Light Reflectors and Optical Resonators

Outline

Review of Wave Reflection

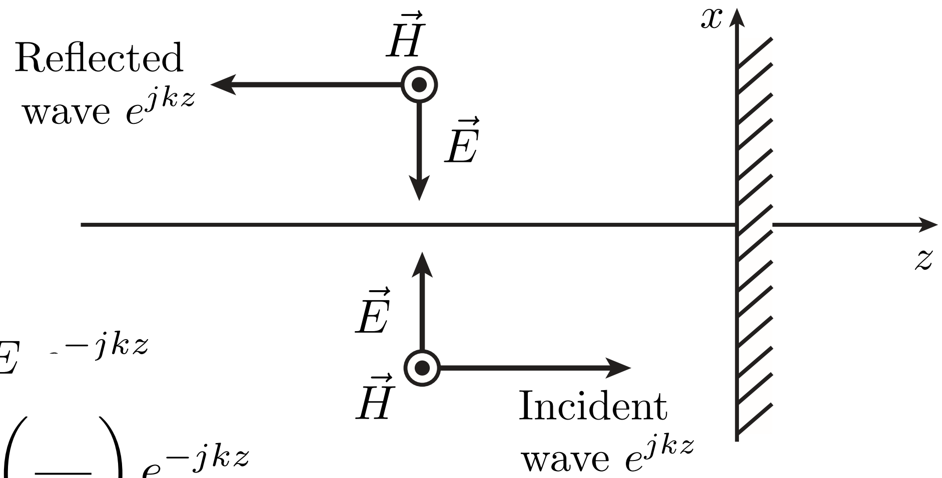
Reflection and Interference

Fiber for Telecommunications

Optical Resonators

Reflection of a Normally Incident EM Wave from a Perfect Conductor

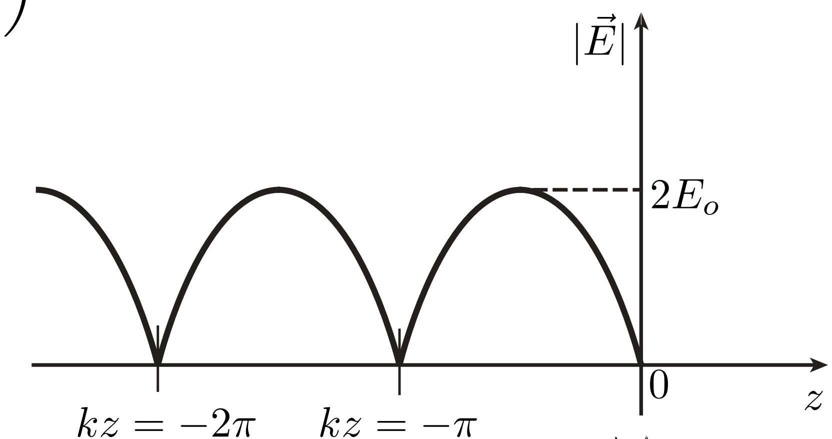
Image by Theogeo http://www.flickr.com/photos/theogeo/1102816166/ on flickr



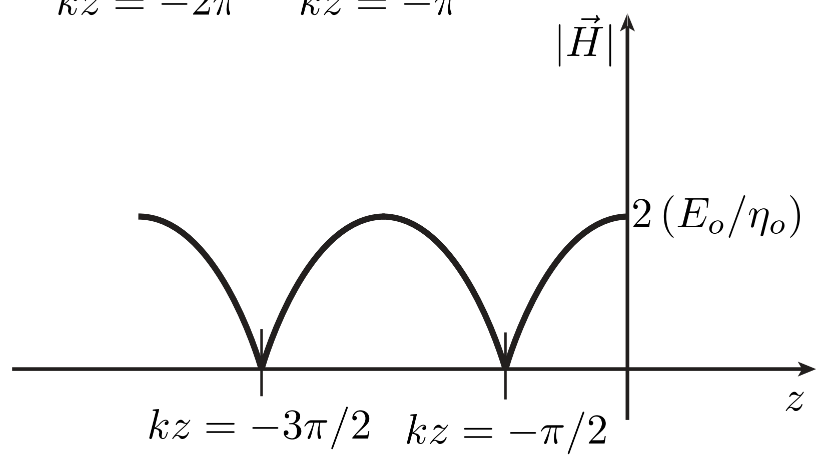
$$\vec{E}^i = \hat{x}E\ e^{-jkz}$$

$$\vec{H}^i = \hat{y}\left(\frac{}{\eta_o}\right)e^{-jkz}$$

Standing wave pattern of the E-field



Standing wave pattern of the H-field



Reflection & Transmission of EM Waves at Boundaries

$$\vec{E}_1 = \vec{E}_i + \vec{E}_r$$
$$= \hat{x}\left(E_o^i\, e^{-jk_1 z} + E_o^r\, e^{+jk_1 z}\right)$$

$$\vec{E}_2 = \vec{E}_t$$
$$= \hat{x} E_o^t\, e^{-jk_2 z}$$

Medium 1 | Medium 2

$$\vec{H}_1 = \vec{H}_i + \vec{H}_r$$
$$= \hat{y}\left(\frac{E_o^i}{\eta_1}\, e^{-jk_1 z} - \frac{E_o^r}{\eta_1}\, e^{+jk_1 z}\right)$$

$$\vec{H}_2 = \vec{H}_r$$
$$= \hat{y}\frac{E_o^t}{\eta_2}\, e^{-jk_2 z}$$

$$\overline{E}_{1(z=0)} = \overline{E}_{2(z=0)}$$
$$\overline{H}_{1(z=0)} = \overline{H}_{2(z=0)}$$

Reflection of EM Waves at Boundaries

$$\vec{E}_1(z=0) = \vec{E}_2(z=0)$$

$$E_o^i + E_o^r = E_o^t$$

$$\vec{H}_1(z=0) = \vec{H}_2(z=0)$$

$$\frac{E_o^i}{\eta_1} - \frac{E_o^r}{\eta_1} = \frac{E_o^t}{\eta_2} \qquad \eta = \sqrt{\frac{\mu}{\epsilon}}$$

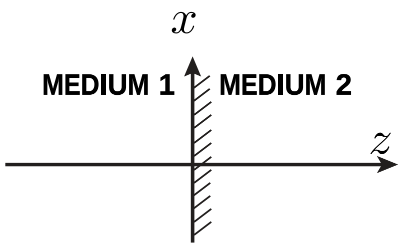
$$r = \frac{E_o^r}{E_o^i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

REFLECTION COEFFICIENT

(note that sign of *r* depends on the relative values of η_2 and η_1)

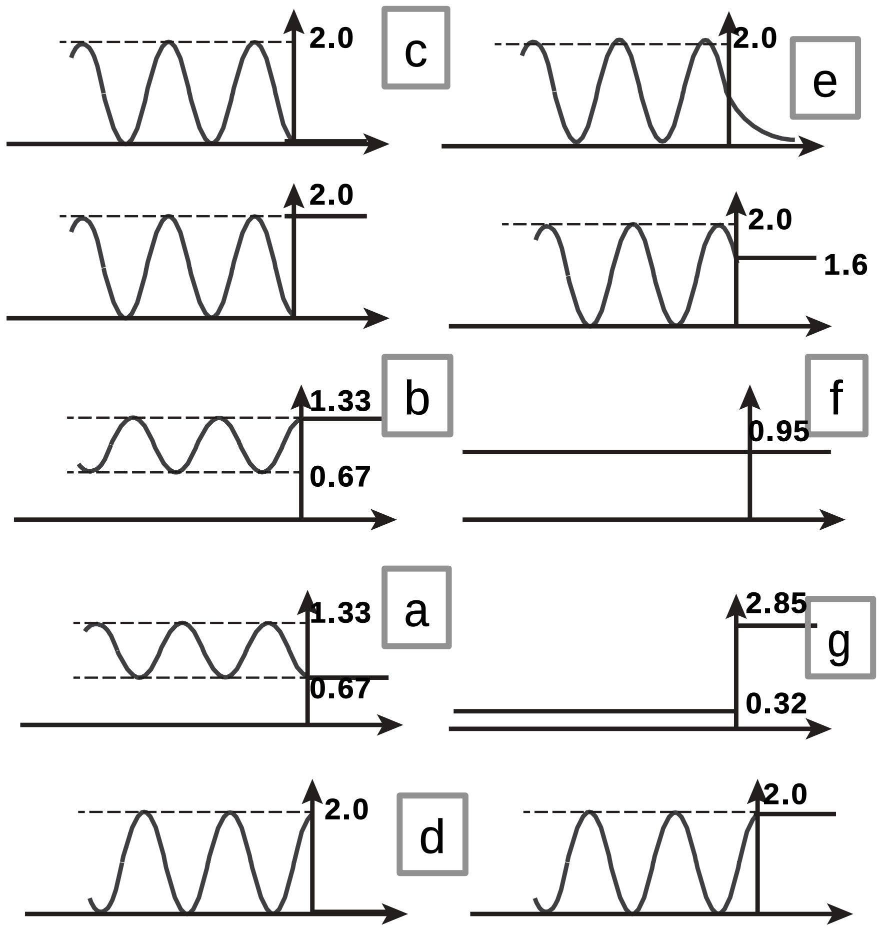
$$t = \frac{E_o^t}{E_o^i} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

TRANSMISION COEFFICIENT



Examples of Light Reflection

The image below indicates the standing wave patterns ($|E_i + E_r|$) resulting when an incident wave in medium 1 with amplitude equal to 1 V/m is incident on an interface. Label the graphs (a)-(g) to match them with the description detailed below.



a) Plot of $|E_{y,total}|$ with medium 1 being air, medium 2, n_2 =2. Normal incidence.

b) Plot of $|E_{y,total}|$ with medium 1 having n_1 = 2, medium 2 being air. Normal incidence.

c) Plot of $|E_{y,total}|$ with medium 1 being air, medium 2 being a perfect conductor

