6.003 (Fall 2009)

Quiz #1 *October 7, 2009*

Name:

Kerberos Username:

Please circle your section number:

Section	*Instructor*	*Time*
1	Marc Baldo	10 am
2	Marc Baldo	11 am
3	Elfar Adalsteinsson	1 pm
4	Elfar Adalsteinsson	2 pm

Partial credit will be given for answers that demonstrate some but not all of the important conceptual issues.

Explanations are not required and will not affect your grade.

You have **two hours.**

Please put your initials on all subsequent sheets.

Enter your answers in the boxes.

This quiz is closed book, but you may use one 8.5×11 sheet of paper (two sides).

No calculators, computers, cell phones, music players, or other aids.

1	/20
2	/20
3	/20
4	/20
5	/20
Total	/100

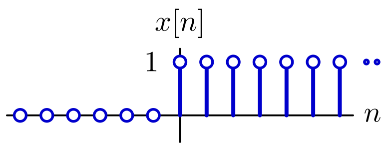
1. Difference equation *[20 points]*

Consider the system described by the following difference equation:

$$y[n] = \alpha x[n] + \beta x[n-1] - y[n-2]\,.$$

a. Assume that the system starts at rest and that the input $x[n]$ is the **unit-step** signal $u[n]$.

$$x[n] = u[n] \equiv \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$



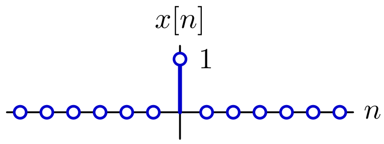
Find $y[119]$ and enter its value in the box below.

$y[119] =$ [ ]

Consider the same system again.

$$y[n] = \alpha x[n] + \beta x[n-1] - y[n-2]$$

b. Let $\alpha = 3$ and $\beta = 4$. Assume that the system starts at rest and that the input $x[n]$ is the **unit-sample** signal.



Determine coefficients A and B so that the response is

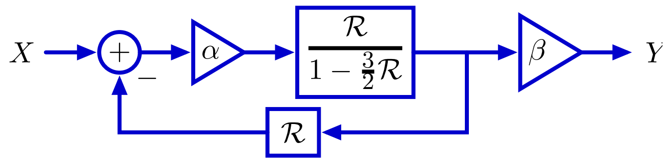
$$Aj^n + B(-j)^n \ ; \quad \text{for } n \geq 0.$$

Enter the coefficients in the boxes below, or enter **none** if no such coefficients can be found.

$A =$ $\boxed{}$ $B =$ $\boxed{}$

2. Feedback *[20 points]*

Consider the following system.



Assume that X is the unit-sample signal, $x[n] = \delta[n]$. Determine the values of α and β for which $y[n]$ is the following sequence (i.e., $y[0]$, $y[1]$, $y[2]$, . . .):

$$0\,,\ 1\,,\ \frac{3}{2}\,,\ \frac{7}{4}\,,\ \frac{15}{8}\,,\ \frac{31}{16}\,,\ \ldots$$

Enter the values of α and β in the boxes below. Enter **none** if the value cannot be determined from the information provided.

$\alpha =$ ☐ $\beta =$ ☐

3. Scaling time *[20 points]*

A system containing only adders, gains, and delays was designed with system functional

$$H = \frac{Y}{X}$$

which is a ratio of two polynomials in $\mathcal{R}$. When this system was constructed, users were dissatisfied with its responses. Engineers then designed three new systems, each based on a different idea for how to modify H to improve the responses.

System H_1: every delay element in H is replaced by a cascade of two delay elements.

System H_2: every delay element in H is replaced by a gain of $\frac{1}{2}$ followed by a delay.

System H_3: every delay element in H is replaced by a cascade of three delay elements.

For each of the following parts, evaluate the truth of the associated statement and enter **yes** if the statement is always true or **no** otherwise.

a. If H has a pole at $z = j = \sqrt{-1}$, then H_1 has a pole at $z = e^{j5\pi/4}$.

Statement is always true (**yes** or **no**): [ ]

b. If H has a pole at $z = p$ then H_2 has a pole at $z = 2p$.

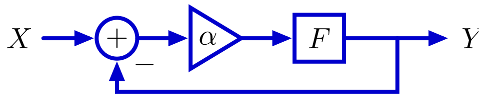
Statement is always true (**yes** or **no**): ☐

c. If H is stable then H_3 is also stable (where a system is said to be stable if all of its poles are inside the unit circle).

Statement is always true (**yes** or **no**): ☐

4. Mystery Feedback *[20 points]*

Consider the following feedback system where F is the system functional for a system composed of just adders, gains, and delay elements.



If $\alpha = 10$ then the closed-loop system functional is known to be

$$\left.\frac{Y}{X}\right|_{\alpha=10} = \frac{1+\mathcal{R}}{2+\mathcal{R}}$$

Determine the closed-loop system functional when $\alpha = 20$.

$$\left.\frac{Y}{X}\right|_{\alpha=20} = \boxed{}$$

5. Ups and Downs *[20 points]*

Use a small number of delays, gains, and 2-input adders (and no other types of elements) to implement a system whose unit-sample response ($h[0]$, $h[1]$, $h[2]$, $\ldots$) (starting at rest) is

1,2,3,1,2,3,1,2,3,...

Draw a block diagram of your system below.