Recitation 15
October 28, 2010

1. Problem 6.14 (a)-(c),(h)-(j), page 330 in text.

   Beginning at time $t = 0$, we begin using bulbs, one at a time, to illuminate a room. Bulbs are replaced immediately upon failure. Each new bulb is selected independently by an equally likely choice between a type-A bulb and a type-B bulb. The lifetime, X, of any particular bulb of a particular type is a random variable, independent of everything else, with the following PDF:

$$\text{for type-A Bulbs:} \quad f_X(x) \;=\; \begin{cases} e^{-x}, & x \geq 0, \\ 0, & \text{otherwise}; \end{cases}$$

$$\text{for type-B Bulbs:} \quad f_X(x) \;=\; \begin{cases} 3e^{-3x}, & x \geq 0, \\ 0, & \text{otherwise}. \end{cases}$$

   (a) Find the expected time until the first failure.

   (b) Find the probability that there are no bulb failures before time t.

   (c) Given that there are no failures until time t, determine the conditional probability that the first bulb used is a type-A bulb.

   (d) Determine the probability that the total period of illumination provided by the first two type-B bulbs is longer than that provided by the first type-A bulb.

   (e) Suppose the process terminates as soon as a total of exactly 12 bulb failures have occurred. Determine the expected value and variance of the total period of illumination provided by type-B bulbs while the process is in operation.

   (f) Given that there are no failures until time t, find the expected value of the time until the first failure.

2. Problem 6.15 (a)-(c), p. 331 in text.

   A service station handles jobs of two types, A and B. (Multiple jobs can be processed simultaneously.) Arrivals of the two job types are independent Poisson processes with parameters $\lambda_A = 3$ and $\lambda_B = 4$ per minute, respectively. Type A jobs stay in the service station for exactly one minute. Each type B job stays in the service station for a random but integer amount of time which is geometrically distributed, with mean equal to 2, and independent of everything else. The service station started operating at some time in the remote past.

   (a) What is the mean, variance, and PMF of the total number of jobs that arrive within a given three-minute interval?

   (b) We are told that during a 10-minute interval, exactly 10 new jobs arrived. What is the probability that exactly 3 of them are of type A?

   (c) At time 0, no job is present in the service station. What is the PMF of the number of type B jobs that arrive in the future, but before the first type A arrival?

3. Let X, Y, and Z be independent exponential random variables with parameters λ, μ, and ν, respectively. Find $\mathbf{P}(X < Y < Z)$.

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6.041 / 6.431 Probabilistic Systems Analysis and Applied Probability
Fall 2010