Appendix A: Numerical Constants

A.1 Fundamental Constants

c	velocity of light	2.998×10^{8} m/s
ε_o	permittivity of free space	8.854×10^{-12} F/m
μ_o	permeability of free space	$4\pi \times 10^{-7}$ H/m
η_o	characteristic impedance of free space	376.7 Ω
e	charge of an electron, (-e.v./Joule)	-1.6008×10^{-19} C
m	mass of an electron	9.1066×10^{-31} kg
m_p	mass of a proton	1.6725×10^{-27} kg
h	Planck constant	6.624×10^{-34} J·s
k	Boltzmann constant	1.3805×10^{-23} J/K
N_o	Avogadro's constant	6.022×10^{23} molec/mole
R	Universal gas constant	8. 31 J/mole·K

A.2 Electrical Conductivity σ, S/m

Silver	6.14×10^{7}	Monel	0.24×10^{7}
Copper	5.80×10^{7}	Mercury	0.1×10^{7}
Gold	4.10×10^{7}	Sea Water	3 – 5
Aluminum	3.54×10^{7}	Distilled Water	2×10^{-4}
Tungsten	1.81×10^{7}	Bakelite	$10^{-8} - 10^{-10}$
Brass	1.57×10^{7}	Glass	10^{-12}
Nickel	1.28×10^{7}	Mica	$10^{-11} - 10^{-15}$
Iron (pure)	1.0×10^{7}	Petroleum	10^{-14}
Steel	$0.5 - 1.0 \times 10^{7}$	Fused Quartz	$<2 \times 10^{-17}$
Lead	0.48×10^{7}		

A.3 Relative Dielectric Constant $\varepsilon/\varepsilon_o$ at 1 MHz

Vacuum	1.00	Vycor glass	3.8
Styrofoam (25% filler)	1.03	Low-loss glass	4.1
Firwood	1.8 – 2.0	Ice	4.15
Paper	2.0 – 3.0	Pyrex glass	5.1
Petroleum	2.1	Muscovite (mica)	5.4
Paraffin	2.1	Mica	5.6 – 6.0
Teflon	2.1	Magnesium silicate	5.7 – 6.4
Vaseline	2.16	Porcelain	5.7
Rubber	2.3 – 4.0	Aluminum oxide	8.8
Polystyrene	2.55	Diamond	16.5
Sandy soil	2.6	Ethyl alcohol	24.5
Plexiglas	2.6 – 3.5	Distilled water	81.1
Fused quartz	3.78	Titanium dioxide	100

A.4 Relative Permeability μ/μ_o

Vacuum	1
Biological tissue	1
Cold steel	2,000
Iron (99.91%)	5,000
Purified iron (99.95%)	180,000
mu metal (FeNiCrCu)	100,000
Supermalloy (FeNiMoMn)	800,000

Appendix B: Complex Numbers and Sinusoidal Representation

Most linear systems that store energy exhibit frequency dependence and therefore are more easily characterized by their response to sinusoids rather than to arbitrary waveforms. The resulting system equations contain many instances of $A\cos(\omega t + \phi)$, where A, ω, and ϕ are the amplitude, frequency, and phase of the sinsusoid, respectively. $A\cos(\omega t + \phi)$ can be replaced by $\underline{A}$ using *complex notation*, indicated here by the underbar and reviewed below; it utilizes the arbitrary definition:

$$j \equiv (-1)^{0.5} \tag{B.1}$$

This arbitrary non-physical definition is exploited by De Moivre's theorem (B.4), which utilizes a unique property of e = 2.71828:

$$e^{\phi} = 1 + \phi + \phi^2/2! + \phi^3/3! + \dots \tag{B.2}$$

Therefore:

$$\begin{aligned} e^{j\phi} &= 1 + j\phi - \phi^2/2! - j\phi^3/3! + \phi^4/4! + j\phi^5/5! - \dots \\ &= \left[1 - \phi^2/2! + \phi^4/4! - \dots\right] + \left[j\phi - j\phi^3/3! + j\phi^5/5! \dots\right] \end{aligned} \tag{B.3}$$

$$e^{j\phi} = \cos\phi + j\sin\phi \tag{B.4}$$

This is a special instance of a general *complex number* $\underline{A}$:

$$\underline{A} = A_r + jA_i \tag{B.5}$$

where the real part is $A_r \equiv R_e\{\underline{A}\}$ and the imaginary part is $A_i \equiv I_m\{\underline{A}\}$.