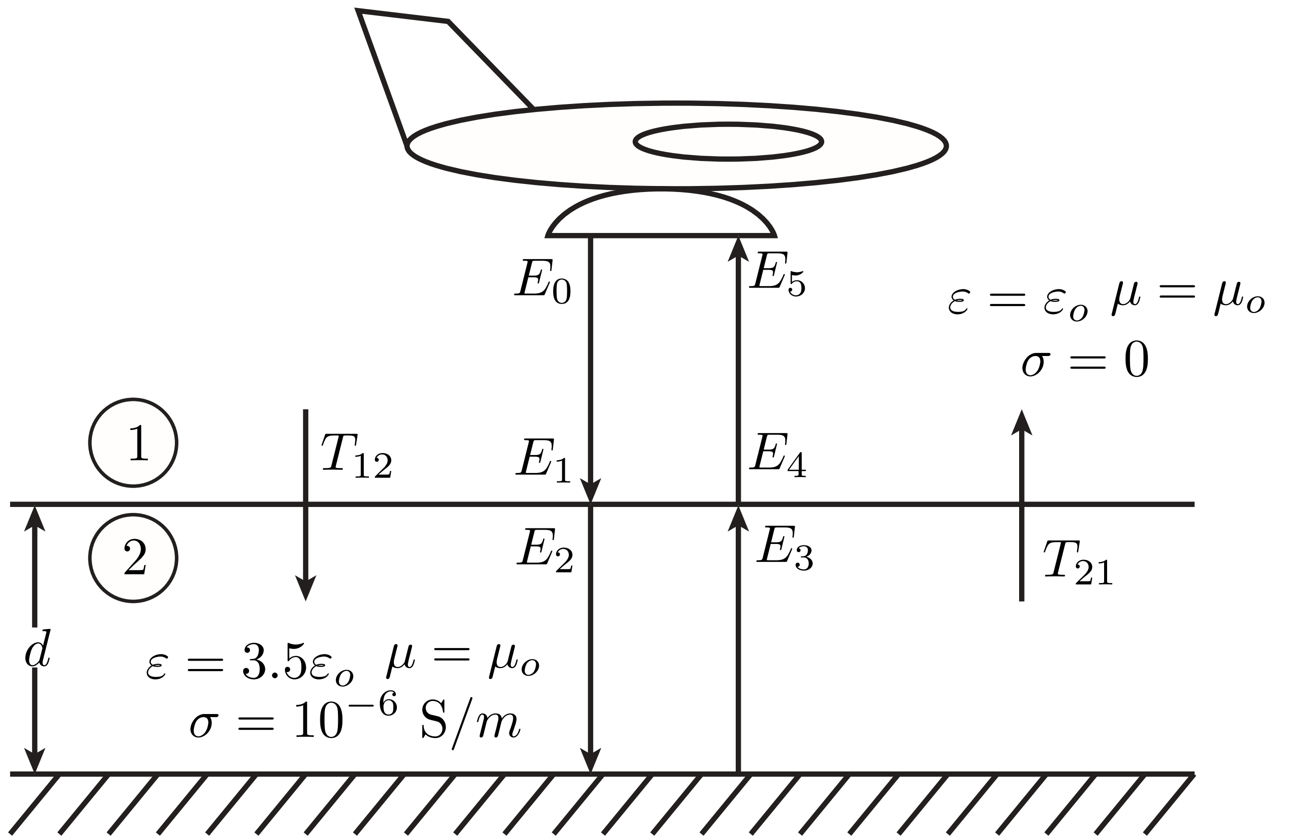
d) Plot of $|H_{y,total}|$ with medium 1 being air, medium 2 being a perfect conductor

e) Plot of $|E_{y,total}|$ with medium 1 having n_1 =2, medium 2 being air. Angle of incidence greater than critical angle.

f) Plot of $|E_{y,total}|$ with angle of incidence equal to the Brewster angle.

g) Plot of $|E_{y,total}|$ with angle of incidence equal to the Brewster angle. Medium 2 is air.

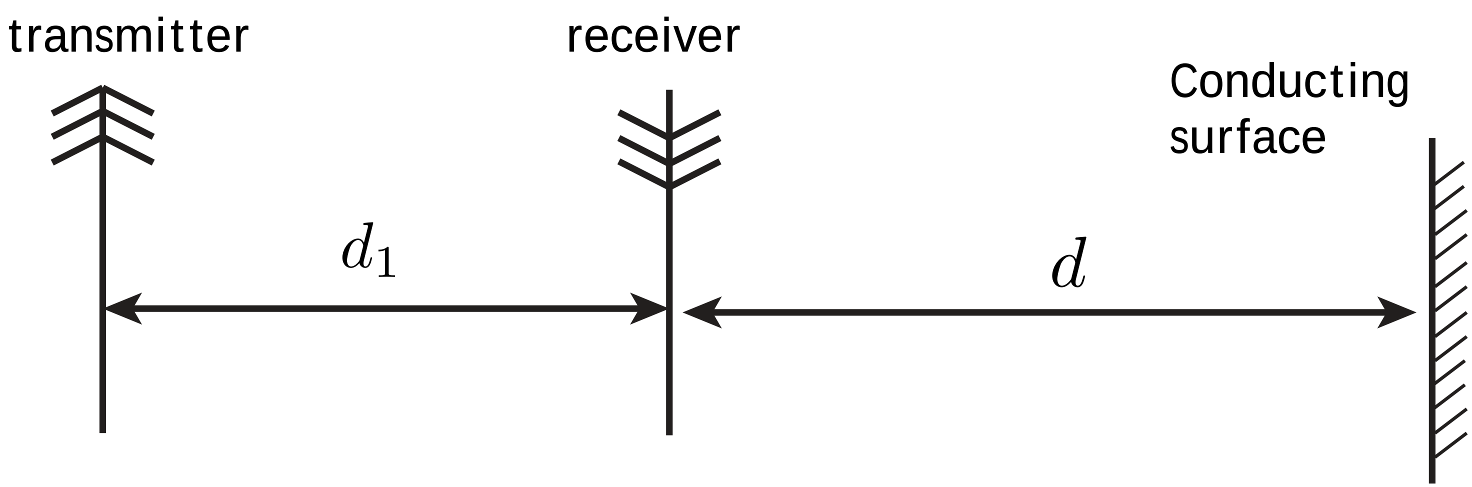
Remote Sensing of the Environment ... ***using radar***



EXAMPLE: MEASUREMENT OF THICKNESS OF POLAR ICE CAPS

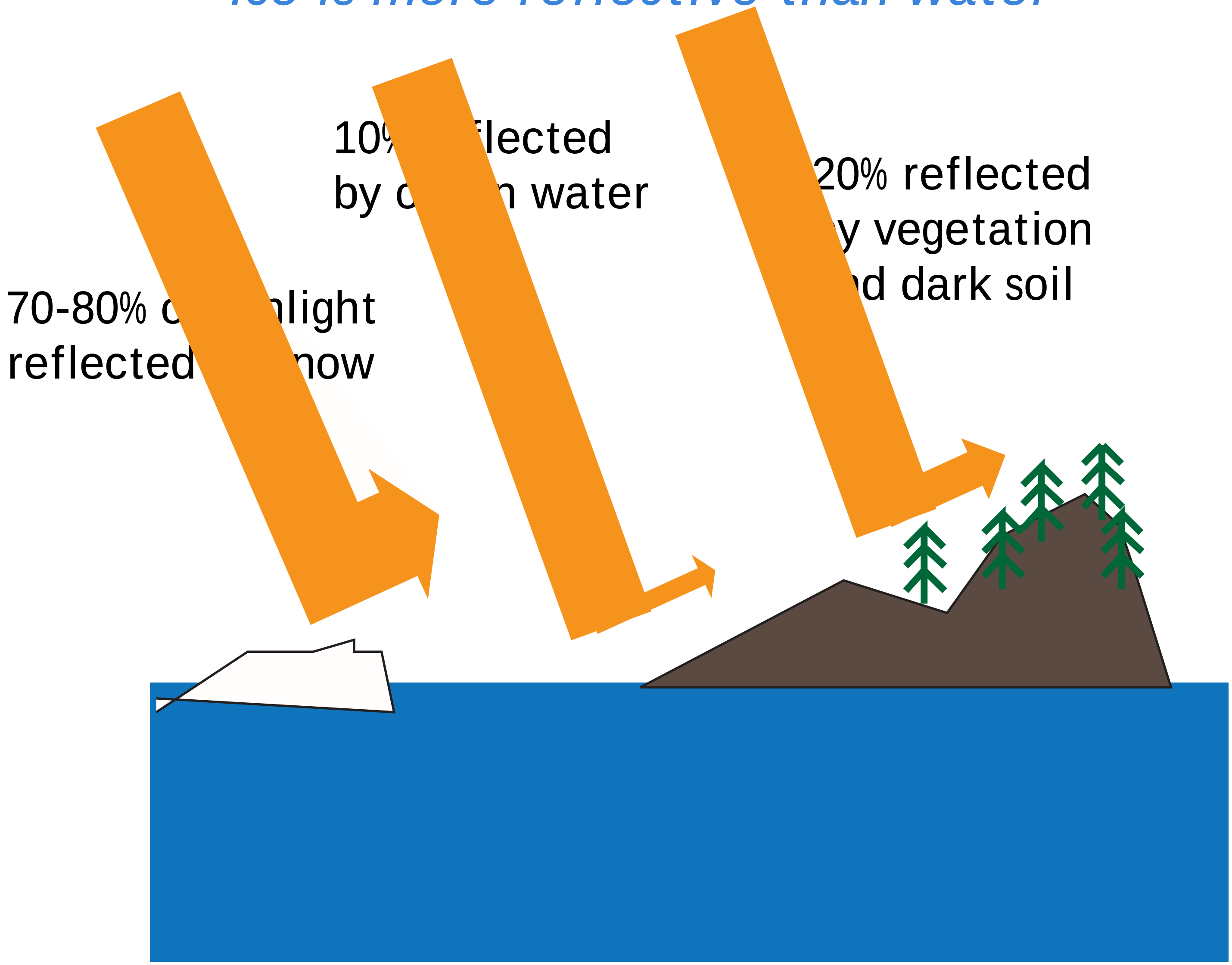
Reflectometry

... measurement of distance to a target by identifying the nodes in the standing wave pattern



