Final Exam: Macroeconomics 14.453

Iván Werning

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You have 2 hours to complete the exam. **Do all the short questions and choose 2 out of the 3 longer questions – do not turn in answers to more than 2 of the longer question!**

Please write your answer to the Shorter questions in the space provided and use your blue book to answer the 2 longer problems. Good luck!

1 Shorter Questions (40 minutes)

1. True or False? Justify your answer. Consider a consumer with uncertain future income that can save at a constant interest rate r and has a concave utility function $u(c)$ satisfying $u'''(c) > 0$. Suppose the individual experiences an unexpected windfall increase in assets. Then due to precautionary savings this consumer would save a higher fraction of this windfall than the CEQ-PIH would predict.

2. True or False? Justify your answer. Consider running the cross sec-

tional regression:

$$\log g^i = \alpha_0 + \alpha_1 \log \left(1 + r^i\right) + u^i$$

where g^i is gross consumption growth and r^i is the net interest rate between two dates for individual i. Assume net interest rates differ permanently across individuals or groups because of exogenous tax differences. The hope is that α_1 identifies the elasticity of substitution preference parameter, σ. However, an income fluctuations model as in Aiyagari (1994) implies that this regression, at a steady state, should yield $\alpha_1 = 0$, independently of the value σ. Thus, in this sense, estimating this equation does not identify σ.

3. Consider a representative agent endowment economy with preferences $E \sum_{t=0}^{\infty} \beta^t u(c_t)$ with a stochastic endowment process $\{y_t\}$. At time t the only information available is the history of income shocks.

(a) Find a formula for the gross short term interest rate process $R_t = 1+r_t$ in terms of the $\{y_t\}$ process. In general, is R_t constant?

(b) Assuming preferences are CRRA $u(c) = c^{1-\sigma}/(1-\sigma)$ find conditions on the $\{y_t\}$ process that ensure that R_t is constant.

(c) Suppose we attempted to test the Euler equation prediction for this economy but assumed in our calculations a constant real interest rate. That is, we test whether or not:

$$1 = \tilde{R}E_t\left[\frac{u'(c_{t+1})}{u'(c_t)}\right]$$

holds for some $\bar{R}$. Should we expect to accept or reject this equation?

4. Suppose preferences are given by,

$$\sum_{t=0}^{\infty} \beta^t \log\left(\frac{c_t}{h_t^\gamma}\right)$$

with $\gamma < 1$, and where h_t is the stock of habits accumulated up to time t,

$$h_t = h_{t-1}^{1-\delta} c_{t-1}^{\delta}$$

(note that if consumption is constant then eventually h_t converges to this constant value). Show that these preferences are equivalent to

$$\sum_{t=0}^{\infty} \beta^t \log(c_t)$$

as long as β, γ are low enough or δ is high enough.

2 Investment with Fixed Costs (40 minutes)

Consider the following problem of a firm under uncertainty in discrete time. At time t current profits net of user costs of capital are given by $\pi(k_t, z_t)$ where $z_t \in Z$ is the current shock to profits and k_t is the amount of capital used during period t. The function $\pi(k, z)$ is assumed to be strictly concave in k. Assume z_t follows a Markov process.

At the beginning of the period the current shock z_t is realized and the firm observes it. The firm then decides whether or not to adjust its capital stock.

If the firm decides not to adjust its capital then its capital remains at the previous level k_{t-1} – we assume no depreciation. If instead the firm chooses to adjust its capital then it pays a fixed cost $c(z_t)$ and selects a new capital level k_t. The costs may depend on the shock z_t but *do not* depend on the new level of capital k_t chosen.

The firms objective is to maximize expected discounted profits net of any costs incurred for changing its capital stock. The firm discounts profits using a constant interest rate r.

a. Argue that k_{t-1}, s_t is the relevant state variable for a firm trying to decide whether or not to adjust capital. Set up the Bellman equation for the value of the firm $v(k_-, z)$ where k_- represents the previous period's capital stock and z represents the current shock.

b. Use your Bellman equation from (a) to argue that the optimal policy can be summarized by a region of inaction $K(z)$ and an optimal adjustment policy $k^*(z)$. For each $z \in Z$ the region of inaction $K(z)$ is a subset of $\mathbb{R}_+$ such that adjustment occurs in the current period if and only if $k_- \notin K(z)$. If adjustment occurs then the firm chooses $k^*(z)$ as its new capital stock. (note: you are not asked to actually solve for $K(z)$ or $k^*(z)$ just to show that the solution takes this form).

c. Is $k^*(z)$ equal to the unconstrained optimum level of capital, i.e. $k^*(z) = \arg\max_k \pi(k, z)$?

d. Can the region of inaction $K(z)$ always be summarized by "sS rules", that is that for each state of the world the inaction region is a single closed interval so that there exists functions $\bar{k}(z)$ and $\underline{k}(z)$ such that

$$K(z) = \{k_- : \bar{k}(z) \geq k_- \geq \underline{k}(z)\}?$$

Why or why not?

e. Show that if the growth rate of z is i.i.d. (so that $z'/z = \varepsilon$ i.i.d.) and $\pi(k, z)$ and $c(z)$ are homogenous of degree one (constant returns to scale) so that $\pi(k, z) = s\pi(k/z, 1)$ and $c(z) = \bar{c}s$ then $v(k_-, z)$, $k^*(z)$, $K(z)$ are homogenous of degree one.

