

Problem Set #2

14.41 Public Economics

DUE: October 1, 2010

Question 1

In a drastic response to the news that Maine students score worse on the SAT than do students from any other state in the Union, shamed Maine politicians have decided to abolish all private provision of education and move to a purely public education system. That is, the state of Maine would directly employ teachers and operate schools and provide education to any resident. The state government has W total resources available to divide between units of educational quality (E) and spending on all other public goods (G). Aggregate preferences within the state over E and G are given by

$$U = \alpha \cdot \ln(G) + (1 - \alpha) \cdot \ln(E)$$

where $\alpha \in (0, 1)$. The cost of each unit of education quality is p_E .

(a) State Provision Only

1. If the state of Maine maximizes the aggregate utility function above, how many units of education quality will Maine provide?
2. What fraction of total resources is spent on education, and what fraction of total resources is spent on other public goods? How do these fractions depend on the price p_E of educational quality and total Maine government resources W ?
3. Demonstrate the optimal choice graphically, using a standard budget constraint and indifference curve analysis.

Part of Maine's new education strategy has been to give students access to top-notch, experienced teachers. But because most good teachers move to warmer states like Florida, Maine has had to increase class sizes in order to achieve its strategy. The Federal Government is concerned that these larger class sizes are detrimental to educational achievement, and thus it has vowed to improve Maine's educational system. To figure out how best to do this, the US Department of Education has (wisely!) asked you to evaluate several proposals.

In parts (b)-(d) below, evaluate the impact of the federal government intervention relative to the setup in (a).

(b) Federal Matching Grant

Suppose the federal government provides Maine with an extremely generous β -for-one matching grant, such that each \$1 in state spending on the education program is matched by β from the federal government.

1. What is Maine's effective price per unit of improving education quality under this proposal?
2. Before solving the problem mathematically, how would you expect the quality provided to change under this proposal, relative to no federal intervention? Should this be an income effect, a substitution effect, or a combination of both? Provide intuition for how each price effect impacts the provided level of quality for the education program.
3. Present the state's revised problem graphically, as above, labeling the original and revised budget constraints as well as the original and revised level of quality provided for the education program and relevant indifference curves.
4. Solve mathematically for the revised level of quality provided for the education program and the total size of the grant.
5. How much does federal spending crowd out state spending?

(c) Federal Block Grant

Suppose instead that the federal government provides Maine with a block grant of equivalent size to the federal grant calculated in (b).

1. What is Maine's effective price per unit of improving education quality under this proposal?
2. How would you expect the quality provided to change under this proposal, relative to no federal intervention? Should this be an income effect, a substitution effect, or a combination of both? Provide intuition for how each price effect impacts the provided level of quality for the education program.
3. Present the state's revised problem graphically, as above, labeling the original and revised budget constraints as well as the original and revised level of quality provided for the education program and relevant indifference curves.
4. Solve mathematically for the revised level of quality provided for the education program and the total size of the grant.

5. The size of the federal grant under this proposal is (by construction) the same as in (b). Explain any difference between the number of education quality units provided here vs. under the grant in (b). In particular, which is larger, and why?

(d) Federal Conditional Block Grant

Suppose instead that the federal government provides Maine with a conditional block grant of equivalent size to the federal grant calculated in (b), whereby the grant is provided to the state with a mandate that the grant be spent only on improving the quality of Maine's education program.

1. How does this problem differ from the unconditional grant in part (c)? For what parameter values will the level of provided education be the same as in (c), and for which values will it differ?
2. Present the state's revised problem relative to no federal government intervention graphically, labeling the original and revised budget constraints as well as the original and revised number of education quality units purchased. Draw two sets of graphs—one for the set of parameter values where the provided education is the same as in (c) and one where provided education differs from that in (c).
3. Solve mathematically for the revised level of quality provided for the education program.
4. Assuming validity of the behavioral 'flypaper effect,' how would you expect the real world response to (d) to compare to the theoretical response assessed above?

(e) Optimal Program

If the federal government is specifically concerned with perceived under-provision of education quality in Maine, which intervention assessed above is most effective? Alternatively, if the federal government is only concerned with the overall welfare of Maine, which intervention assessed above is most effective. Provide intuition for similarities or differences in your responses.

