14.30 Introduction to Statistical Methods in Economics
Spring 2009

Problem Set #6

14.30 - Intro. to Statistical Methods in Economics

Instructor: Konrad Menzel

Due: Tuesday, April 7, 2009

Question One

Let X be a random variable that is uniformly distributed on $[0, 1]$ (i.e. $f(x) = 1$ on that interval and zero elsewhere). In Problem Set #4, you use the "2-step"/CDF technique and the transformation method to determine the PDF of each of the following transformations, $Y = g(X)$. Now that you have the PDFs, compute (a) $\mathbb{E}[g(X)]$, (b) $g(\mathbb{E}[X])$, (c) $Var(g(X))$ and (d) $g(Var(X))$ for each of the following transformations:

1. $Y = X^{\frac{1}{4}}$, $f_Y(y) = 4y^3$ on $[0, 1]$ and zero otherwise.
2. $Y = e^{-X}$, $f_Y(y) = \frac{1}{y}$ on $[\frac{1}{e}, 1]$ and zero otherwise.
3. $Y = 1 - e^{-X}$, $f_Y(y) = \frac{1}{1-y}$ on $[0, 1 - \frac{1}{e}]$ and zero otherwise.
4. How does (a) $\mathbb{E}[g(X)]$ compare to (b) $g(\mathbb{E}[X])$ and (c) $Var(g(X))$ to (d) $g(Var(X))$ for each of the above transformations? Are there any generalities that can be noted? Explain.

Question Two

Compute the expectation and the variance for each of the following PDF's.

1. $f_X(x) = ax^{a-1}$, $0 < x < 1$, $a > 0$.
2. $f_X(x) = \frac{1}{n}$, $x = 1, 2, \ldots, n$, where n is an integer.
3. $f_X(x) = \frac{3}{2}(x-1)^2$, $0 < x < 2$.

Question Three

Suppose that X, Y, and Z are independently and identically distributed with mean zero and variance one. Calculate the following:

1. $\mathbb{E}[3X + 2Y + Z]$
2. $Var[5X - 3Y - 2Z]$
3. $Cov[X - Y + 4, 2X + 3Y + Z]$
4. $E[3XY]$

Question Four

Simplify the following expressions for random variables X and Y and scalar constants $a, b \in \mathbb{R}$:

1. $Var(aX + b)$
2. $Cov(aX + c, bY + d)$
3. $Var(aX + bY)$

Question Five

(Bain/Engelhardt p.190)

Suppose X and Y are continuous random variables with joint PDF $f(x, y) = 4(x - xy)$ if $0 < x < 1$ and $0 < y < 1$, and zero otherwise.

1. Find $\mathbb{E}[X^2Y]$.
2. Find $\mathbb{E}[X - Y]$.
3. Find $Var(X - Y)$.
4. What is the value of the correlation coefficient, $\rho_{XY} = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$, of X and Y?
5. What is $\mathbb{E}[Y|x]$?

Question Six

(Bain/Engelhardt p. 191)

Let X and Y have joint pdf $f(x, y) = e^{-y}$ if $0 < x < y < \infty$ and zero otherwise. Find $\mathbb{E}[X|y]$.

Question Seven

Let X be a uniform random variable defined over the interval (a, b), i.e. $f(x) = \frac{1}{b-a}$. The k^{th} central moment of X is defined as $\mu_k = \mathbb{E}[(X - \mathbb{E}[X])^k]$. The standardized central moment is defined as $\frac{\mu_k}{(\mu_2)^{\frac{k}{2}}}$. Find an expression for the k^{th} standardized central moment of X.