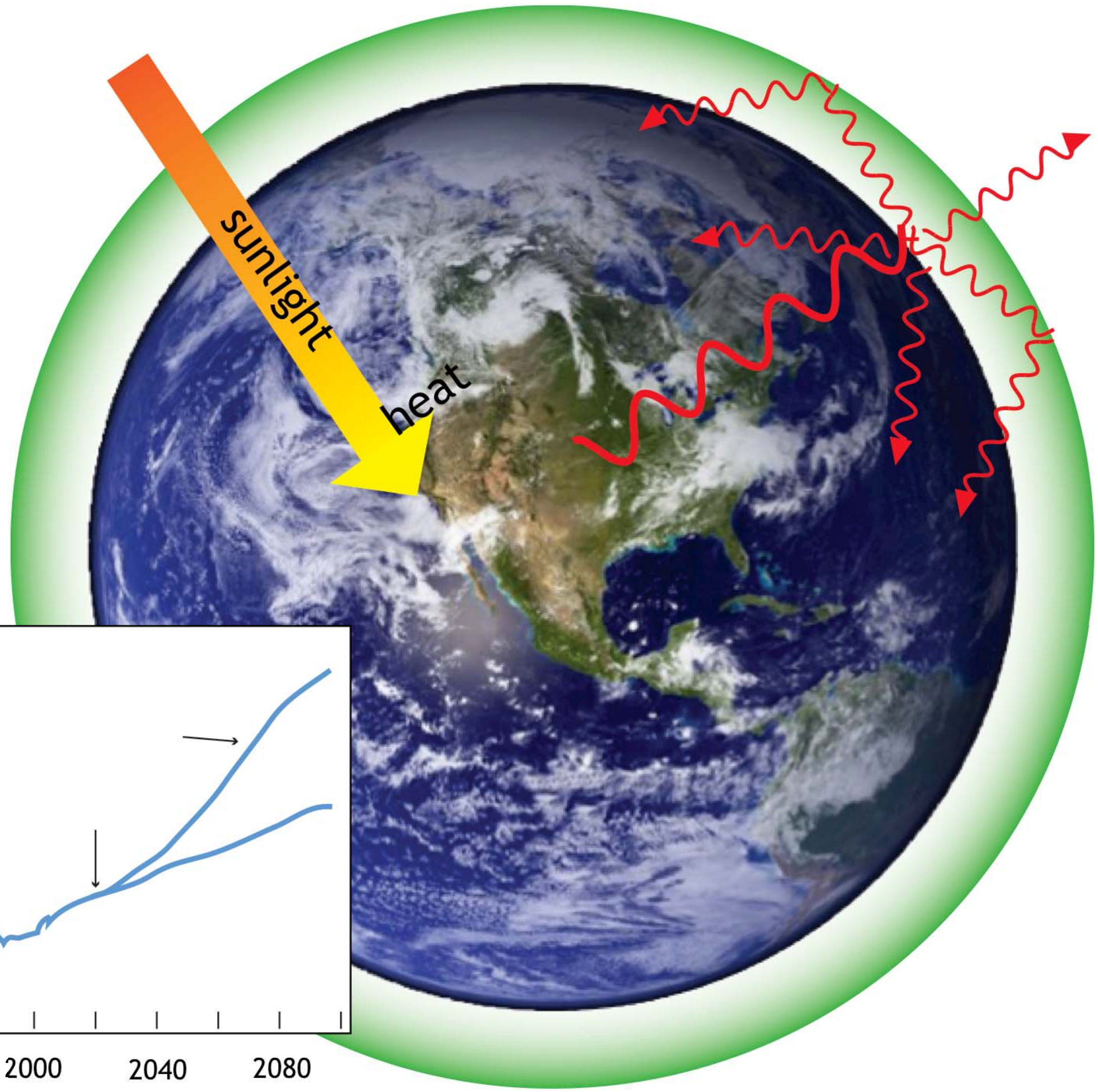
Today’s Culture Moment

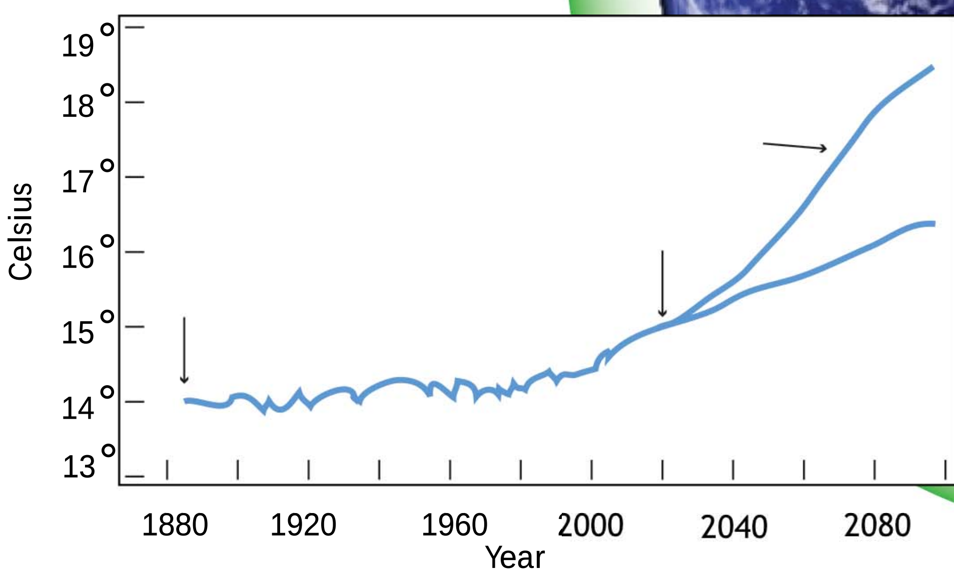
Ice is more reflective than water



The Greenhouse Effect

Sunlight is reradiated as heat and trapped by greenhouse gasses such as carbon dioxide. Too much carbon dioxide, however, causes the planet to heat up more than usual.





Deploy Aluminum Rafts over Dead Ocean Areas ?

Net excess energy input into planet Earth 1.6 W/m^2.
Illuminance on ground level is ~1000 W/m^2
→ **We need to reflect 1.6/1000 of energy back to Balance the Energy IN/OUT**

Oceans are 90% absorptive (10% reflective)
Aluminum is 88% reflective on the shiny side and 80% reflective on the dull side.
(Frosted silica might also be able to be used as a reflector)

How much of ocean area do we need to cover
with 80% reflective sheets of aluminum to balance the energy IN/OUT ?
(1.6/1000) / (80% - 10%) = 0.23% (of the Earth's surface area)

Earth surface area = 5100 million km^2

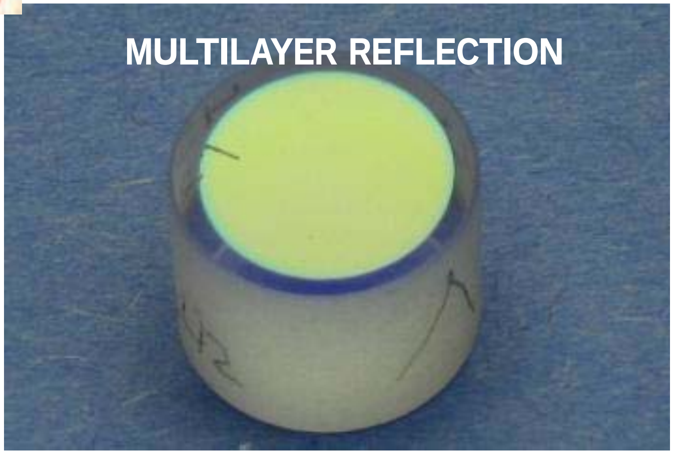
We need to cover = 1.2 million km^2
→ equivalent to ~100 years
of today's Aluminum production
(assuming 50 μm thick Al foil)

Image is in the public domain

Dead ocean zones = 0.24 million km^2

Ice fields:
North Pole = 9 to 12 million km^2
Greenland ice sheet = 1.7 million km^2
South Pole = 14 million km^2

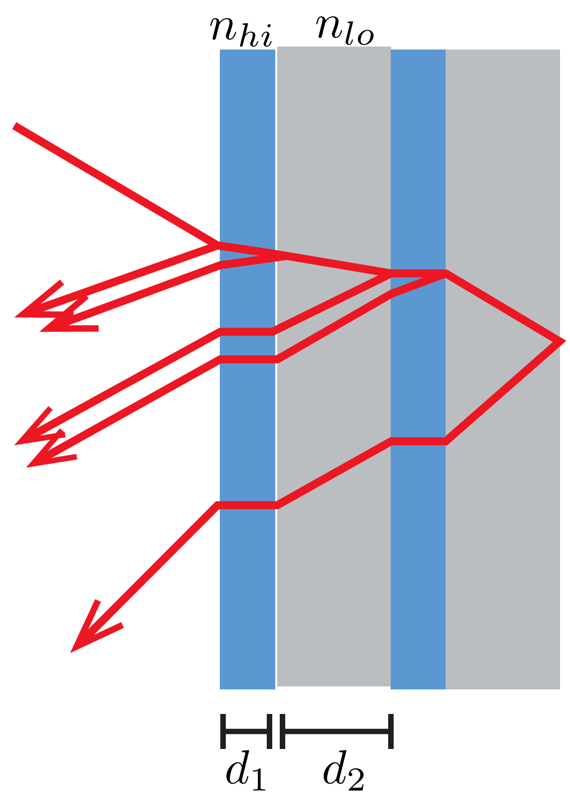
Three Ways to Make a Mirror



Dielectric Mirrors

... can be >99% reflective

Simple dielectric mirrors consist of stacked of layers of high and low refractive index. The layers are chosen such the path-length differences of reflections from low to high index layers are integer multiples of wavelengths. Similarly, reflections from low-index layers have path length difference of half a wavelength, but add constructively because of 180 degree phase shift from the reflection. For normal incidence, these optimized thicknesses are a quarter of a wavelength



$$R = \left(\frac{1-b}{1+b}\right)^2 \qquad b = \Pi_{i=0}^{N}\left[\frac{n_{LO}}{n_{HI}}\right]$$

$$d_1 = \lambda/(4n_{hi})$$
$$d_2 = \lambda/(4n_{lo})$$

Thin layers with a high refractive index n_{HI} *are interleaved with thicker layers with a lower refractive index* n_{LO}*.*
The path lengths l_A *and* l_B *differ by exactly one wavelength, which leads to constructive interference.*