It is now easy to use (B.4) and (B.5) to show that[76]:

$$A\cos(\omega t + \phi) = R_e\left\{Ae^{j(\omega t + \phi)}\right\} = R_e\left\{Ae^{j\phi}e^{j\omega t}\right\} = R_e\left\{\underline{A}e^{j\omega t}\right\} = A_r\cos\omega t - A_i\sin\omega t \tag{B.6}$$

where:

$$\underline{A} = Ae^{j\phi} = A\cos\phi + jA\sin\phi = A_r + jA_i \tag{B.7}$$

[76] The physics community differs and commonly defines $A\cos(\omega t + \phi) = R_e\{Ae^{-j(\omega t + \phi)}\}$ and $A_i \equiv -A\sin\phi$, where the rotational direction of ϕ is reversed in Figure B.1. Because phase is reversed in this alternative notation, the impedance of an inductor L becomes $-j\omega L$, and that of a capacitor becomes $j/\omega C$. In this notation j is commonly replaced by -i.

$$A_r \equiv A\cos\phi, \quad A_i \equiv A\sin\phi \tag{B.8}$$

The definition of $\underline{A}$ given in (B.8) has the useful geometric interpretation shown in Figure B.1(a), where the magnitude of the *phasor* $\underline{A}$ is simply the given amplitude A of the sinusoid, and the angle ϕ is its phase.

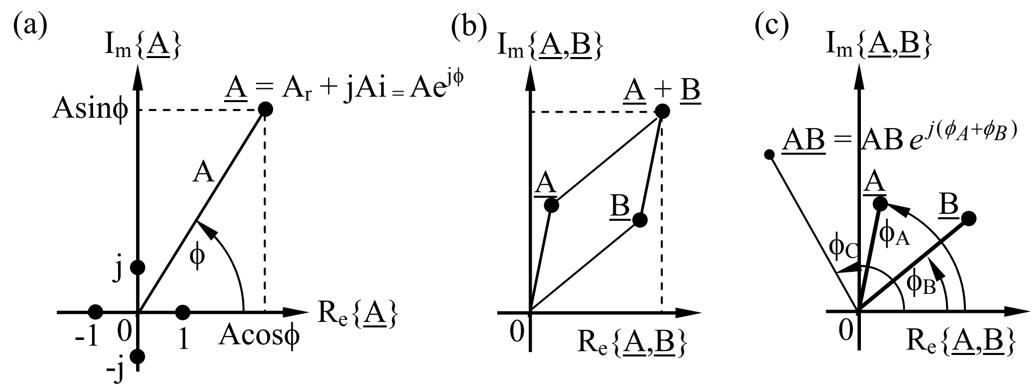


Figure B.1 Representation of phasors in the complex plane.

When $\phi = 0$ we have $R_e\{\underline{A}e^{j\omega t}\} = A\cos\omega t$, and when $\phi = \pi/2$ we have $-A\sin\omega t$. Advances in time alter the phasor $\underline{A}$ in the same sense as advances in ϕ; the phasor rotates counterclockwise. The utility of this diagram is partly that the signal of interest, $R_e\{\underline{A}e^{j\omega t}\}$, is simply the projection of the phasor $\underline{A}e^{j\omega t}$ on the real axis. It also makes clear that:

$$A = \left(A_r^2 + A_i^2\right)^{0.5} \tag{B.9}$$

$$\phi = \tan^{-1}\left(A_i/A_r\right) \tag{B.10}$$

It is also easy to see, for example, that $e^{j\pi} = -1$, and that $\underline{A} = jA$ corresponds to $-A\sin\omega t$.

Examples of equivalent representations in the time and complex domains are:

$$\begin{aligned} A\cos\omega t &\leftrightarrow A \\ -A\sin\omega t &\leftrightarrow jA \\ A\cos(\omega t + \phi) &\leftrightarrow Ae^{j\phi} \\ A\sin(\omega t + \phi) &\leftrightarrow -jAe^{j\phi} = Ae^{j(\phi - \pi/2)} \end{aligned}$$

Complex numbers behave as vectors in some respects, where addition and multiplication are also illustrated in Figure B.1(b) and (c), respectively:

$$\underline{A}+\underline{B}=\underline{B}+\underline{A}=A_r+B_r+j(A_i+B_i) \tag{B.11}$$

$$\underline{AB}=\underline{BA}=(A_rB_r-A_iB_i)+j(A_rB_i+A_iB_r)=AB\,e^{j(\phi_A+\phi_B)} \tag{B.12}$$

$$\underline{A}^*=A_r-jA_i=A\,e^{-j\phi_A} \tag{B.13}$$

We can easily solve for the real and imaginary parts of $\underline{A}$:

$$A_r=\left(\underline{A}+\underline{A}^*\right)/2, \qquad A_i=\left(\underline{A}-\underline{A}^*\right)/2 \tag{B.14}$$

