Problem Set 7

14.462 Topics in Macro

Spring 2007

Problem 1
Consider the model in Lorenzoni and Walentin, that we have seen in class (see slides for notation). Consider a simplified version where $\gamma = 0$ and $\beta_E = \beta_C = \beta$ and $\xi = 0$ (no adjustment costs). The production function is Cobb-Douglas.
Consider the case where there is no aggregate shocks: $A = 1$. Suppose the initial condition for the economy is that the entrepreneurs have initial capital K_0 and zero liabilities $B_0 = 0$. We will study the transitional dynamics of this economy. We will use the same approach as in the paper defining a recursive equilibrium, let's try the simple state space $X = K$ (this is almost right, except at date 0, you can fix this problem as you prefer).

1. Define the entrepreneurs problem in recursive term and derive the optimality conditions and envelope condition.

2. Let K^* be such that $\beta\left[(K^*)^{\alpha-1} + 1 - \delta\right] = 1$. Let us conjecture that $H(K)$ (the law of motion for aggregate capital) is a non-decreasing function and that $\hat{K}$ is such that $H\left(\hat{K}\right) = K^*$. Argue that $\phi(K)$ is a non-increasing function, with $\phi(K) = 1$ for $K \geq \hat{K}$, and show that the optimal contract involves

$$k' = \frac{(1-\theta) R(H(K)) k}{1 - \beta R(H(K))}$$

for $K < \hat{K}$, and is indeterminate if $K \geq \hat{K}$.

3. Show that in equilibrium the economy converges in finite time to K^*.

4. Suppose $\gamma > 0$ (there is a positive probability of death), show that the conclusion in (3) survives.

5. Suppose $\gamma > 0$ AND the entrepreneurs' debt is not allowed to be contingent on death ($b_L = b$ in the notation of the paper). Show that the model can have a steady state with $K < K^*$.

Problem 2

Consider the same model as in Problem 1. Now we allow for a binary, i.i.d. productivity shock. Each period with probability π^h the productivity is A^h and with probability π^l is A^l. The initial condition is still K_0 and zero liabilities.

1. Define a recursive equilibrium with the appropriate state space (I suggest you still use only K and s, instead of K, B and s, and fix the first period at the end).
2. Define the entrepreneur's problem and derive the first order conditions.
3. Construct a recursive equilibrium where $\phi(K, h)$ and $\phi(K, l)$ are both decreasing functions and where $\phi\left(\hat{K}^s, s\right) = 1$ for two cutoffs $\hat{K}^s$, $s = l, h$.
4. Show that in equilibrium there is a region (K_1, K_2) such that the optimal contract has
$$b'(s')(k, X) < \theta R(H(X))$$
and
$$\phi(X) = \phi(H(X, s'))$$
if $X = (K, s)$, $s = l$, and $s' = h$.
5. Characterize the economy's dynamics, showing that (in some sense) volatility falls as the economy grows.