Source: wikipedia.com

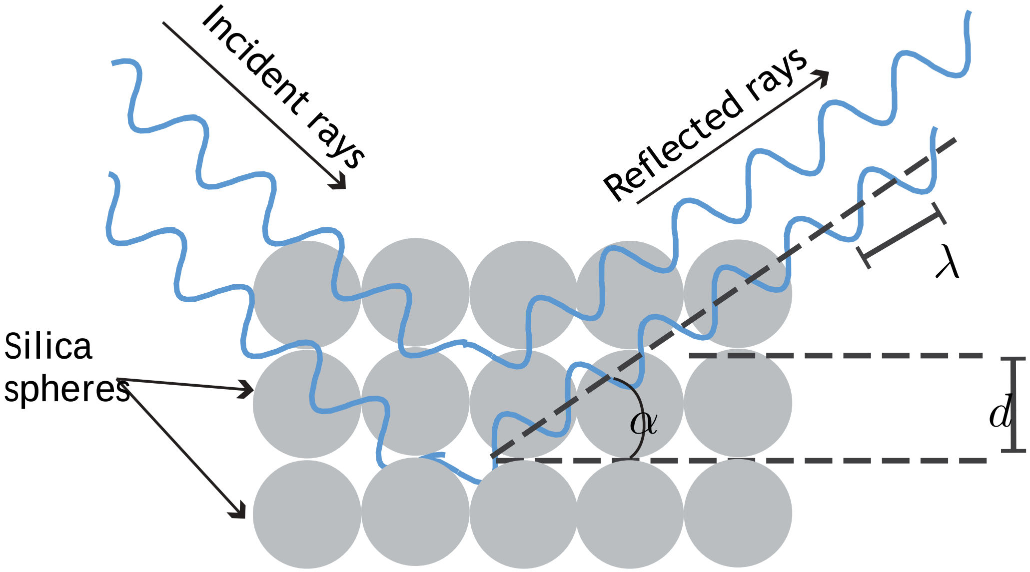
Opals

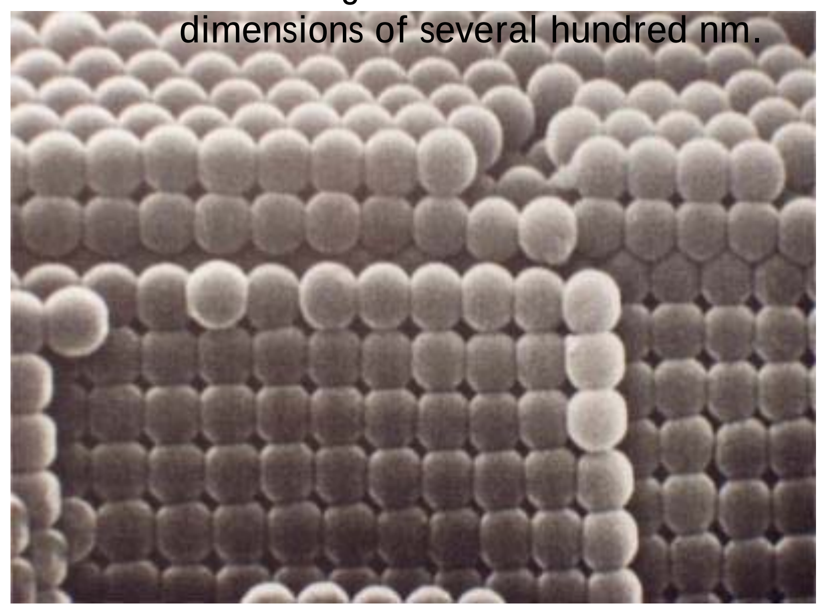
… are an example of dielectric mirrors

Colors with $\lambda = 2dsin(\alpha)$ have constructive interference

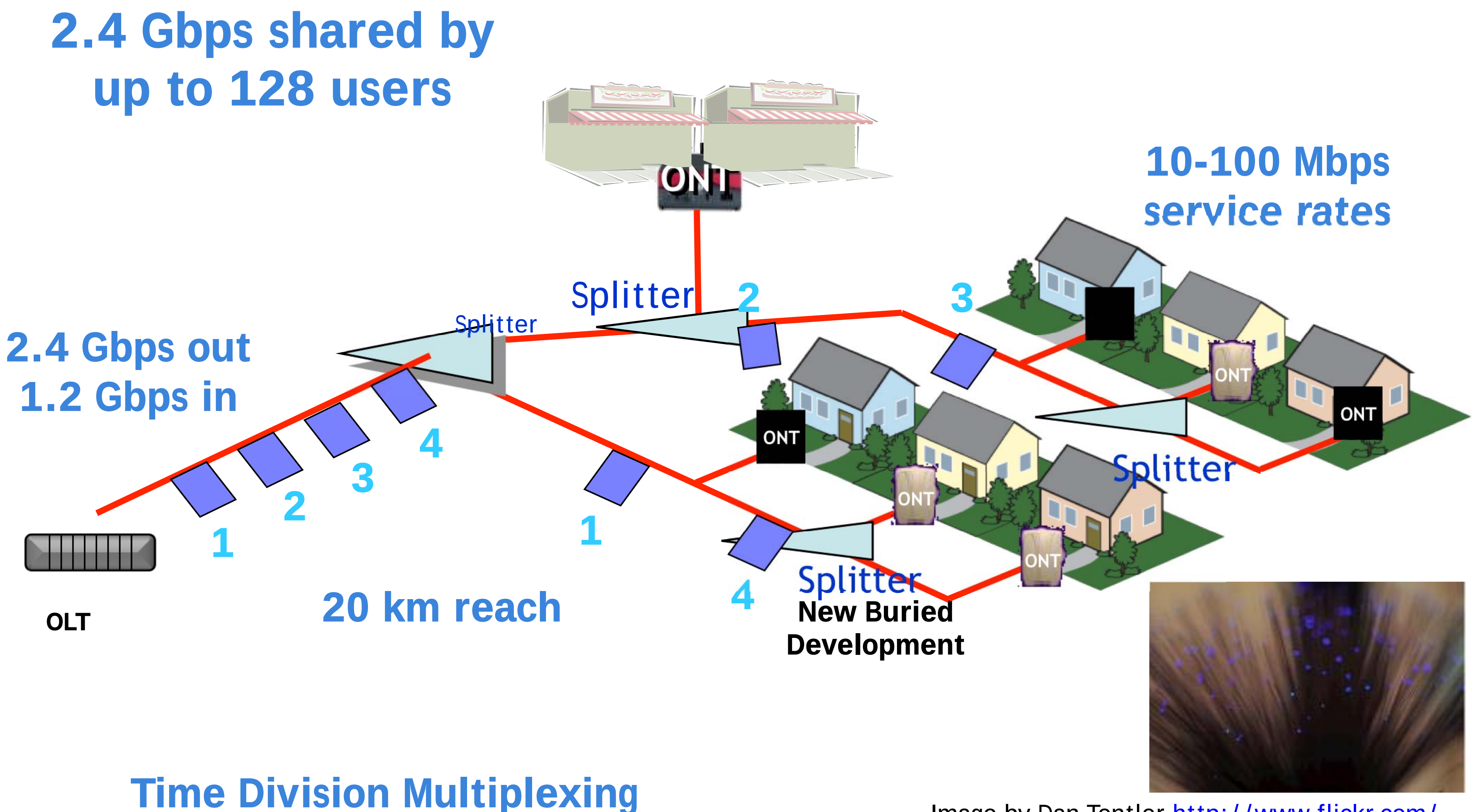
Image is in the public domain

Precious opal consists of spheres of silica of fairly regular size, packed into close-packed planes that are stacked together with characteristic dimensions of several hundred nm.





Fiber to the Home



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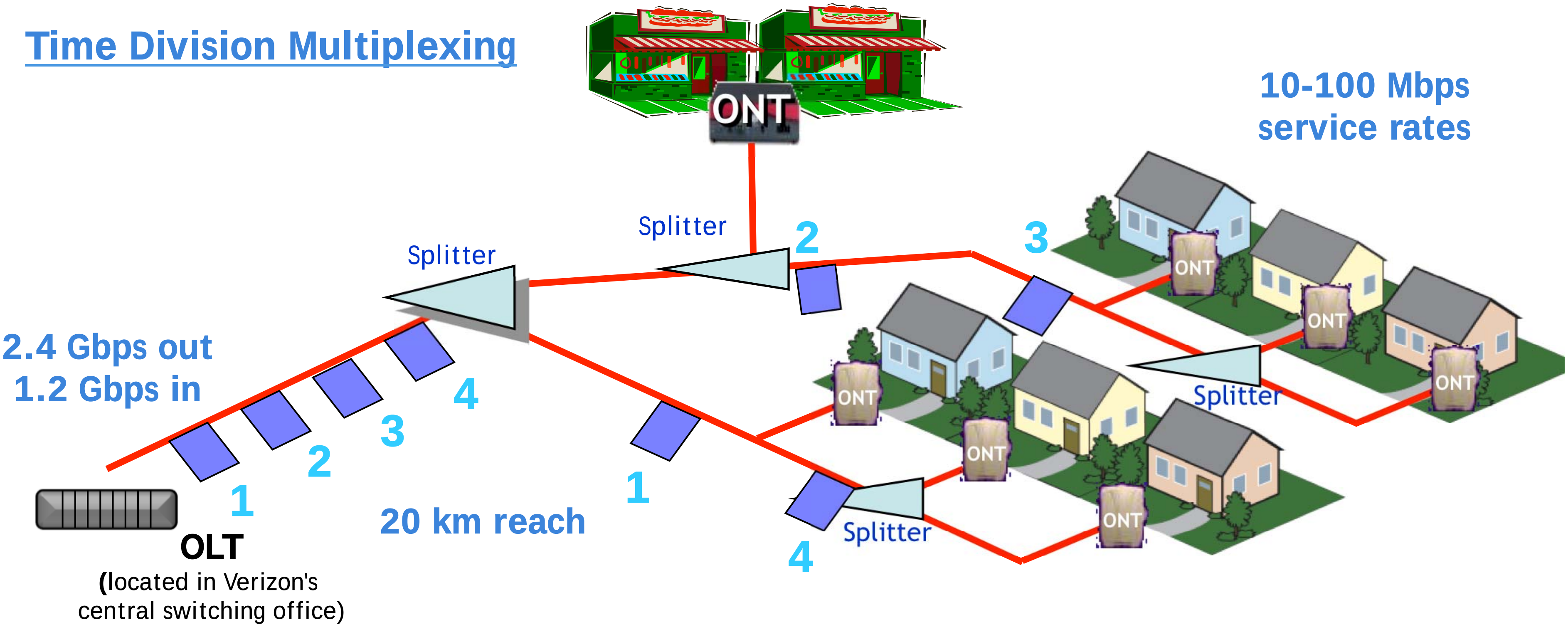
Fiber to the Home



An ONT (Optical Network Terminal) is a media converter that is installed by Verizon either outside or inside your premises, during FiOS installation. The ONT converts fiber-optic light signals to copper/electric signals. Three wavelengths of light are used between the ONT and the OLT (Optical Line Terminal):

- λ = 1310 nm voice/data transmit
- λ = 1490 nm voice/data receive
- λ = 1550 nm video receive

Each ONT is capable of delivering:
Multiple POTS (plain old telephone service) lines, Internet data, Video

Image by uuzinger
http://www.flickr.com/photos/uuzinger/411425452/ on flickr

Fiber to the Home

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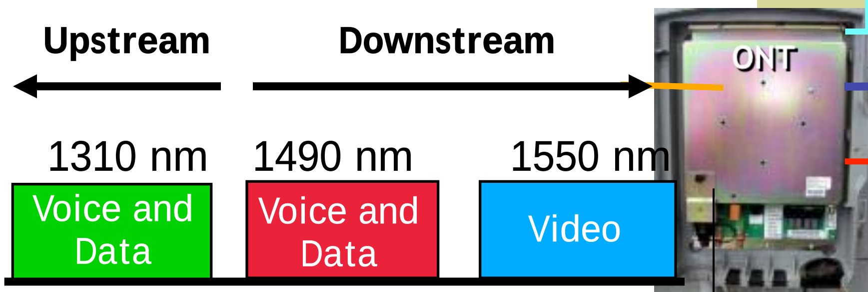
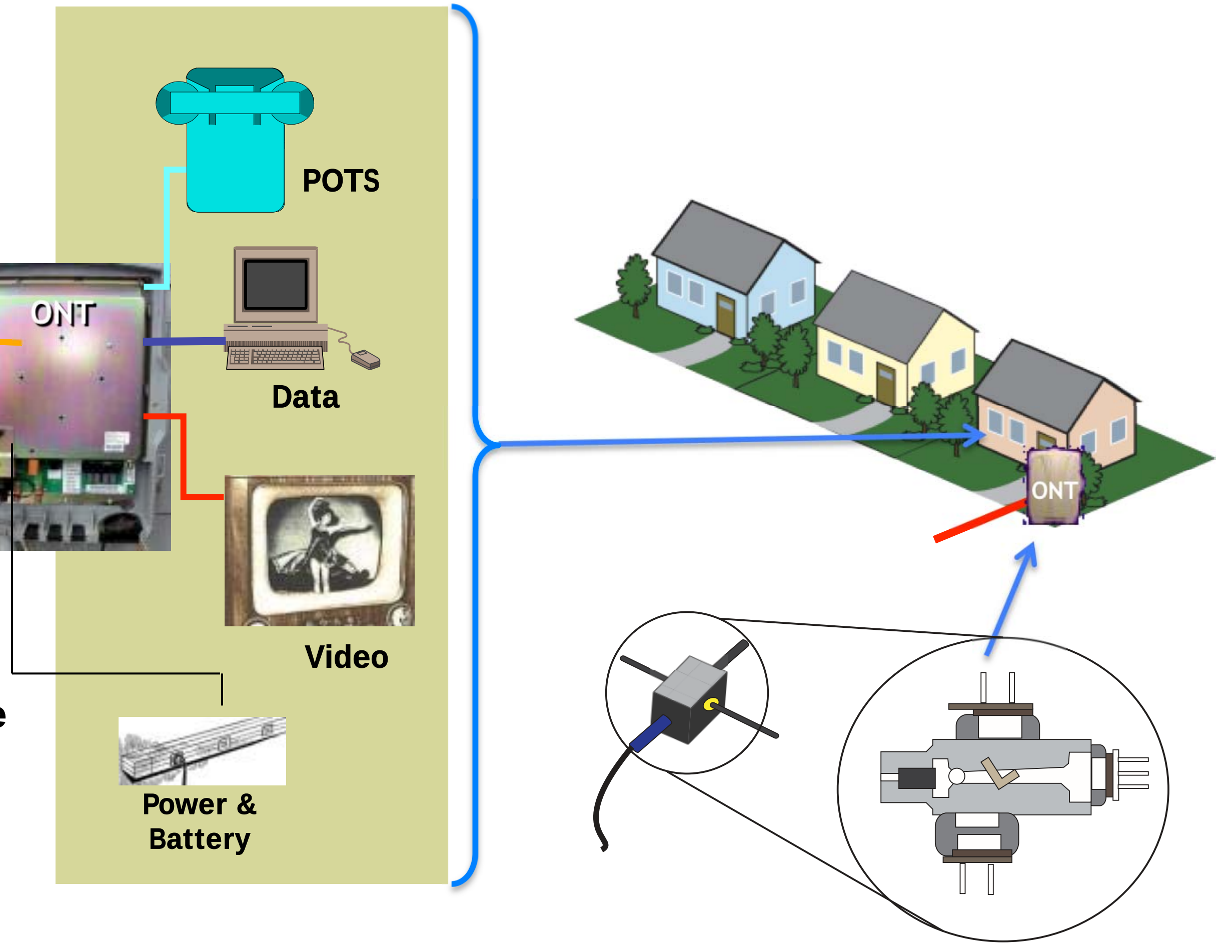
Bandwidths & Services