Ratios of complex numbers can also be readily computed:

$$\underline{A}/\underline{B}=(A/B)\,e^{j(\phi_A-\phi_B)}=\underline{A}\underline{B}^*/\underline{B}\underline{B}^*=\underline{A}\underline{B}^*/|\underline{B}|^2 \tag{B.15}$$

Even an n^{th} root of $\underline{A} = Ae^{j\phi}$ can be simply found:

$$\underline{A}^{1/n}=A^{1/n}e^{j\phi/n} \tag{B.16}$$

where n legitimate roots exist and are:

$$\underline{A}^{1/n}=A^{(1/n)}e^{(j\phi/n)}e^{(j2\pi m/n)} \tag{B.17}$$

for m = 0, 1, …, n – 1.

Appendix C: Mathematical Identities

$$\overline{A} = \hat{x}A_x + \hat{y}A_y + \hat{z}A_z$$

$$\overline{A}\bullet\overline{B} = A_xB_x + A_yB_y + A_zB_z = \hat{a}\times\hat{b}\left|\overline{A}\right|\left|\overline{B}\right|\cos\theta$$

$$\overline{A}\times\overline{B} = \det\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{x}\left(A_yB_z - A_zB_y\right) + \hat{y}\left(A_zB_x - A_xB_z\right) + \hat{z}\left(A_xB_y - A_yB_x\right)$$

$$= \hat{a}\times\hat{b}\left|\overline{A}\right|\left|\overline{B}\right|\sin\theta$$

$$\overline{A}\bullet(\overline{B}\times\overline{C}) = \overline{B}\bullet(\overline{C}\times\overline{A}) = \overline{C}\bullet(\overline{A}\times\overline{B})$$

$$\overline{A}\times(\overline{B}\times\overline{C}) = (\overline{A}\bullet\overline{C})\overline{B} - (\overline{A}\bullet\overline{B})\overline{C}$$

$$(\overline{A}\times\overline{B})\bullet(\overline{C}\times\overline{D}) = (\overline{A}\bullet\overline{C})(\overline{B}\bullet\overline{D}) - (\overline{A}\bullet\overline{D})(\overline{B}\bullet\overline{C})$$

$$\nabla\times\nabla\Psi = 0$$

$$\nabla\bullet(\nabla\times\overline{A}) = 0$$

$$\nabla\times(\nabla\times\overline{A}) = \nabla(\nabla\bullet\overline{A}) - \nabla^2\overline{A}$$

$$-\overline{A}\times(\nabla\times\overline{A}) = (\overline{A}\bullet\nabla)\overline{A} - \frac{1}{2}\nabla(\overline{A}\bullet\overline{A})$$

$$\nabla(\Psi\Phi) = \Psi\nabla\Phi + \Phi\nabla\Psi$$

$$\nabla\bullet(\Psi\overline{A}) = \overline{A}\bullet\nabla\Psi + \Psi\nabla\bullet\overline{A}$$

$$\nabla\times(\Psi\overline{A}) = \nabla\Psi\times\overline{A} + \Psi\nabla\times\overline{A}$$

$$\nabla^2\Psi = \nabla\bullet\nabla\Psi$$

$$\nabla(\overline{A}\bullet\overline{B}) = (\overline{A}\bullet\nabla)\overline{B} + (\overline{B}\bullet\nabla)\overline{A} + \overline{A}\times(\nabla\times\overline{B}) + \overline{B}\times(\nabla\times\overline{A})$$

$$\nabla\bullet(\overline{A}\times\overline{B}) = \overline{B}\bullet(\nabla\times\overline{A}) - \overline{A}\bullet(\nabla\times\overline{B})$$

$$\nabla\times(\overline{A}\times\overline{B}) = \overline{A}(\nabla\bullet\overline{B}) - \overline{B}(\nabla\bullet\overline{A}) + (\overline{B}\bullet\nabla)\overline{A} - (\overline{A}\bullet\nabla)\overline{B}$$

Cartesian Coordinates (x,y,z):

$$\nabla\Psi = \hat{x}\frac{\partial\Psi}{\partial x} + \hat{y}\frac{\partial\Psi}{\partial y} + \hat{z}\frac{\partial\Psi}{\partial z}$$

$$\nabla\bullet\overline{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\overline{A} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2\Psi = \frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2}$$

