# In-Class Problems Week 11, Fri.

**Problem 1.**

Solve the following problems using the pigeonhole principle. For each problem, try to identify the *pigeons*, the *pigeonholes*, and a *rule* assigning each pigeon to a pigeonhole.

**(a)** In a certain Institute of Technology, every ID number starts with a 9. Suppose that each of the 75 students in a class sums the nine digits of their ID number. Explain why two people must arrive at the same sum.

**(b)** In every set of 100 integers, there exist two whose difference is a multiple of 37.

**(c)** For any five points inside a unit square (not on the boundary), there are two points at distance *less than* $1/\sqrt{2}$.

**(d)** Show that if $n + 1$ numbers are selected from $\{1, 2, 3, \ldots, 2n\}$, two must be consecutive, that is, equal to $k$ and $k + 1$ for some $k$.

**Problem 2.**

To ensure password security, a company requires their employees to choose a password. A length 10 word containing each of the characters:

a, d, e, f, i, l, o, p, r, s,

is called a *cword*. A password can be a cword which does not contain any of the subwords "fails", "failed", or "drop."

For example, the following two words are passwords: adefiloprs, srpolifeda,
but the following three cwords are not: a**drop**eflis, **failedrop**s, **dropefails**.

**(a)** How many cwords contain the subword "drop"?

**(b)** How many cwords contain both "drop" and "fails"?

**(c)** Use the Inclusion-Exclusion Principle to find a simple arithmetic formula involving factorials for the number of passwords.

**Problem 3.**

How many paths are there from point $(0, 0)$ to $(50, 50)$ if each step along a path increments one coordinate and leaves the other unchanged? How many are there when there are impassable boulders sitting at points $(10, 11)$ and $(21, 20)$? (You do not have to calculate the number explicitly; your answer may be an expression involving binomial coefficients.)

*Hint:* Inclusion-Exclusion.

## Supplemental problems

**Problem 4.** **(a)** Prove that every positive integer divides a number such as 70, 700, 7770, 77000, whose decimal representation consists of one or more 7's followed by one or more 0's.

*Hint:* $7, 77, 777, 7777, \ldots$

**(b)** Conclude that if a positive number is not divisible by 2 or 5, then it divides a number whose decimal representation is all 7's.

**Problem 5.**
Show that for any set of 201 positive integers less than 300, there must be two whose quotient is a power of three (with no remainder).