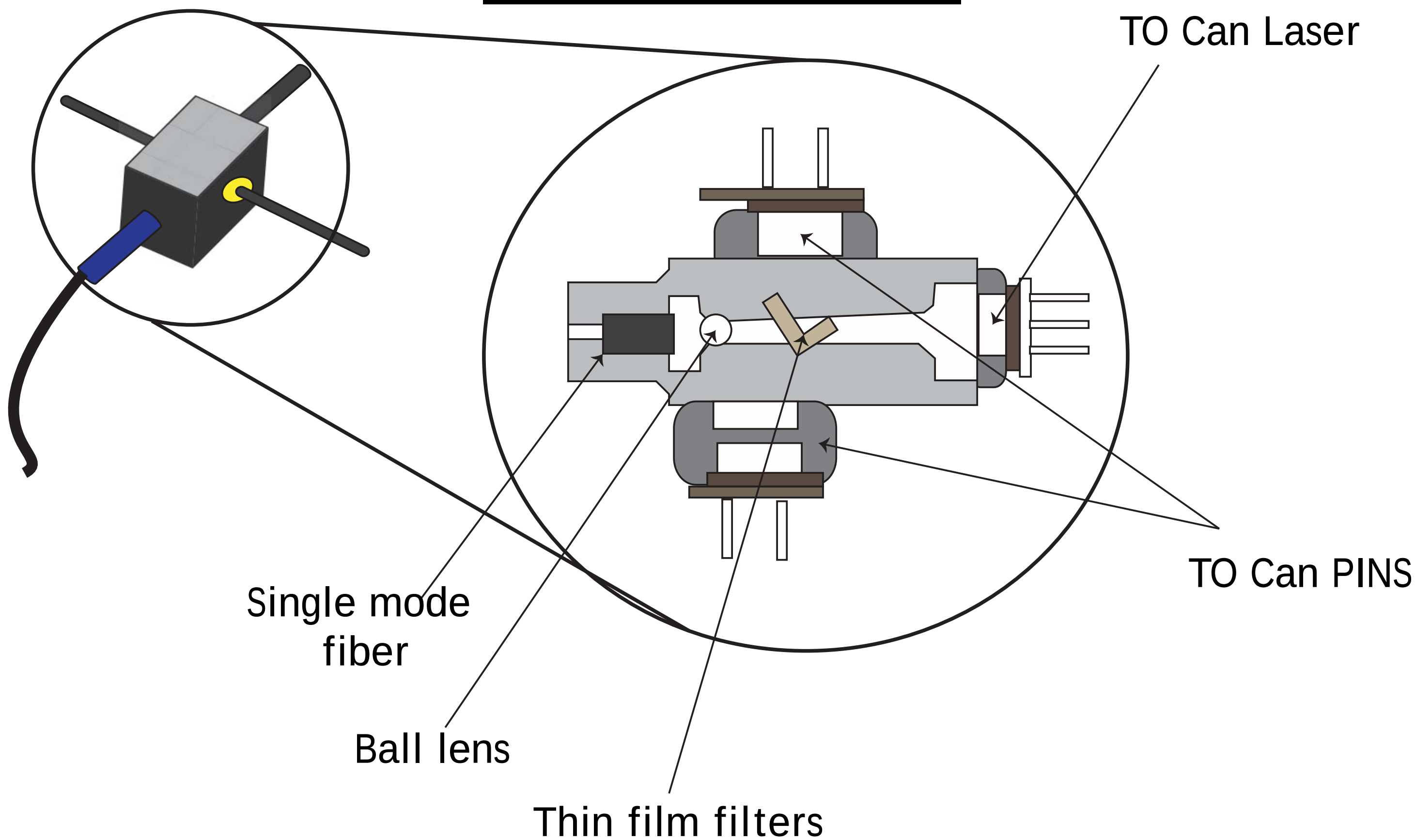




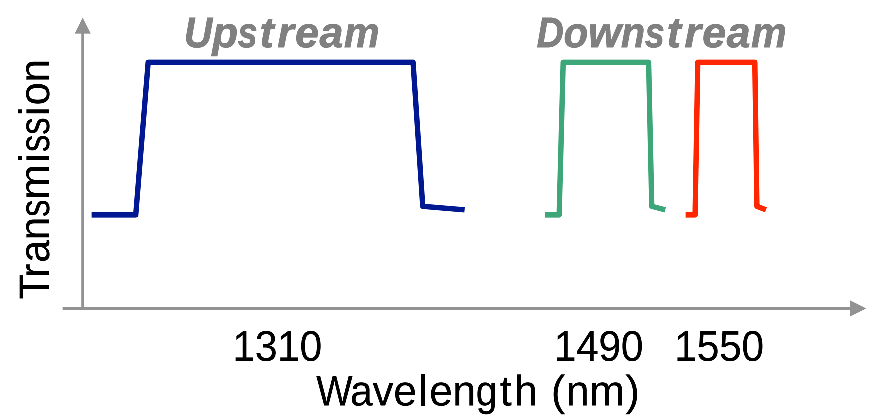
- **Channels downstream to each home**
  λ = 1490 and λ = 1550 nm
- **Channel upstream from each home**
  λ = 1310 nm

Image of ONT by Josh Bancroft
http://www.flickr.com/photos/joshb/87167324/ on flickr

Optical Assembly

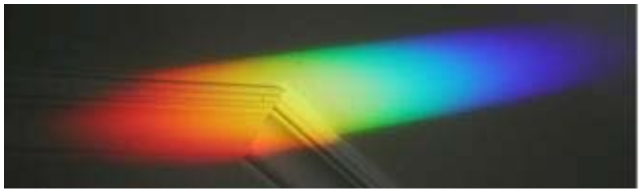


- **Channels downstream to each home**
  - λ = 1490 and λ = 1550 nm
- **Channelλupstream from each home**
  - λ = 1310 nm



Separating Wavelengths

Dispersion



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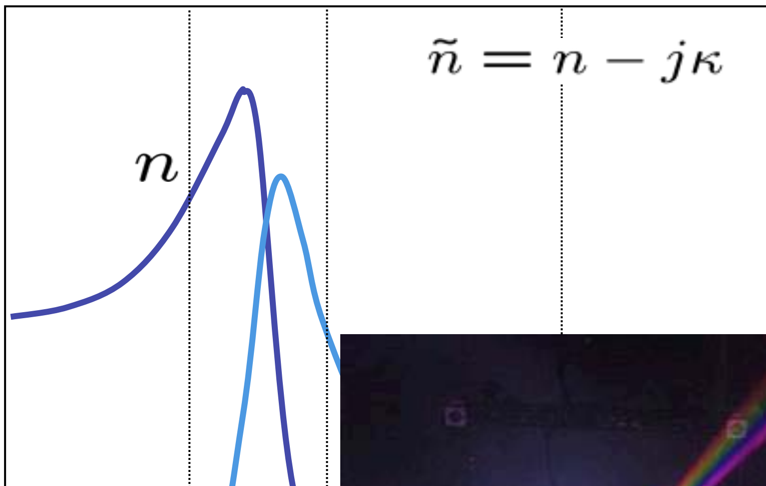

Image by wonker
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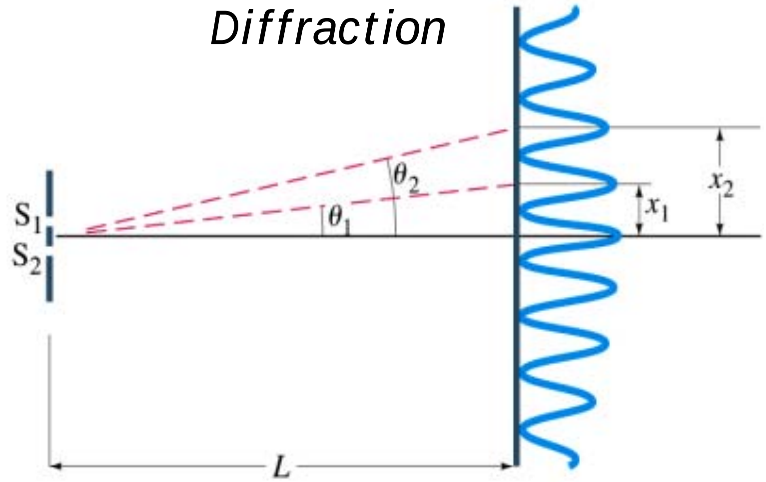
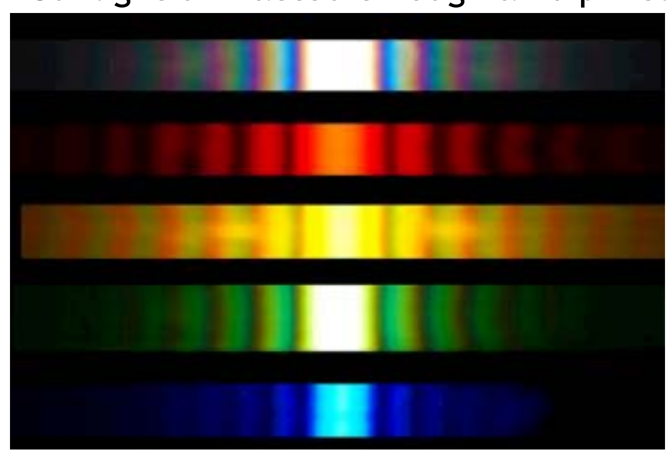
Diffraction



