14.454 - Macroeconomics IV
Problem Set 2

04/24/2011

Due: Monday 5/02/2011

1   Question 1 - Allen & Gale (2000)

Take an economy as in the Diamond-Dygvig model seen in class, where agents have utility function:

$$U(c_1, c_2) = \begin{cases} u(c_1) & \text{with probability } \omega \\ u(c_2) & \text{with probability } 1-\omega \end{cases}$$

as we have seen in class, where $u(c) \equiv \frac{c^{1-\sigma}}{1-\sigma}$ with $\sigma > 1$. We will call the event in which the agent only derives utility from $t = 1$ consumption a "liquidity shock". However, in this model ω is a random variable, that takes values in $\{\omega_L, \omega_H\}$ with $0 < \omega_L < \omega_H < 1$. Moreover, the economy is divided in four different regions : A, B, C, D, each with a continuum of measure 1 of consumers, and each of them may have different realizations of the probability parameter ω. Let ω^j be a particular realization of ω in region $j \in \{A, B, C, D\}$

More specifically, there is a random variable $S \in \{S_1, S_2\}$ that governs the process for ω :

- With probability $\frac{1}{2}$, $S = S_1$ and $\omega^A = \omega^C = \omega_H$ and $\omega^B = \omega^D = \omega_L$

- With probability $\frac{1}{2}$, $S = S_2$ and $\omega^A = \omega^C = \omega_L$ and $\omega^B = \omega^D = \omega_H$

This can be more easily seen in the following table

	A	B	C	D
S_1	ω_H	ω_L	ω_H	ω_L
S_2	ω_L	ω_H	ω_L	ω_H

See that in the aggregate there's no risk: this model is identical to the one seen in class, in which the probability of being impatient is just $\gamma \equiv \frac{1}{2}(\omega_H + \omega_L)$
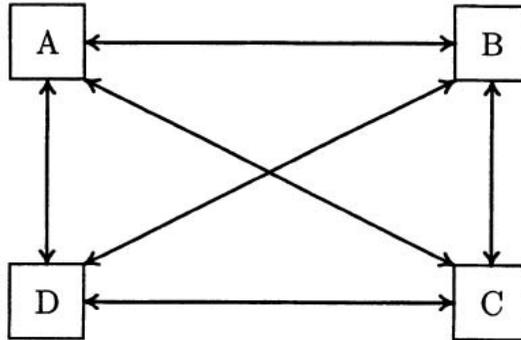
(a) Set up and solve the Planner's problem of this economy, finding bundles $\left(c_1^j(S), c_2^j(S)\right)$ with $j \in \{A, B, C, D\}$ (which may depend on the state of nature realized). Show that

$$c_1^j(S) = c_1^* \text{ for all } j, S \text{ and } c_2^j(S) = c_2^*$$

where c_1^* and c_2^* are the First Best consumption levels of the Diamond-Dygvig model, where $\Pr(\text{agent } i \text{ is impatient}) = \gamma$. Also, find x^* and y^*, the first best investment levels in the storage and long technologies respectively.
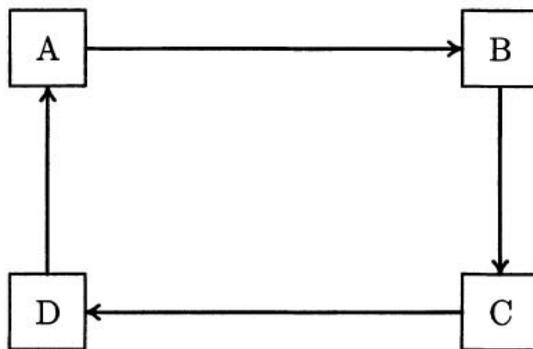
(b) Suppose that in each region, there's a bank that offers deposit contracts, as in the Diamond-Dygvig model. Show that the First Best (c_1^*, c_2^*) cannot be implemented. (**Hint:** The bank must be able to make the payments **in all states of nature**)

(c) Now imagine that there's a market of interbank deposits: at $t = 0$ banks can make deposits on any banks in the other regions of the economy. More explicitly, a bank on region j can make deposits $z_{i,j} \geq 0$ on banks of region $i \neq j$ at $t = 0$, and can withdraw any fraction of those deposits at any time. The financial network in this economy can be expressed in the following picture

where the arrows between regions i and j mean that banks in region i can make deposits in banks of region j. We will say that this market structure is "Complete" (because banks can make deposits in banks of all other regions). Find deposits $z_{i,j}$ such that the first best can be implemented (**Hint:** Try a case in which $z_{i,j} = \overline{z}$ for all $i \neq j$)

(d) Now suppose that banks cannot make deposits in any region, but rather that they can make deposits only on neighboring regions, as shown in the following figure

So, for example, banks in region A can only make deposits in banks of region B, and banks in region B can only make deposits in banks of region C (this market structure is similar to the one in Caballero and Simsek (2011)). We will call this market structure "incomplete". Show that in this case, we can also implement the first best, and find the associated deposits $z_{i,j}$ of each region. (**Hint:** try $z_{i,j} = \omega_H - \gamma$)

(e) Suppose the interbank market is incomplete (as in **(d)**) with the implementation suggested, and imagine now that there is an unforeseen state $\overline{S}$ which is perceived to have zero probability (so it does not affect the investment decisions nor deposit decisions at $t = 0$, nor the first best levels of consumption (c_1^*, c_2^*)). In the following table we see how liquidity shocks are affected by this new state:

	A	B	C	D
S_1	ω_H	ω_L	ω_H	ω_L
S_2	ω_L	ω_H	ω_L	ω_H
$\overline{S}$	$\gamma + \epsilon$	γ	γ	γ

So there will be asymmetries across regions: in particular, only region A has a higher need for funds than all the other regions. We will make 3 more assumptions:

1. Assume that σ is such that $1 < \frac{c_2^*}{c_1^*} < \frac{R}{L}$, so that banks that need to liquidate assets to make payments to early consumers, prefer to liquidate interbank deposits first and the long asset second.

2. If a bank goes bankrupt (i.e. it does not have enough funds to make payments to consumers and banks) then it pays each depositor a proportional share on the liquidation of its assets (so consumers and banks are treated equally)

3. For simplicity, assume that there will be bank runs if and only if it is optimal for late consumers to do so; i.e. there are bank runs only if $c_2 < c_1$ after liquidation.

Prove Proposition 2 in Section V in Allen F and Gale, D; "Financial Contagion", *The Journal of Political Economy*, Vol. 108, No. 1 (Feb., 2000), pp. 1-33

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