Cylindrical coordinates (r,ϕ,z):

$$\nabla\Psi = \hat{\rho}\frac{\partial\Psi}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial\Psi}{\partial y} + \hat{z}\frac{\partial\Psi}{\partial z}$$

$$\nabla\bullet\overline{A} = \frac{1}{r}\frac{\partial(rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\overline{A} = \hat{r}\left(\frac{1}{r}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{z}\frac{1}{r}\left(\frac{\partial(rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial\phi}\right) = \frac{1}{r}\det\begin{vmatrix} \hat{r} & r\hat{\phi} & \hat{z} \\ \partial/\partial r & \partial/\partial\phi & \partial/\partial z \\ A_r & rA_\phi & A_z \end{vmatrix}$$

$$\nabla^2\Psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\Psi}{\partial\phi^2} + \frac{\partial^2\Psi}{\partial z^2}$$

Spherical coordinates (r,θ,ϕ):

$$\nabla\Psi = \hat{r}\frac{\partial\Psi}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial\Psi}{\partial\theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial\Psi}{\partial\phi}$$

$$\nabla\bullet\overline{A} = \frac{1}{r^2}\frac{\partial(r^2A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta A_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\times\overline{A} = \hat{r}\frac{1}{r\sin\theta}\left(\frac{\partial(r\sin\theta A_\phi)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right) + \hat{\theta}\left(\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{1}{r}\frac{\partial(rA_\phi)}{\partial r}\right) + \hat{\phi}\frac{1}{r}\left(\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right)$$

$$= \frac{1}{r^2\sin\theta}\det\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

$$\nabla^2\Psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Psi}{\partial\phi^2}$$

Gauss' Divergence Theorem:

$$\int_V \nabla \bullet \overline{G}\ dv = \oint_A \overline{G} \bullet \hat{n}\ da$$

Stokes' Theorem:

$$\int_A (\nabla \times \overline{G}) \bullet \hat{n}\ da = \oint_C \overline{G} \bullet d\overline{\ell}$$

Fourier Transforms for pulse signals h(t):

$$\underline{H}(f) = \int_{-\infty}^{\infty} h(t) e^{-j2\pi ft}\ dt$$

$$h(t) = \int_{-\infty}^{\infty} \underline{H}(f) e^{+j2\pi ft}\ df$$

Appendix D: Basic Equations for Electromagnetics and Applications

Fundamentals

$\overline{f} = q\left(\overline{E} + \overline{v} \times \mu_o \overline{H}\right)[N]$

$\nabla \times \overline{E} = -\partial \overline{B}/\partial t$

$\oint_c \overline{E} \bullet d\overline{s} = -\frac{d}{dt}\int_A \overline{B} \bullet d\overline{a}$

$\nabla \times \overline{H} = \overline{J} + \partial \overline{D}/\partial t$

$\oint_c \overline{H} \bullet d\overline{s} = \int_A \overline{J} \bullet d\overline{a} + \frac{d}{dt}\int_A \overline{D} \bullet d\overline{a}$

$\nabla \bullet \overline{D} = \rho \rightarrow \oint_A \overline{D} \bullet d\overline{a} = \int_V \rho dv$

$\nabla \bullet \overline{B} = 0 \rightarrow \oint_A \overline{B} \bullet d\overline{a} = 0$

$\nabla \bullet \overline{J} = -\partial \rho/\partial t$

$\overline{E}$ = electric field (Vm^{-1})

$\overline{H}$ = magnetic field (Am^{-1})

$\overline{D}$ = electric displacement (Cm^{-2})

$\overline{B}$ = magnetic flux density (T)

Tesla (T) = Weber m^{-2} = 10,000 gauss

ρ = charge density (Cm^{-3})

$\overline{J}$ = current density (Am^{-2})

σ = conductivity (Siemens m^{-1})

$\overline{J}_s$ = surface current density (Am^{-1})

ρ_s = surface charge density (Cm^{-2})

$\varepsilon_o = 8.85 \times 10^{-12}\ Fm^{-1}$

$\mu_o = 4\pi \times 10^{-7}\ Hm^{-1}$

$c = (\varepsilon_o\mu_o)^{-0.5} \cong 3 \times 10^8\ ms^{-1}$

$e = -1.60 \times 10^{-19}$ C

$\eta_o \cong 377$ ohms $= (\mu_o/\varepsilon_o)^{0.5}$

$\left(\nabla^2 - \mu\varepsilon\partial^2/\partial t^2\right)\overline{E} = 0$ [Wave Eqn.]