Sunlight diffracted through a 20 μm slit



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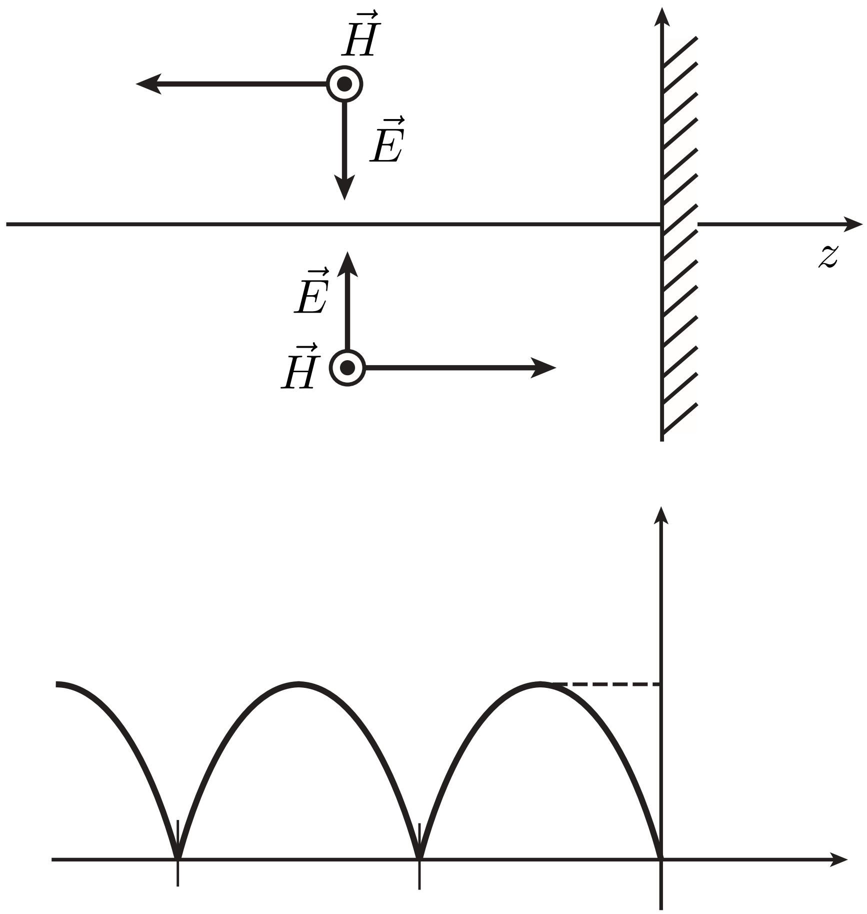
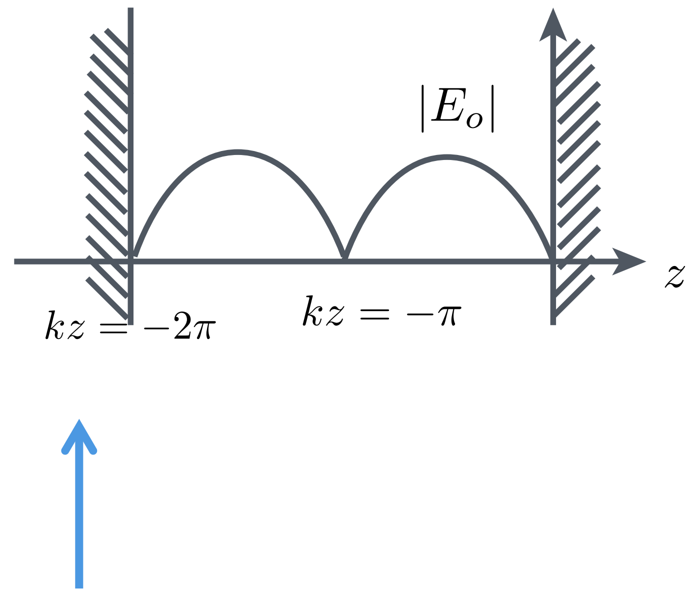
Resonators

STANDING WAVE



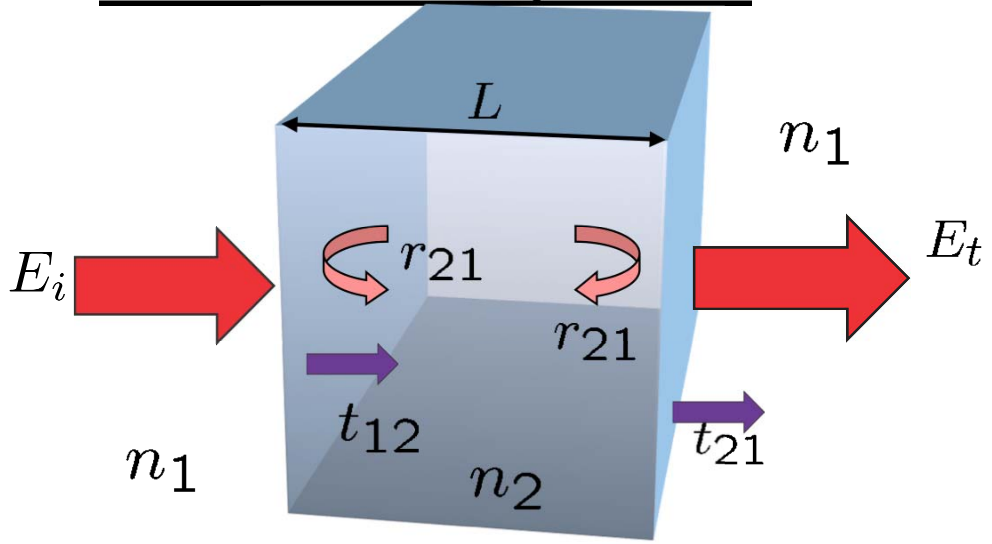
RESONATORS

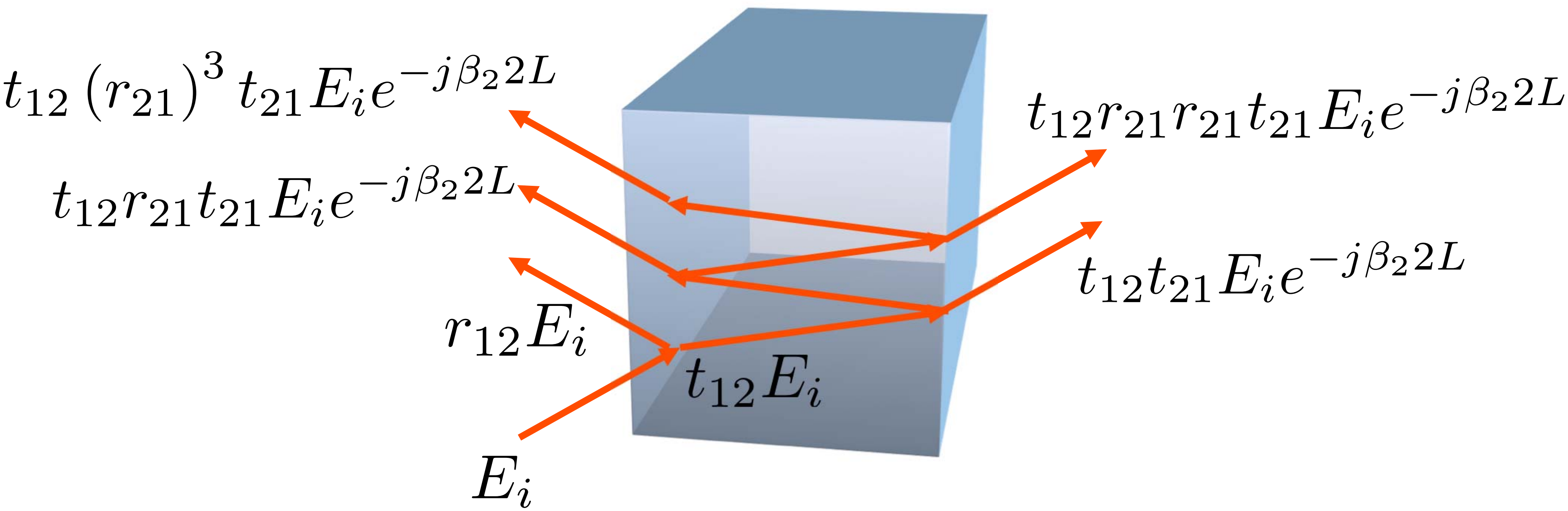


Terminate the standing wave with a second wall to form a resonator

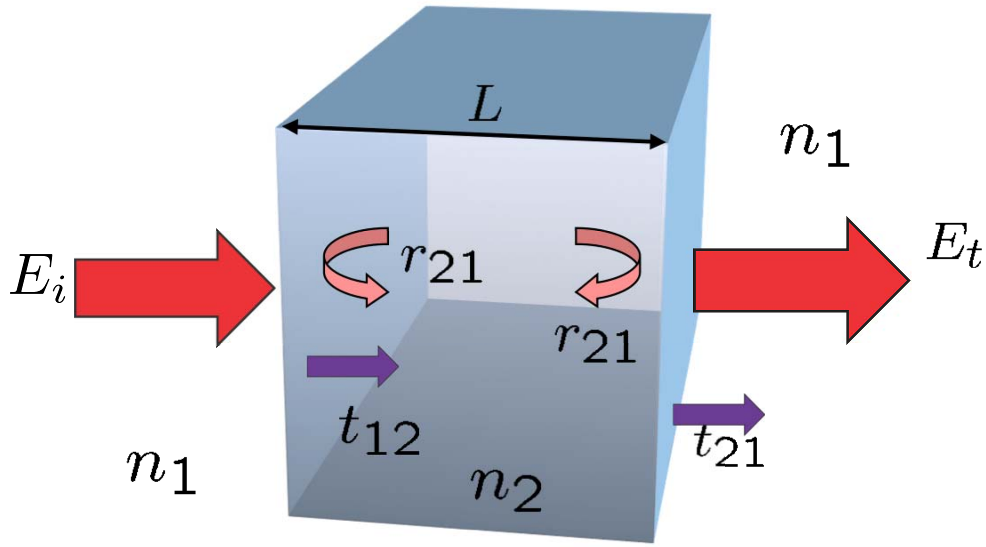
Image by Yoko Nekonomania http://www.flickr.com/photos/nekonomania/4827035737/ on flickr

Thin Film Interference





Optical Resonator



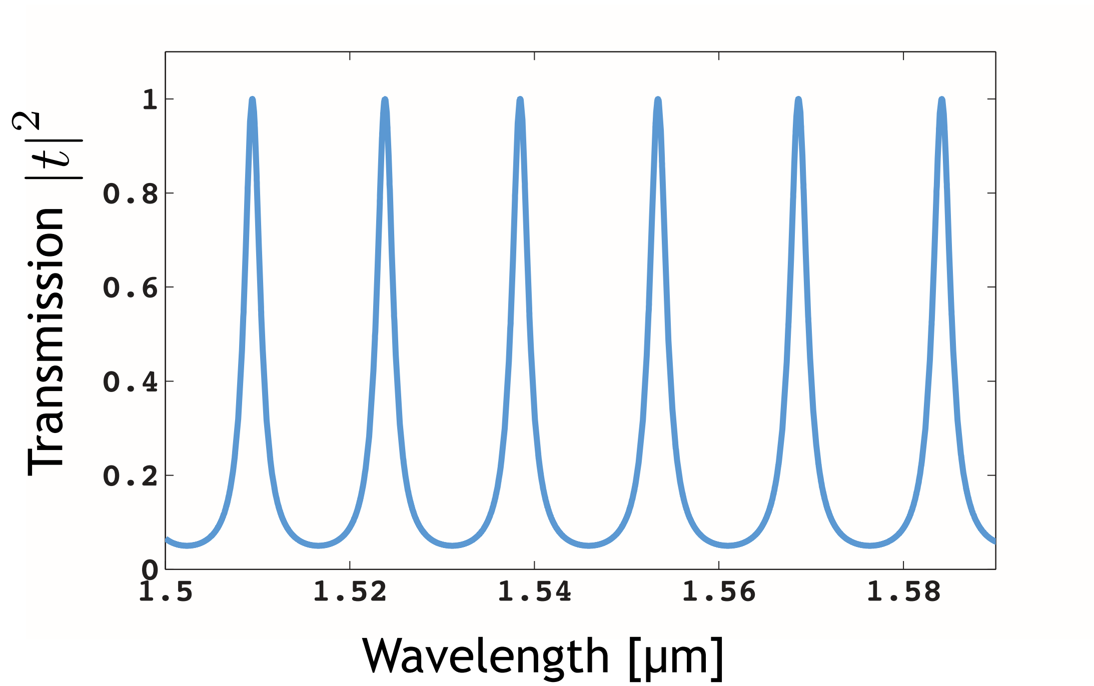
$$E_t = \left[t_{12}t_{21}e^{-j\beta_2 L} + t_{12}e^{-j\beta_2 L}r_{21}e^{-j\beta_2 L}r_{21}e^{-j\beta_2 L}t_{21}...\right] E_i$$

$$= \left[t_{12}t_{21}e^{-j\beta L}\left(1 + r_{12}r_{21}e^{-2j\beta L} + \left(r_{12}r_{21}e^{-2j\beta L}\right)^2 ...\right)\right] E_i$$