Question 2

The remote island nation of Wheredat has two types of citizens: earls and plebs. Wheredat is a small, beautiful island composed of a bunch of villages connected to each other by dirt walking paths. Due to rough seas and dangerous shoals around the island, the citizens use these paths as their sole method for transportation in the country. Unfortunately, Wheredat is plagued by the vine-like weed *accumula*, which if left alone quickly over-grows the walking paths and makes walking along the paths dangerous, especially at night.

It is possible to clear weeds off the paths, but doing so costs $p=\$1$ to clear out 1 unit of *accumula* each day. The earls in Wheredat own the businesses, and often have to travel considerably to work out business deals with earls in other villages. The plebs, on the other hand, engage in manual labor and usually work in a village close to home, and thus do not travel as often on the paths. Hence, these two groups of citizens get different amounts of utility from the provision of cleared weeds.

Plebs have a daily income of \$10, while the daily income of earls is \$15. Let X_i denote each individual's spending on private goods, and let A denote the total amount of weed clearing provided. The plebs' and earls' utilities are, respectively,

$$\begin{aligned} \text{Pleb} : \quad & U_p = \ln(X_p) + \ln(A) \\ \text{Earl} : \quad & U_e = \ln(X_e) + 2 \ln(A) \end{aligned}$$

Even though plural terms “earls” and “plebs” are used below, when solving the problems below you should assume for simplicity that there is only one pleb and one earl living on Wheredat.

1. What requirements must be satisfied for a good to be a pure public good? Evaluate to what extent the provision of clearing *accumula* fits these requirements.
2. Suppose a competitive market supplies weed clearing services, and suppose that the clearing of *accumula* is a pure public good.
 - (a) Set up the maximization problems for the pleb and the earl. Let A_p and A_e denote the amount of weed clearing demanded by the pleb and the earl, respectively. Without doing any math, describe whether you expect A_p and A_e to be equal or different, and give two reasons for your answer.
 - (b) Solve mathematically for A_p and A_e ? What is the resulting utility of the pleb and the earl?
 - (c) Relate the result of this problem to the “mansion and shack” example Professor Gruber discussed in class.
3. The government of Wheredat is concerned that there is a market failure in the provision of weed clearing services and is considering a public provision option financed by taxes on the plebs and earls. However, when the tax collector shows up to collect taxes, it is easy for earls to put on dirty clothes and pretend to be a pleb. Hence, the government is restricted to taxing everyone the same amount to finance the weed clearing. I.e., if A units of clearing are provided, everyone is charged $A/2$ in taxes.
 - (a) Under this scenario, what amount of daily weed clearing should the government provide to maximize social surplus? What would be the resulting utility of the pleb and the earl under this level of provision? Discuss any differences from the utilities in part 2 above, and also comment on any changes in social surplus.

- (b) Does the sum of the individuals' marginal rates of substitution equal the price ratio? Why or why not?
- (c) Now suppose that the tax collector can distinguish between earls and plebs by subtle differences in their accents, thus allowing differential taxation. Now how much of A does the government provide, and how is the tax burden divided? Calculate the sum of the individuals' marginal rates of substitution, and compare with the previous point. Also calculate resulting individual and social surplus.
4. A politician in government has proposed dividing the island of Wheredat into two cities: High and Low. The High city would include all villages on one side of the island, and the Low city all villages on the other side. Citizens would be free to move to whichever town they please as long as there is room—no more than 75% of the population can fit in the same city. If all the earls lived in the High city, there would be no need for them to travel into the Low city to conduct business deals. Likewise, if all plebs lived in the Low city, their jobs could also be located in the Low city, and there would be no reason for them to travel into the High city.

Under the proposal, cities would take care of their own weed clearing. A city's provision of this public good would only affect the residents of their respective city. Public good spending in each town would be financed with a uniform tax on that city's residents. Of course, because a city's weed clearing is now concentrated in that city only, a single unit of clearing has twice the impact on a city's roads compared to the impact in previous parts where the clearing was spread evenly across the country. Thus, if city J provides A_J units of weed removal, residents receive utility

$$\begin{aligned} \text{Pleb :} \quad U_p &= \ln(X_i) + \ln(2A_J) \\ \text{Earl :} \quad U_e &= \ln(X_i) + 2 \ln(2A_J) \end{aligned}$$

- (a) What rates of provision of weed removal A_H and A_L should the High and Low cities provide, respectively to maximize social surplus in the country? If these rates differ, let the High city have the higher rate.
- (b) In which city do the earls and plebs choose to live?

