



6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Number Theory: Die Hard Unique factorization



Albert R Meyer

March 5, 2012

lec 5M.1



6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Generalized Die Hard

Did it with buckets:

3 gal. & 5 gal.

3 gal. & 9 gal.

Now a gal. & b gal.?



Albert R Meyer

March 5, 2012

lec 5M.2



6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Generalized Die Hard

Under Die Hard rules,
gal.'s in each bucket are

linear combinations
of a and b



Albert R Meyer

March 5, 2012

lec 5M.3



6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Generalized Die Hard

Under Die Hard rules,
gal.'s in each bucket are

multiples of $\gcd(a,b)$
of a and b



Albert R Meyer

March 5, 2012

lec 5M.4

4	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Generalized Die Hard

Claim: Can get **any multiple** of $\gcd(a, b)$ into a bucket
(if there's room for it).



Albert R Meyer

March 5, 2012

lec 5M.5

4	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Generalized Die Hard

Claim: Can get **any linear combination** of a, b into a bucket (if there's room for it).
Namely, say $0 \leq sa + tb < b$.
Get $sa + tb$ into the b gal.
bucket as follows:



Albert R Meyer

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4	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Generalized Die Hard

assume $s > 0$. do s times:
fill bucket a , pour into b
— if b fills, empty it.
total fills = sa
 $0 \leq$ amount left $< b$
b emptyings must be $-t$



Albert R Meyer

March 5, 2012

lec 5M.7

4	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Generalized Die Hard

In fact, no need to count:
fill bucket a , pour into b
— if b fills, empty it
— until **desired gal.'s** in b !



Albert R Meyer

March 5, 2012

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