Mathematics for Computer Science

MIT 6.042J/18.062J

Number Theory: Die Hard

Unique factorization

Generalized Die Hard

Did it with buckets:

3 gal. & 5 gal.

3 gal. & 9 gal.

Now a gal. & b gal.?

Generalized Die Hard

Under Die Hard rules, gal.'s in each bucket are

linear combinations

of a and b

Generalized Die Hard

Under Die Hard rules, gal.'s in each bucket are

multiples of gcd(a,b)

of a and b

Generalized Die Hard

Claim: Can get any multiple of gcd(a, b) into a bucket

(if there's room for it).

Generalized Die Hard

Claim: Can get any linear combination of a, b into a bucket (if there's room for it).

Namely, say $0 \leq sa + tb < b$.

Get $sa + tb$ into the b gal.

bucket as follows:

Generalized Die Hard

assume $s > 0$. do s times:

fill bucket a, pour into b

— if b fills, empty it.

total fills = sa

$0 \leq$ amount left $< b$

b emptyings must be $-t$

Generalized Die Hard

In fact, no need to count:

fill bucket a, pour into b

—if b fills, empty it

— until desired gal.'s in b !

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