$E_y(z,t) = E_+(z-ct) + E_-(z+ct) = R_e\{\underline{E}_y(z)e^{j\omega t}\}$

$H_x(z,t) = \eta_o^{-1}[E_+(z-ct)-E_-(z+ct)]$ [or(ωt-kz) or (t-z/c)]

$\int_A \left(\overline{E} \times \overline{H}\right) \bullet d\overline{a} + (d/dt)\int_V \left(\varepsilon|\overline{E}|^2/2 + \mu|\overline{H}|^2/2\right)dv$

$= -\int_V \overline{E} \bullet \overline{J}\ dv$ (Poynting Theorem)

Media and Boundaries

$\overline{D} = \varepsilon_o \overline{E} + \overline{P}$

$\nabla \bullet \overline{D} = \rho_f,\ \tau = \varepsilon/\sigma$

$\nabla \bullet \varepsilon_o \overline{E} = \rho_f + \rho_p$

$\nabla \bullet \overline{P} = -\rho_p,\ \overline{J} = \sigma\overline{E}$

$\overline{B} = \mu\overline{H} = \mu_o\left(\overline{H} + \overline{M}\right)$

$\varepsilon = \varepsilon_o\left(1 - \omega_p{}^2/\omega^2\right)$

$\omega_p = \left(Ne^2/m\varepsilon_o\right)^{0.5}$

$\varepsilon_{eff} = \varepsilon(1 - j\sigma/\omega\varepsilon)$

skin depth $\delta = (2/\omega\mu\sigma)^{0.5}$ [m]

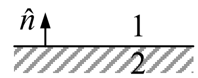
$\overline{E}_{1//} - \overline{E}_{2//} = 0$

$\overline{H}_{1//} - \overline{H}_{2//} = \overline{J}_s \times \hat{n}$

$B_{1\perp} - B_{2\perp} = 0$

$(D_{1\perp} - D_{2\perp}) = \rho_s$

$\hookrightarrow 0 =$ if $\sigma = \infty$



Electromagnetic Quasistatics

$\nabla^2\Phi = 0$

KCL : $\sum_i I_i(t) = 0$ at node

KVL : $\sum_i V_i(t) = 0$ around loop

$C = Q/V = A\varepsilon/d$ [F]

$L = \Lambda/I$

$i(t) = C\ dv(t)/dt$

$v(t) = L\ di(t)/dt = d\Lambda/dt$

$C_{parallel} = C_1 + C_2$

$C_{series} = (C_1^{-1} + C_2^{-1})^{-1}$

$w_e = Cv^2(t)/2;\ w_m = Li^2(t)/2$

$L_{solenoid} = N^2\mu A/W$

$\tau = RC,\ \tau = L/R$

$\Lambda = \int_A \overline{B} \bullet d\overline{a}$ (per turn)

$\overline{f} = q\left(\overline{E} + \overline{v} \times \mu_o \overline{H}\right)[N]$

$f_z = -dw_T/dz$

$\overline{F} = \overline{I} \times \mu_o \overline{H}\ [Nm^{-1}]$

$\overline{E}_e = -\overline{v} \times \mu_o \overline{H}$ inside wire

$P = \omega T = W_T dV_{olume}/dt$ [W]

Max $f/A = B^2/2\mu_o,\ D^2/2\varepsilon_o\ [Nm^{-2}]$

$vi = \frac{dw_T}{dt} + f\frac{dz}{dt}$

Electromagnetic Waves

$\left(\nabla^2 - \mu\varepsilon\partial^2/\partial t^2\right)\overline{E} = 0$ [Wave Eqn.]

$\left(\nabla^2 + k^2\right)\underline{\overline{E}} = 0,\ \underline{\overline{E}} = \underline{\overline{E}}_o e^{-j\overline{k}\bullet\overline{r}}$

$k = \omega(\mu\varepsilon)^{0.5} = \omega/c = 2\pi/\lambda$

$k_x{}^2 + k_y{}^2 + k_z{}^2 = k_o{}^2 = \omega^2\mu\varepsilon$

$v_p = \omega/k,\ v_g = (\partial k/\partial\omega)^{-1}$

$\theta_r = \theta_i$

$\sin\theta_t / \sin\theta_i = k_i / k_t = n_i / n_t$

$\theta_c = \sin^{-1}(n_t / n_i)$

$\theta > \theta_c \Rightarrow \underline{\overline{E}}_t = \underline{\overline{E}}_i \underline{T} e^{+\alpha x - jk_z z}$