Question 3

The primary intent of medical malpractice law is to protect patients against professional negligence by a health care provider, which results in injury or death to the patient. However, proponents of medical liability reform argue that in fact these laws limit patient access to health care by driving doctors out of business or encouraging doctors not to use high-risk but potentially beneficial procedures. On June 11, 2003, Texas Governor Perry signed House Bill 4, a medical liability reform that greatly limited the amount of damages for which a physician could be held liable. You are provided the data table below, which indicates the number of doctors per 100,000 patients for Texas as well as for states neighboring the Lone Star State (nickname for TX).

Table 1: Docs per 100,000 Patients

State	Year	Doctors
Texas	1998	152
Texas	2002	158
Texas	2006	175
Neighbors	1998	196
Neighbors	2002	189
Neighbors	2006	180

1. Propose a time-series estimator for the impact of Bill 4 on the doctor to patient ratio in TX.
 - (a) Provide the symbolic formula for the estimator as well as the numerical estimate. Provide a brief description of the estimator.
 - (b) Discuss the key assumption required for the estimator to be valid (no bias).
 - (c) Discuss a scenario under which each assumption would be violated (bias).

2. Propose a cross-sectional estimator for the impact of Bill 4 on the doctor to patient ratio in TX.
 - (a) Provide the symbolic formula for the estimator as well as the numerical estimate. Provide a brief description of the estimator.
 - (b) Discuss the key assumption required for the estimator to be valid (no bias).
 - (c) Discuss a scenario under which each assumption would be violated (bias).

3. Instead, construct a difference-in-difference estimator for the impact of Bill 4 on the doctor to patient ratio in TX that addresses the issues raised with the time-series and cross-sectional estimators.
 - (a) Provide the symbolic formula for the estimator as well as the numerical estimate. Provide a brief description of the estimator.
 - (b) Explain the key assumption required for the estimator to be valid.
 - (c) Discuss a scenario under which this assumption would be violated.
 - (d) Set up a test of your key assumption above using the available data. Does the assumption appear valid?

Question 4

The town of Concord must decide how much to spend on its local schools, and it decides to do so by holding a town-wide meeting to discuss the issue and vote. Suppose that the

town can spend \$ X million on the schools, where $X \in [0, 10]$. Suppose further that we can characterize the population of Concord as consisting solely of three types of households (A , B , and C), with N households of each type. Preferences over X for each type of household are given by

$$\begin{aligned} A : U_A(X) &= 3X - X^2 \\ B : U_B(X) &= 5X - X^2 \\ C : U_C(X) &= 9X - X^2. \end{aligned}$$

1. Before the town-wide meeting, the town selectmen solicit nominations for values of X . Which values of X will each type of household nominate? Label these X_A^* , X_B^* , and X_C^* for household types A , B , and C , respectively.
2. At the town meeting, the selectmen lead the town through pair-wise voting. They vote on X_A^* versus X_B^* ; then on the “winner” of that versus X_C^* ; then on the “winner” of that election with the loser of the first election; and so on.
 - (a) Write out the outcome of each election.
 - (b) Does the town eventually choose a consistent “winner”? If so, which option do they choose?
3. Now suppose that the town is composed of different households of types D , E , and F , and that the selectmen decide on nominations themselves. The selectmen choose three possible outcomes: $X = 3$, $X = 6$, or $X = 8$. Moreover, the three types of households with N households of each type, have preference rankings

$$\begin{aligned} D : 8 \succ^D 6 \succ^D 3 \\ E : 3 \succ^E 8 \succ^E 6 \\ F : 6 \succ^F 3 \succ^F 8 \end{aligned}$$

- (a) What is the outcome of these elections?
 - (b) Why is this a very different case than in the town’s previous elections?
4. One selectman nominates himself as the “agenda setter.” This selectman chooses a pair-wise vote, and “winner” of that pair-wise vote is put up against the third option, and the winner of this second pairwise vote is implemented. Demonstrate that the “agenda setter” can determine the final provision level chosen, assuming that all households vote sincerely.
5. Suppose that there are many towns and each town has an agenda-setter who manipulates the town to one eventual value of X . Explain why this might actually be okay; household will not be dissatisfied with their town’s level of school funding.

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