Define $\hat{k}_- = k_-/z$. What do these results imply about the region of inaction $\hat{K}(z)$ and the optimum adjustment $\hat{k}^*(z)$ for $\hat{k}_-$?

f. Consider an industry with a continuum of ex-ante identical firms facing their own idiosyncratic z-shocks that are independent over time and across firms. Suppose these firms use sS rules for k_- as described in part (d). Briefly discuss how you would think of a steady state in such an economy – what kind of object is a possible equilibruim and what does it have to satisfy? What determines the steady state level of aggregate investment?

3 The American Dream (40 minutes)

A consumer has preferences:

$$\int_0^\infty e^{-\rho t} u(c(t))\, dt + e^{-\rho T} V$$

where $u(c)$ is an increasing concave differentiable utility function over non-durable consumption c. Here T represents the moment the agent buys a house – an indivisable durable good which yields a stock utility V from its ownership. We allow "$T = \infty$" and interpret this as never buying a house.

The value of a house is constant over time and equal to $H > 0$. The consumer receives a constant labor income flow y, owns initial assets a_0 and faces a constant interest rate $r = \rho$. We assume initially that the consumer can borrow and lend freely at this interest rate.

(a) Derive the present value budget constraint that the consumption path, $c(t)$, and the time of the house purchase, T, must satisfy.

(b) Using the change in variables $x \equiv e^{-rT}$ so that $x \in [0, 1]$ set up the Lagrangian for the consumer's problem over x and $c(t)$. Characterize fully the optimal $x \in [0, 1]$ and path $c(t)$ as a function of the parameters y and a_0.

(c) We now impose the following borrowing constraint: if the consumer buys a house at time T then his accumulated wealth at time T must be greater or equal to H. Write this constraint as an inequality involving T and $c(t)$ for $t \leq T$ only (as well as the parameters r, y and H). Rewrite this constraint in terms of x and $c(t)$ for $t \leq T$.

(d) Using the change in variable $x = e^{-rT}$ set up the the consumer's problem as maximizing utility subject to the present value constraint found in (a) and the borrowing constraint derived in (c).

Set up the associated Lagrangian and derive the necessary first order conditions for $x \in [0, 1]$ and for an interior $c(t)$. For $0 < x < 1$ be careful to consider separately the f.o.c. for $c(t)$ for (i) $t < T$ (before buying a house); (ii) $t \geq T$ (after buying the house).

(e) Show that there is *always* a solution to the first order condtions with $x = 0$ and that there may be another solution with $x > 0$. Characterize these paths as much as possible. Show that solutions with $x > 0$ involve a discontinuity in consumption at T. Why?

(f) We now compare the two possible solutions to the first order conditions found in (e). For some given y, let $V(a_0)$ and $W(a_0)$ denote the lifetime utility attained as a function of a_0 of using the $x = 0$ plan and the $x > 0$ plan (if one exists given a_0 and y), respectively.

Characterize $V(a_0)$ and $W(a_0)$ and show that there exists a cutoff level of initial assets a_0^* such that $V(a_0) \leq W(a_0)$ if and only if $a_0 \geq a_0^*$. Thus, the consumer buys a house eventually if $a_0 \geq a_0^*$ and never buys a house if $a_0 < a_0^*$ (note: a_0^* depends on y and may be zero).

4 Hand-to-Mouth Workers and Growth (40 minutes)

Consider the following variation of the standard neoclassical growth model. Half of the population, the 'hand-to-mouth' consumers (type 1), simply consume any labor income they earn each period – they never own any assets whatsoever. The other half, the 'savers' (type 2), have preferences and choices as in the standard neoclassical model. There is no population growth and we conveniently normalize the total population to be (a continuum) of size 2.

The preferences for the savers are standard:

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t^2\right) \tag{1}$$

for some $\beta < 1$, and u twice continuously differentiable, increasing and strictly concave with the INADA condition $\lim_{c \to 0} u'(c) = \infty$.

Technology is given by the production function $Y_t = K_t^{\alpha} L_t^{1-\alpha}$. Labor, L_t, is supplied inelastically by both types of agents with total labor supply normalized to 1 – the savers and the hand-to-mouth agents each supply 1/2 unit. The resource constraint is,

$$C_t + K_{t+1} = Y_t + (1 - \delta) K_t$$

where $C_t \equiv c_t^1 + c_t^2$ is aggregate consumption and c^1 and c^2 represents consumption of hand to mouth consumers savers, respectively.

(a) Set up the standard description of markets for labor and capital, stating the budget constraints faced by savers and hand to mouth consumers, the (static) problem of the firm. Define a competitive equilibrium.

(b) Show that in equilibrium the labor income and consumption of the hand-to-mouth agents is a constant fraction λ of output Y_t. Determine λ.

(c) Argue that the competitive equilibrium is not optimal for the 'savers' in the following sense. It **does not** maximize (1) subject to, $c_t^2 + K_{t+1} = (1 - \lambda) K_t^{\alpha} + (1 - \delta) K_t$. Here λ is the constant fraction of output that goes to the hand-to-mouth agents found in (b). [Hint: compare what the competitive equilbrium and the above optimization imply for $u'\left(c_t^2\right) / \beta u'\left(c_{t+1}^2\right)$]

(d) How does the steady state level of capital with hand-to-mouth workers compare with the steady state level in the standard neoclassical growth model?

(e) Consider the case with $u(c) = \log(c)$ and $\delta = 1$. Guess and verify that at the equilibrium $K_{t+1} = sK_t^{\alpha}$. Determine s. [Hint: substitute this guess into the relevant Euler equation and solve for s]. How does the introduction of the hand-to-mouth consumers affect the equilibrium dynamics of consumption, output and capital in this case?