$\underline{\overline{k}} = \overline{k}' - j\overline{k}''$

$\underline{\Gamma} = \underline{T} - 1$

$\underline{T}_{TE} = 2 / (1 + [\eta_o \cos\theta_t / \eta_t \cos\theta_i])$

$\underline{T}_{TM} = 2 / (1 + [\eta_t \cos\theta_t / \eta_i \cos\theta_i])$

$\theta_B = \tan^{-1}(\varepsilon_t / \varepsilon_i)^{0.5}$ for TM

$P_d \cong |\underline{\overline{J}}_S|^2 / 2\sigma\delta \ [Wm^{-2}]$

$\overline{E} = -\nabla\phi - \partial\overline{A}/\partial t, \quad \overline{B} = \nabla \times \overline{A}$

$\underline{\Phi}(r) = \int_{V'} \left(\underline{\rho}(\overline{r}) e^{-jk|\overline{r}' - \overline{r}|} / 4\pi\varepsilon_o |\overline{r}' - \overline{r}|\right) dv'$

$\underline{\overline{A}}(r) = \int_{V'} \left(\mu_o \underline{\overline{J}}(\overline{r}) e^{-jk|\overline{r}' - \overline{r}|} / 4\pi |\overline{r}' - \overline{r}|\right) dv'$

$\underline{\overline{E}}_{ff} = \hat{\vartheta}(j\eta_o k \underline{I} d / 4\pi r) e^{-jkr} \sin\theta$

$\nabla^2 \underline{\Phi} + \omega^2 \mu_o \varepsilon_o \underline{\Phi} = -\rho / \varepsilon_o$

$\nabla^2 \underline{\overline{A}} + \omega^2 \mu_o \varepsilon_o \underline{\overline{A}} = -\mu_o \underline{\overline{J}}$

Forces, Motors, and Generators

$\overline{f} = q(\overline{E} + \overline{v} \times \mu_o \overline{H}) [N]$

$f_z = -dw_T / dz$

$\overline{F} = \overline{I} \times \mu_o \overline{H} \ [Nm^{-1}]$

$\overline{E}_e = -\overline{v} \times \mu_o \overline{H}$ inside wire

$P = \omega T = W_T dV_{olume}/dt$ [W]

Max $f/A = B^2/2\mu_o$, $D^2/2\varepsilon_o$ [Nm^{-2}]

$vi = \frac{dw_T}{dt} + f\frac{dz}{dt}$

$f = ma = d(mv)/dt$

$x = x_o + v_o t + at^2/2$

$P = fv$ [W] $= T\omega$

$w_k = mv^2/2$

$T = I \, d\omega/dt$

$I = \sum_i m_i r_i^2$

Circuits

$\text{KCL}: \sum_i I_i(t) = 0$ at node

$\text{KVL}: \sum_i V_i(t) = 0$ around loop

$C = Q/V = A\varepsilon/d$ [F]

$L = \Lambda/I$

$i(t) = C \, dv(t)/dt$

$v(t) = L \, di(t)/dt = d\Lambda/dt$

$C_{parallel} = C_1 + C_2$

$C_{series} = (C_1^{-1} + C_2^{-1})^{-1}$

$w_e = Cv^2(t)/2$; $w_m = Li^2(t)/2$

$L_{solenoid} = N^2 \mu A/W$

$\tau = RC$, $\tau = L/R$

$\Lambda = \int_A \overline{B} \bullet d\overline{a}$ (per turn)

$\underline{Z}_{series} = R + j\omega L + 1/j\omega C$

$\underline{Y}_{par} = G + j\omega C + 1/j\omega L$

$Q = \omega_o w_T / P_{diss} = \omega_o / \Delta\omega$

$\omega_o = (LC)^{-0.5}$

$\langle v^2(t) \rangle / R = kT$

Limits to Computation Speed

$dv(z)/dz = -L di(z)/dt$

$di(z)/dz = -C dv(z)/dt$

$d^2v/dz^2 = LC \, d^2v/dt^2$

$v(z,t) = f_+(t - z/c) + f_-(t + z/c)$

$\quad = g_+(z - ct) + g_-(z - ct)$

$i(t,z) = Y_o[f_+(t - z/c) - f_-(t + z/c)]$

$c = (LC)^{-0.5} = 1/\sqrt{\mu\varepsilon}$

$Z_o = Y_o^{-1} = (L/C)^{0.5}$

$\Gamma_L = f_-/f_+ = (R_L - Z_o)/(R_L + Z_o)$

$v(z,t) = g_+(z - ct) + g_-(z + ct)$

$V_{Th} = 2f_+(t)$, $R_{Th} = Z_o$

Power Transmission

$(d^2/dz^2 + \omega^2 LC)\underline{V}(z) = 0$

$\underline{V}(z) = \underline{V}_+ e^{-jkz} + \underline{V}_- e^{+jkz}$

$\underline{I}(z) = Y_o[\underline{V}_+ e^{-jkz} - \underline{V}_- e^{+jkz}]$

$k = 2\pi/\lambda = \omega/c = \omega(\mu\varepsilon)^{0.5}$

$\underline{Z}(z) = \underline{V}(z)/\underline{I}(z) = Z_o \underline{Z}_n(z)$

$\underline{Z}_n(z) = [1 + \underline{\Gamma}(z)] / [1 - \underline{\Gamma}(z)] = R_n + jX_n$

