Recitation 5
September 23, 2010

1. (a) Derive the expected value rule for functions of random variables $\mathbf{E}[g(X)] = \sum_x g(x)p_X(x)$.
   (b) Derive the property for the mean and variance of a linear function of a random variable $Y = aX + b$.
   $$\mathbf{E}[Y] = a\mathbf{E}[X] + b, \qquad \text{var}(Y) = a^2\text{var}(X).$$
   (c) Derive $\text{var}(X) = \mathbf{E}[X^2] - (\mathbf{E}[X])^2$

2. A marksman takes 10 shots at a target and has probability 0.2 of hitting the target with each shot, independently of all other shots. Let X be the number of hits.
   (a) Calculate and sketch the PMF of X.
   (b) What is the probability of scoring no hits?
   (c) What is the probability of scoring more hits than misses?
   (d) Find the expectation and the variance of X.
   (e) Suppose the marksman has to pay \$3 to enter the shooting range and he gets \$2 dollars for each hit. Let Y be his profit. Find the expectation and the variance of Y.
   (f) Now let's assume that the marksman enters the shooting range for free and gets the number of dollars that is equal to the square of the number of hits. Let Z be his profit. Find the expectation of Z.

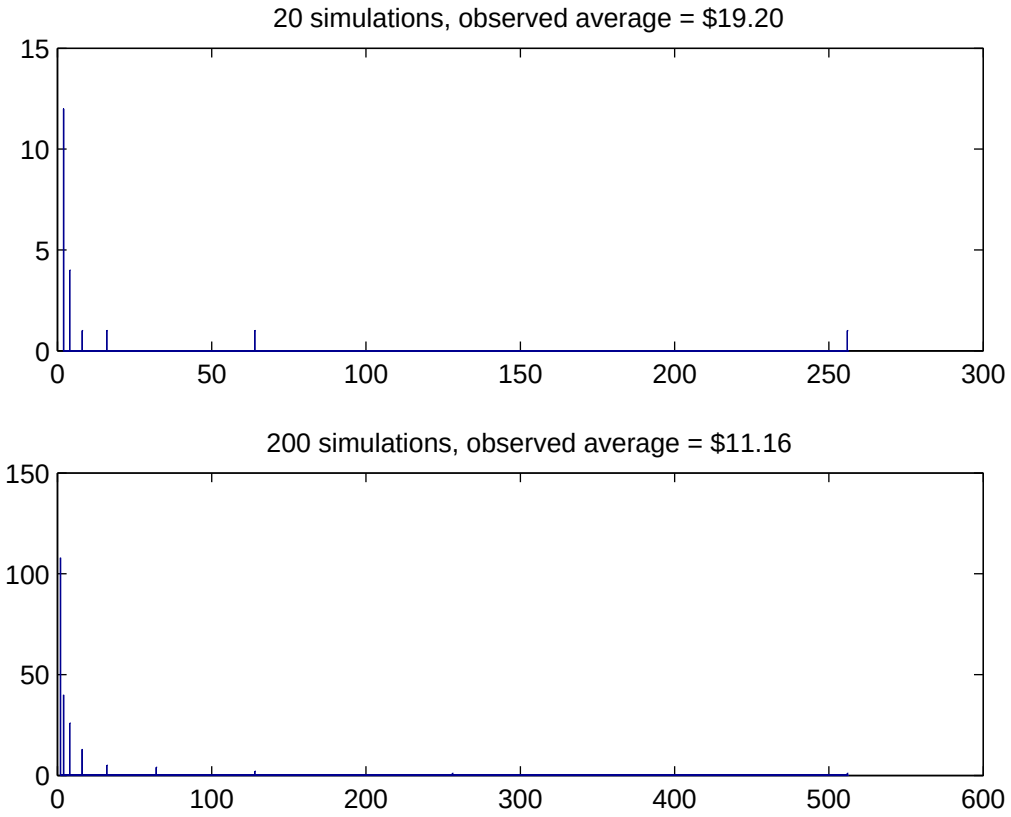
3. 4 buses carrying 148 job-seeking MIT students arrive at a job convention. The buses carry 40, 33, 25, and 50 students, respectively. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying this randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on his bus.
   (a) Which of $E[X]$ or $E[Y]$ do you think is larger? Give your reasoning in words.
   (b) Compute $E[X]$ and $E[Y]$.

4. Problem 2.21, page 123 in the text.

   St. Petersburg paradox. You toss independently a fair coin and you count the number of tosses until the first tail appears. If this number is n, you receive 2^n dollars. What is the expected amount that you will receive? How much would you be willing to pay to play this game?

Recitation 5: Extra Handout
September 23, 2010

1. To show some relavant computations to Problem 4, the results (plotted as histograms) of simulations of this game have been plotted below for various numbers of simulations.



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