$$= \frac{t_{12}t_{21}e^{-j\beta_2 L}}{1 - r_{12}r_{21}e^{-2j\beta L}} E_i$$

Fabry-Perot Resonance

$$t = \frac{t_{12}t_{21}e^{-jkL}}{1 - r_{12}r_{21}e^{-2jkL}}$$



Fabry-Perot Resonance: $\max\{e^{-2jk_2L}\} = 1$ maximum transmission

$\min\{e^{-2jk_2L}\} = -1$ minimum transmission

Total Internal Reflection

Beyond the critical angle, θ_c , a ray within the higher index medium cannot escape at shallower angles

$$n_2 sin\theta_2 = n_1 sin\theta_1 \quad \theta_c = sin^{-1}(n_1/n_2)$$

For glass, the critical internal angle is 42°

For water, it is 49°

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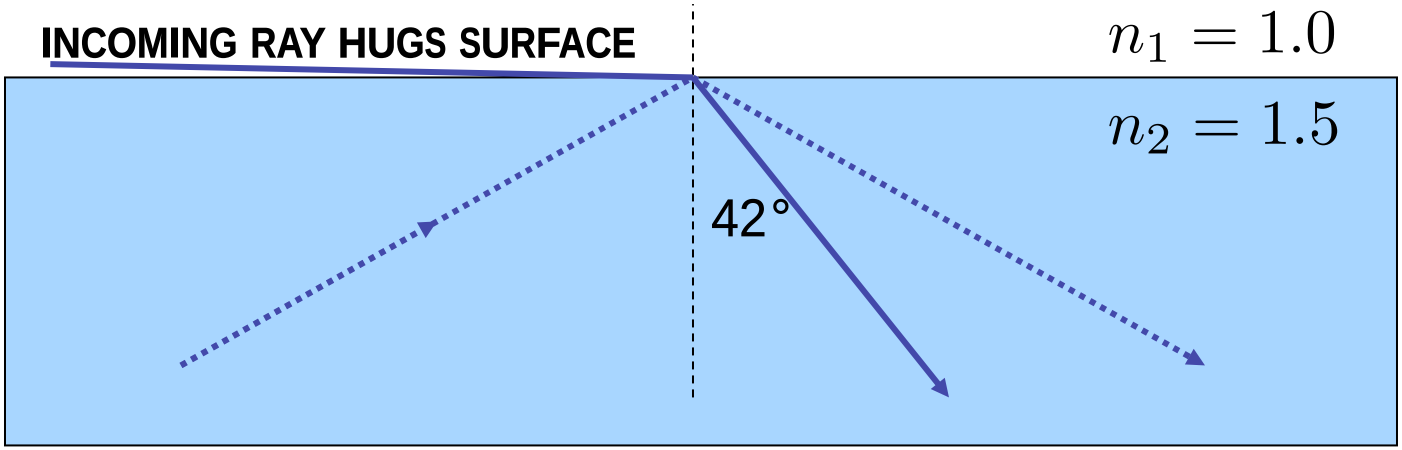
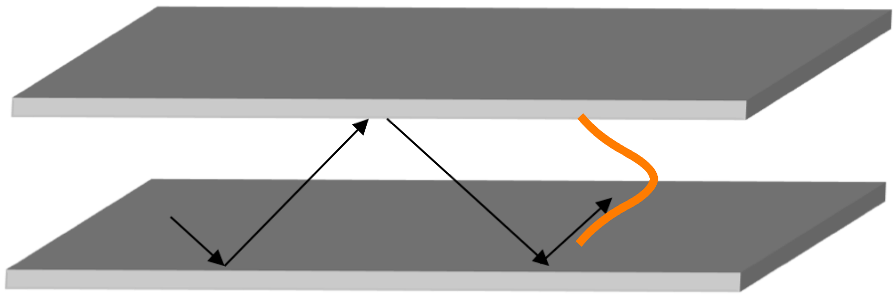
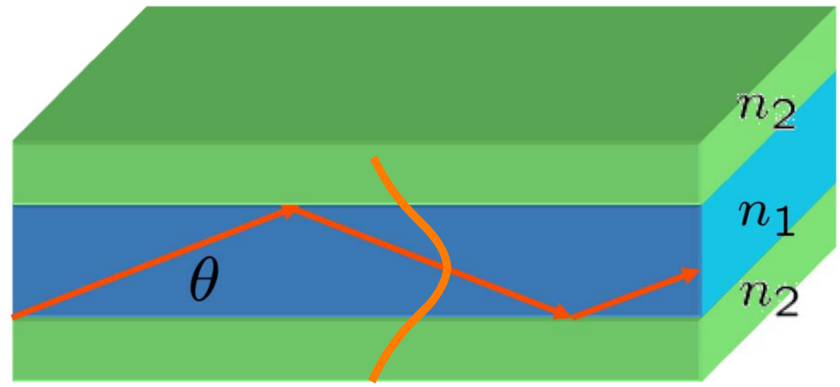


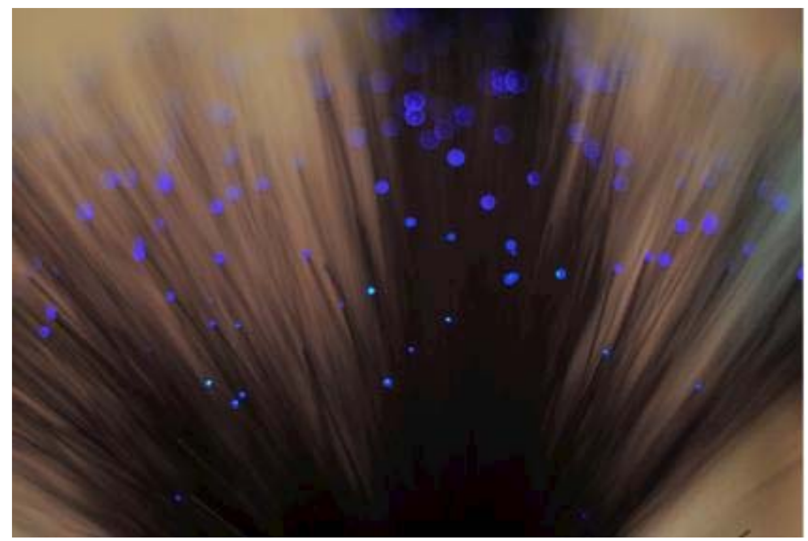
Waveguide Transport Light Between Mirrors

Metal waveguides



Dielectric waveguides





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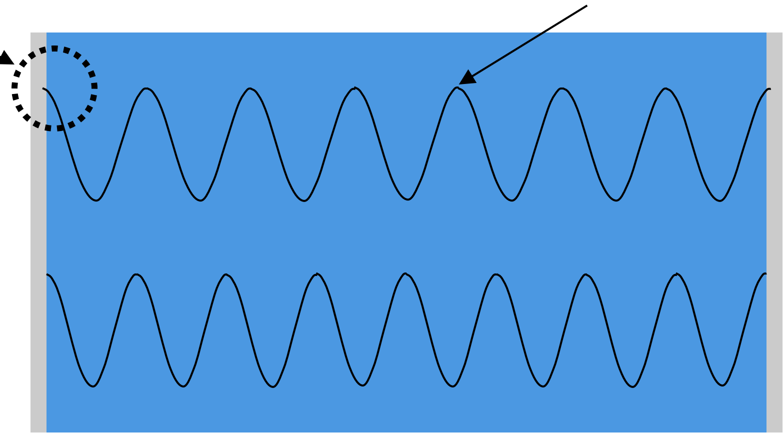
So what kind of waveguide are the optical fibers ?

Fabry-Perot Modes



$$r_{21} = \frac{n_2 - n_1}{n_2 + n_1}$$

$$E_r = rE_i$$

$$\lambda_{o1} = \frac{2n_1 L}{m}$$

$$\lambda_{o2} = \frac{2n_2 L}{m+1}$$

$$\Rightarrow \Delta\lambda_o = 2n_2 L \left(\frac{1}{m} - \frac{1}{m+1} \right) = \frac{2n_2 L}{m(m+1)}$$

$$\lambda_o = 1\mu m, \;\; n_2 = 3.5, \;\; L = 300\mu m$$

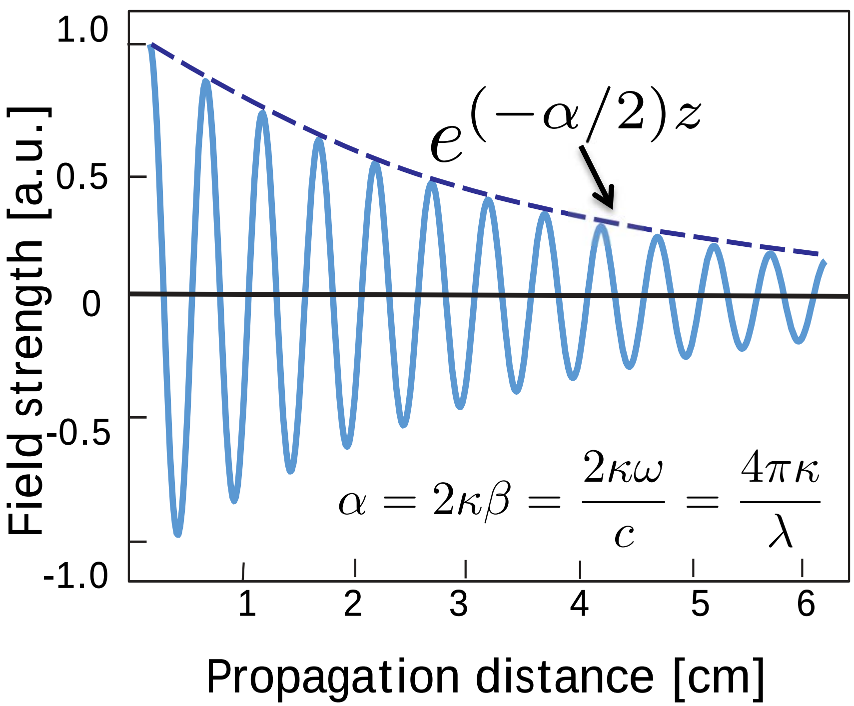
$$\lambda_1 = 1\mu m \Rightarrow m = 2100$$

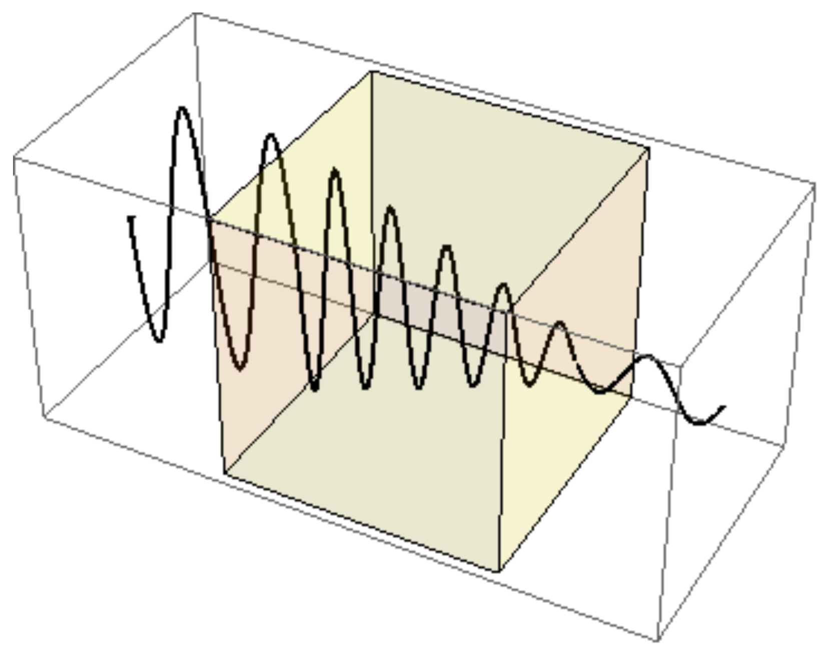
$$\Delta\lambda = 5\mathring{A}$$

Plane Waves in Lossy Materials

$$E_y = \mathrm{Re}\{A_1 e^{j(\omega t \tilde{k} z)}\} + \mathrm{Re}\{A_2 e^{j(\omega t \tilde{k} z)}\}$$

$$E_y(z,t) = A_1 e^{-\alpha/2z} \cos(\omega t - kz) + A_2 e^{+\alpha/2z} \cos(\omega t + kz)$$