$\underline{\Gamma}(z) = (\underline{V}_- / \underline{V}_+) e^{2jkz} = [\underline{Z}_n(z) - 1] / [\underline{Z}_n(z) + 1]$

$\underline{Z}(z) = Z_o (\underline{Z}_L - jZ_o \tan kz) / (\underline{Z}_o - jZ_L \tan kz)$

$\text{VSWR} = |\underline{V}_{max}| / |\underline{V}_{min}| = R_{max}$

Wireless Communications and Radar

$G(\theta,\phi) = P_r/(P_R/4\pi r^2)$

$P_R = \int_{4\pi} P_r(\theta, \phi, r) r^2 \sin\theta d\theta d\phi$

$P_{rec} = P_r(\theta,\phi)A_e(\theta,\phi)$

$A_e(\theta,\phi) = G(\theta,\phi)\lambda^2/4\pi$

$R_r = P_R/\langle i^2(t)\rangle$

$E_{ff}(\theta \cong 0) = \left(je^{jkr}/\lambda r\right)\int_A E_t(x,y)e^{jk_x x + jk_y y}dxdy$

$P_{rec} = P_R\left(G\lambda/4\pi r^2\right)^2 \sigma_s/4\pi$

$\underline{E} = \sum_i a_i \underline{E}_i e^{-jkr_i}$ = (element factor)(array f)

$E_{bit} \geq \sim 4 \times 10^{-20}$ [J]

$\underline{Z}_{12} = \underline{Z}_{21}$ if reciprocity

$(d^2/dz^2 + \omega^2 LC)\underline{V}(z) = 0$

$\underline{V}(z) = \underline{V}_+ e^{-jkz} + \underline{V}_- e^{+jkz}$

$\underline{I}(z) = Y_o[\underline{V}_+ e^{-jkz} - \underline{V}_- e^{+jkz}]$

$k = 2\pi/\lambda = \omega/c = \omega(\mu\varepsilon)^{0.5}$

$\underline{Z}(z) = \underline{V}(z)/\underline{I}(z) = Z_o \underline{Z}_n(z)$

$\underline{Z}_n(z) = [1+\underline{\Gamma}(z)]/[1-\underline{\Gamma}(z)] = R_n + jX_n$

$\underline{\Gamma}(z) = (\underline{V}_-/\underline{V}_+)e^{2jkz} = [\underline{Z}_n(z)-1]/[\underline{Z}_n(z)+1]$

$\underline{Z}(z) = Z_o\left(\underline{Z}_L - jZ_o \tan kz\right)/\left(\underline{Z}_o - jZ_L \tan kz\right)$

$VSWR = |\underline{V}_{max}|/|\underline{V}_{min}| = R_{max}$

$\theta_r = \theta_i$

$\sin\theta_t/\sin\theta_i = k_i/k_t = n_i/n_t$

$\theta_c = \sin^{-1}(n_t/n_i)$

$\theta > \theta_c \Rightarrow \underline{E}_t = \underline{E}_i \underline{T} e^{+\alpha x - jk_z z}$

$\underline{\bar{k}} = \bar{k}' - j\bar{k}''$

$\underline{\Gamma} = \underline{T} - 1$

At ω_o, $\langle w_e\rangle = \langle w_m\rangle$

$\langle w_e\rangle = \int_V\left(\varepsilon|\underline{\bar{E}}|^2/4\right)dv$

$\langle w_m\rangle = \int_V\left(\mu|\underline{\bar{H}}|^2/4\right)dv$

$Q_n = \omega_n w_{Tn}/P_n = \omega_n/2\alpha_n$

$f_{mnp} = (c/2)\left([m/a]^2 + [n/b]^2 + [p/d]^2\right)^{0.5}$

$s_n = j\omega_n - \alpha_n$

Optical Communications

E = hf, photons or phonons

hf/c = momentum [kg ms^{-1}]

