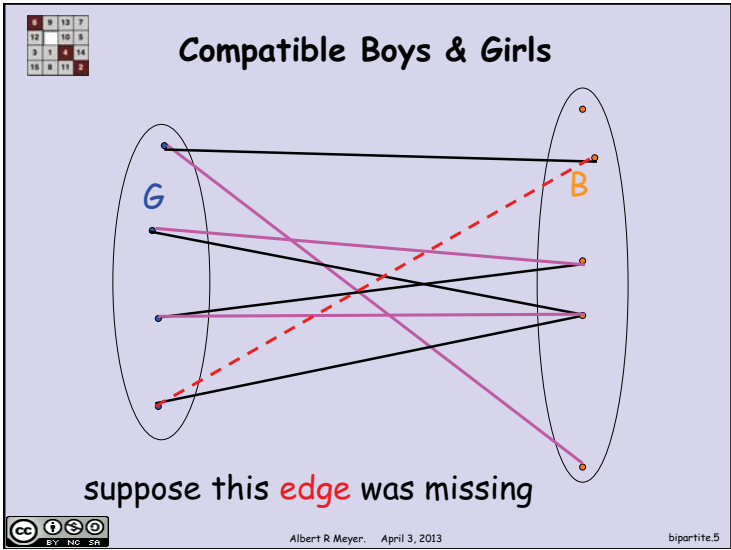
Albert R Meyer. April 3, 2013 bipartite.3

6	9	13	7
12	10	5	
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Compatible Boys & Girls

a matching

Albert R Meyer. April 3, 2013 bipartite.4



6	9	13	7
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15	8	11	2

No match is possible!

$|S| = 3 > 2 = |E(S)|$

Albert R Meyer. April 3, 2013 bipartite.9

6	9	13	7
12	10	5	
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15	8	11	2

a bottleneck

$|S| > |E(S)|$

Albert R Meyer. April 3, 2013 bipartite.10

6	9	13	7
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Bottleneck Lemma

Bottleneck: a set S of girls without enough boys.

$E(S) ::=$ boys adjacent to at least one girl in S .

$|S| > |E(S)|$

Albert R Meyer. April 3, 2013 lec 7M.11

6	9	13	7
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Bottleneck Lemma

If there **is** a **bottleneck**,
then **no match** is possible
obviously

Albert R Meyer. April 3, 2013 bipartite.12

6	9	13	7
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Hall's Theorem

Conversely, if there are
no bottlenecks, then
there is a match.

Not obvious. Discuss next.



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