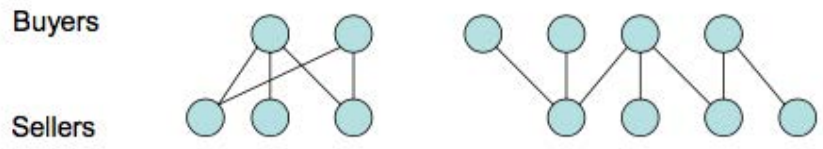
Problem Set 6

Not Due

Problem 1: Consider the following buyer-seller network. What are the predicted payoffs in the model from lecture 19?



Problem 2: Consider the input-output model introduced in lecture 19, with labor share α in the production function and n sectors. Assume the sector level shocks ϵ_i all have variance σ^2. Suppose the network W is a directed circle. That is, suppose $w_{ij} = 1$ if $j = i + 1$ or if $i = n$ and $j = 1$, and $w_{ij} = 0$ otherwise.

(a) Compute the influence vector.

(b) What is the volatility of aggregate output?

Suppose we add a firm $n + 1$ to the network which supplies some small amount to all others: $w_{n+1,j} = \delta$ for all j. Correspondingly, we reweight $w_{ij} = 1 - \delta$ for the old links. Repeat the exercise for the new network.

Problem 3: Recall Selten's horse from the lecture.

(a) Find the set of pure strategy Nash equilibria of Selten's horse

(b) Find the set of pure strategy perfect Bayesian equilibria of Selten's horse (including the corresponding belief systems)

Problem 4: Consider a variant of the classic herding model. There are two possible states $\theta \in \{0, 1\}$, each equally likely, and agents $n = 1, 2, ...$ make binary choices $x_n \in \{0, 1\}$ in sequence. Agent n earns 1 if $x_n = \theta$ and 0 otherwise. Prior to making a choice, agent n observes the signal $s_n \in \{0, 1\}$ which is equal to the true state with probability $g > \frac{1}{2}$ and equal to the other state with probability $1 - g$. Signals are independent and identically distributed conditional on the state.

Partition the agents into three sets: S_1 contains agents with indices $3k+1$ for some $k \in \mathbb{N}$, S_2 contains agents with indices of the form $3k + 2$, and S_3 contains agents with indices of the form $3k$. Suppose we have the following observation structure:

- Agent $n \in S_1$ observes the actions of all $m \in S_1$ with $m < n$
- Agent $n \in S_2$ observes the actions of all $m \in S_2$ with $m < n$
- Agent $n \in S_3$ observes the actions of $n-1$ and $n-2$

Answer the following:

(a) What is the maximum asymptotic utility agents in S_1 obtain in a perfect Bayesian equilibrium? Describe the equilibrium strategies.

(b) What is the maximum asymptotic utility agents in S_3 obtain in a perfect Bayesian equilibrium? Describe the equilibrium strategies.

Problem 5: Consider a DeGroot learning model with matrix

$$W = \begin{pmatrix} 0.6 & 0.1 & 0.3 \\ 0.5 & 0 & 0.5 \\ 1.0 & 0 & 0 \end{pmatrix}$$

(a) Compute the vector of eigenvalue centralities.

(b) If initial beliefs are $\mathbf{x}(0) = [1.0, 0.5, 0]$, compute the conensus belief.

(c) How many periods does it take for all beliefs to be within 0.01 of the eventual conensus?

Suppose we adjust the network so that agent 3 puts full weight on his own belief:

$$W = \begin{pmatrix} 0.6 & 0.1 & 0.3 \\ 0.5 & 0 & 0.5 \\ 0 & 0 & 1.0 \end{pmatrix}$$

How does this change the eventual consensus?

14.15J/6.207J Networks