$dn_2/dt = -\left[An_2 + B(n_2 - n_1)\right]$

Acoustics

$P = P_o + p,\ \bar{U} = \bar{U}_o + u \quad (\bar{U}_o = 0 \text{ here})$

$\nabla p = -\rho_o \partial\bar{u}/\partial t$

$\nabla \bullet \bar{u} = -(1/\gamma P_o)\partial p/\partial t$

$\left(\nabla^2 - k^2\partial^2/\partial t^2\right)p = 0$

$k^2 = \omega^2/c_s^2 = \omega^2\rho_o/\gamma P_o$

$c_s = v_p = v_g = (\gamma P_o/\rho_o)^{0.5}$ or $(K/\rho_o)^{0.5}$

$\eta_s = p/u = \rho_o c_s = (\rho_o\gamma P_o)^{0.5}$ gases

$\eta_s = (\rho_o K)^{0.5}$ solids, liquids

$p, \bar{u}_\perp$ continuous at boundaries

$\underline{p} = \underline{p}_+ e^{-jkz} + \underline{p}_- e^{+jkz}$

$\underline{u}_z = \eta_s^{-1}(\underline{p}_+ e^{-jkz} - \underline{p}_- e^{+jkz})$

$\int_A \bar{u}p \bullet d\bar{a} + (d/dt)\int_V\left(\rho_o|\bar{u}|^2/2 + p^2/2\gamma P_o\right)dV$

Mathematical Identities

$\sin^2\theta + \cos^2\theta = 1$

$\cos\alpha + \cos\beta = 2\cos[(\alpha+\beta)/2]\cos[(\alpha-\beta)/2]$

$\underline{H}(f) = \int_{-\infty}^{+\infty} h(t)e^{-j\omega t}dt$

$e^x = 1 + x + x^2/2! + x^3/3! + \ldots$

$\sin\alpha = \left(e^{j\alpha} - e^{-j\alpha}\right)/2j$

$\cos\alpha = \left(e^{j\alpha} + e^{-j\alpha}\right)/2$

Vector Algebra

$\nabla = \hat{x}\partial/\partial x + \hat{y}\partial/\partial y + \hat{z}\partial/\partial z$

$\bar{A} \bullet \bar{B} = A_x B_x + A_y B_y + A_z B_z$

$\nabla^2\phi = \left(\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2\right)\phi$

$\nabla \bullet (\nabla \times \bar{A}) = 0$

$\nabla \times (\nabla \times \bar{A}) = \nabla(\nabla \bullet \bar{A}) - \nabla^2\bar{A}$

Gauss and Stokes' Theorems

$\oiiint_V(\nabla \bullet \bar{G})dv = \oiint_A \bar{G} \bullet d\bar{a}$

$\oiint_A(\nabla \times \bar{G}) \bullet d\bar{a} = \oint_c \bar{G} \bullet d\bar{s}$

Complex Numbers and Phasors

$v(t) = R_e\{\underline{V}e^{j\omega t}\}$ where $\underline{V} = |V|e^{j\phi}$

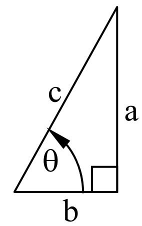
$e^{j\omega t} = \cos\omega t + j\sin\omega t$

Spherical Trigonometry

$\int_{4\pi} r^2 \sin\theta\, d\theta d\phi = 4\pi$

Appendix E: Frequently Used Trigonometric and Calculus Expressions

$\sin\theta = a/c$



$\cos\theta = b/c$

$\tan\theta = a/b$

$a^2 + b^2 = c^2$

$\sin^2\theta + \cos^2\theta = 1$

$e^{j\theta} = \cos\theta + j\sin\theta$

$(d/d\theta)\sin\theta = \cos\theta$

$(d/d\theta)\cos\theta = -\sin\theta$

$(d/dx)e^{f(x)} = [df(x)/dx]\ e^{f(x)}$

$a^x = (e^{\ln a})^x$

$(d/dx)x^n = nx^{n-1}$

$(d/dx)AB = A(dB/dx) + B(dA/dx)$

$(d/dx)f_1[f_2(\theta)] = [df_1/df_2][df_2(\theta)/d\theta]d\theta/dx$

$(d/dx)\sin[f(\theta)] = \cos[f(\theta)][df(\theta)/d\theta]d\theta/dx$

$\int\sin\theta\ d\theta = -\cos\theta$

$\int\cos\theta\ d\theta = \sin\theta$

$\int e^{ax}\ dx = e^{ax}/a$

$\int x^n\ dx = x^{n+1}/(n+1)$

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