Resonators with Internal Loss

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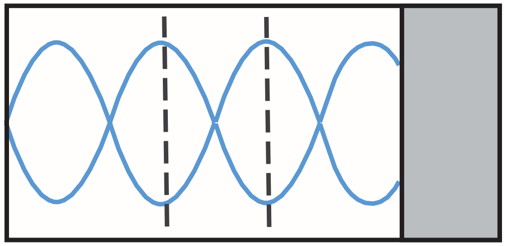


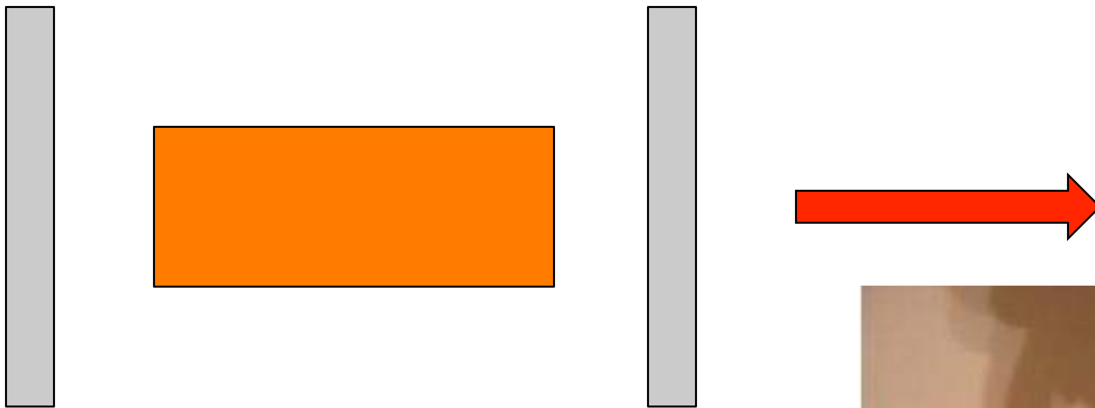
$$\tilde{r} = \frac{\tilde{n}_1 - \tilde{n}_2}{\tilde{n}_1 + \tilde{n}_2}$$

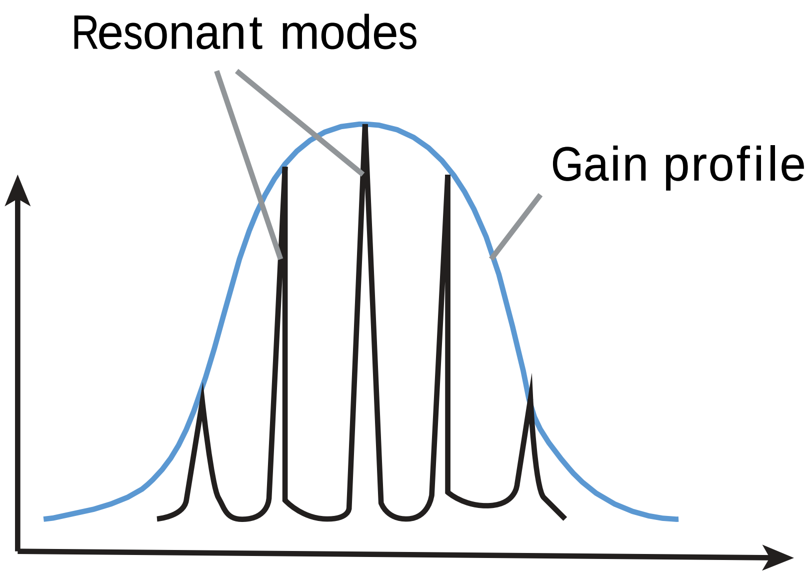
$$\tilde{t} = \frac{2\tilde{n}_1}{\tilde{n}_1 + \tilde{n}_2}$$

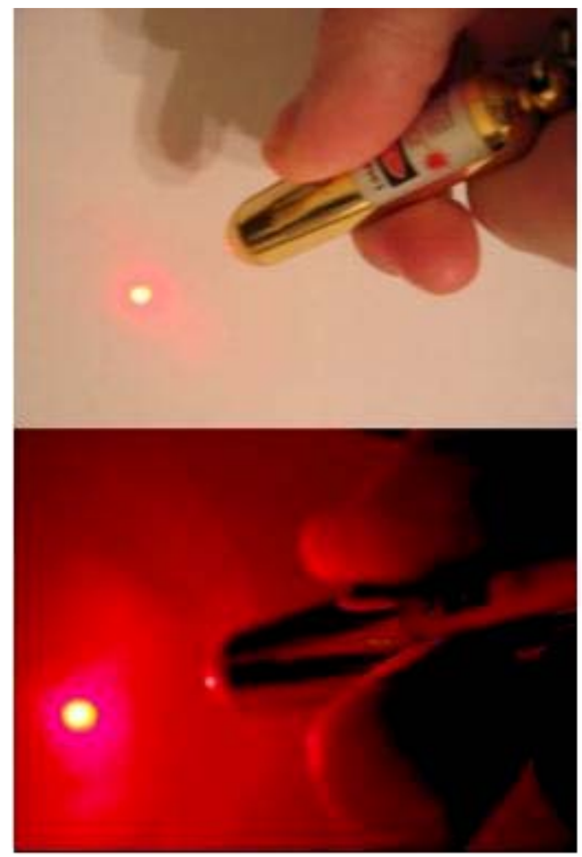
$$\boxed{\frac{E_t}{E_i} = \frac{\tilde{t}_1 \tilde{t}_2 e^{-j\tilde{k}L}}{1 - \tilde{r}_1 \tilde{r}_2 e^{-2j\tilde{k}L}} = \frac{\tilde{t}_1 \tilde{t}_2 e^{-jk_r L} e^{-\alpha L}}{1 - \tilde{r}_1 \tilde{r}_2 e^{-2jk_r L} e^{-2\alpha L}}}$$

…the EM wave loss is what heats the water inside the food

Laser Using Fabre-Perot Cavity







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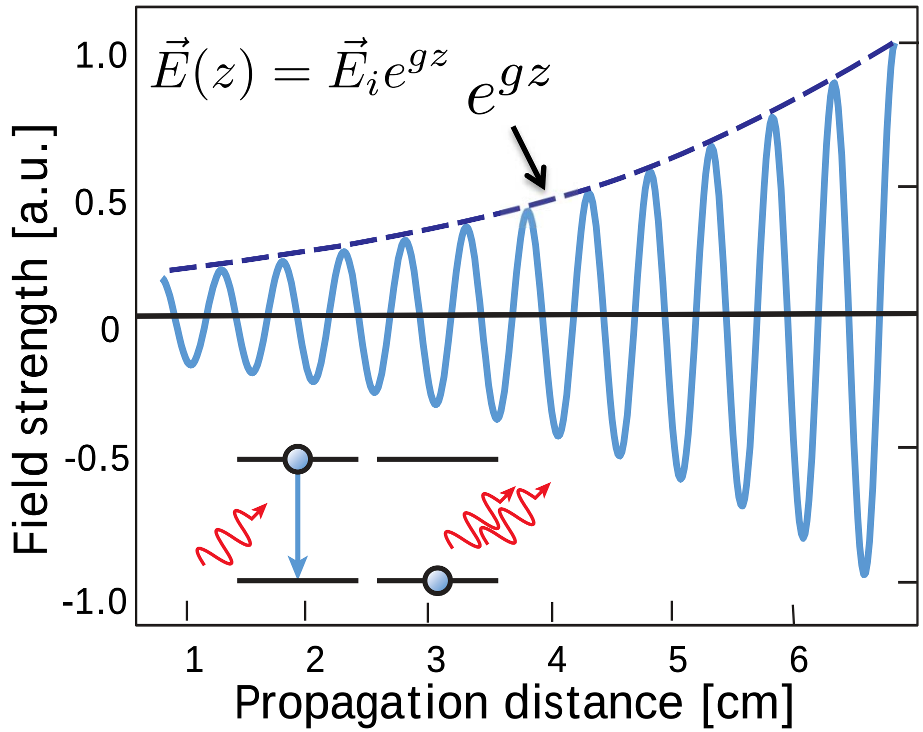
Resonators with Internal Gain

What if it was possible to make a material with "negative absorption" so the field grew in magnitude as it passed through a material?



$$\frac{E_t}{E_i} = \frac{\tilde{t}_1 \tilde{t}_2 e^{-j\tilde{k}L}}{1 - \tilde{r}_1 \tilde{r}_2 e^{-2j\tilde{k}L}} = \frac{\tilde{t}_1 \tilde{t}_2 e^{-jk_r L} e^{-\alpha L}}{1 - \tilde{r}_1 \tilde{r}_2 e^{-2jk_r L} e^{-2\alpha L}}$$

Resonance:

$$e^{2jkL} = 1$$

Lasers: Something for Nothing (almost)

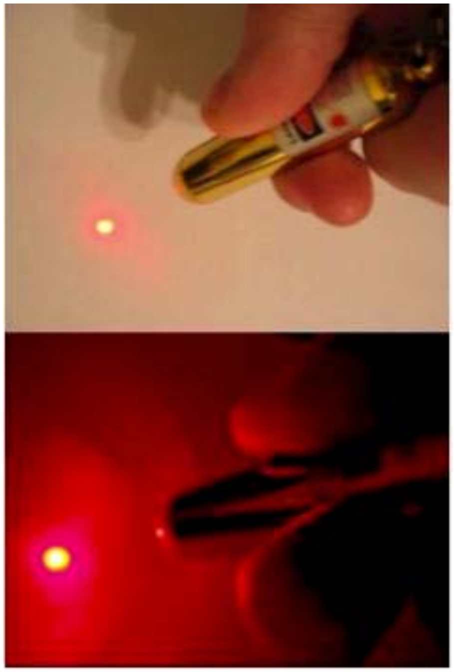
at resonance $e^{2jkL} = 1$

$$\frac{E_t}{E_i} = \frac{\tilde{t}_1 \tilde{t}_2 e^{-j\tilde{k}L}}{1 - \tilde{r}_1 \tilde{r}_2 e^{-2j\tilde{k}L}} = \frac{\tilde{t}_1 \tilde{t}_2 e^{-jk_r L} e^{-\alpha L}}{1 - \tilde{r}_1 \tilde{r}_2 e^{-2jk_r L} e^{-2\alpha L}}$$

singularity at

$$1 = r_1 r_2 e^{\Gamma g L} e^{-\alpha_i L} \Leftrightarrow 1 = R_1 R_2 e^{2\Gamma g L} e^{-2\alpha_i L}$$

$$\frac{E_t}{E_i} \to \infty$$



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6.007 Electromagnetic Energy: From Motors to Lasers
Spring 2011