Recitation 6: Midterm Review

Will Rafey

Outline

Midterm consists of three parts:

i. True/false
    - State true, false, or uncertain
    - Always explain answer carefully
    - Need to provide intuition
ii. Multiple choice
iii. Short answer (similar to problem set)

Most important resources:

- lecture + recitation slides
- problem sets and solutions.

True-False: Example 1

T/F. Consider individuals with "β, δ" preferences, who only differ by their present bias, $\beta \in [0, 1]$. Suppose there is a commitment savings device available. Willingness to pay for this commitment device strictly decreases in β.

False. Why?

- Individuals may be naïve
- Commitment device may not be effective
- Even if individuals are fully sophisticated and the device is effective, willingness to pay may not be strictly decreasing.
  - Individuals would be willing to pay 0 for $\beta = 0$ and for $\beta = 1$, but willing to pay a positive amount for $\beta \in (0, 1)$.

True-False: Example 2

T/F. Fully sophisticated individuals can experience large welfare losses from their present bias.

True. Why?

- Awareness of present bias (i.e. sophistication) does *not* remove present bias
- Sophisticates that lack commitment devices may still make suboptimal decisions

True-False: Example 3

T/F. Present-biased individuals will always have positive demand for commitment devices.

False. Why?

- Three conditions must be met for positive demand for commitment:
  1. Individuals must be present-biased.
  2. Individuals must be aware of their present-bias (i.e. they can't be fully naive).
  3. Individuals must perceive the commitment device as effective in helping overcome the self-control problem.
- When only the first is met, we cannot be sure there will be positive demand for commitment.

Multiple Choice: Example 1

Pierre-Luc is writing a problem set for 14.13. He gets utility $u(q)$ from the number of questions he writes. He has reference dependent preferences around his goal of writing 10 questions. Normalize $u(10) = 0$. Which of the following would be consistent with loss aversion?

(a) $u(8) = -2, u(12) = 2$
(b) $u(8) = -2, u(12) = 1$
(c) $u(8) = -1, u(12) = 2$

(b). Why?

- Loss aversion means losses hurt more than gains help
- With preferences in (b), Pierre-Luc would have a utility cost of 2 from falling short of his goal by 2 questions, but only gain 1 util from exceeding his goal by 2 questions.

Multiple Choice: Example 2

Q: Maddie is walking home and passes a bakery. She suddenly decides to buy a pastry. Prior to purchasing the pastry, her maximum willingness to pay for the pastry was p_0. She then runs into Pierre-Luc who asks to buy the pastry from her. She offers him the lowest price she is willing to accept, p_1. Which of the following comparisons between p_0 and p_1 is consistent with an endowment effect?

(a) $p_0 > p_1$
(b) $p_0 = p_1$
(c) $p_0 < p_1$

(c). Why?

- Consistent with an endowment effect, $p_0 < p_1$ implies Maddie values the pastry more after she has bought it than prior to buying it.

Q: Now suppose that Maddie first notices the pastry has gone stale, before she offers Pierre-Luc a price. Maddie always prefers fresher pastries. Which of (a)–(c) is consistent with the endowment effect?

Long Question: Example 1

Present Bias

Setup. Assume 14.13 students are present biased with $\beta < 1$ and $\delta = 1$. All students have the same $\beta < 1$ and $\delta = 1$ but differ in the value they derive from using laptops in class, L_i.

L_i is uniformly distributed across students i on the interval [0,1].

Each lecture generates no immediate utility, but does give a future benefit V. Using a laptop reduces the long-run benefit by D. Both V and D are the same for all students.

In summary, a student that uses a laptop in class gets immediate utility L_i and future (undiscounted) utility $V - D$. A student that does not use a laptop gets immediate utility 0 and future (undiscounted) utility V.

The social planner is not present biased and seeks to maximize the utility of 14.13 students.

Present bias

1(a). Show that a student i^* is just indifferent between using and not using their laptop in the current class if $L_{i^*} = \beta D$. Explain why students with lower values of L_i (i.e. $L_i < \beta D$) don't use laptops in class, while students with higher values of L_i (i.e. $L_i > \beta D$) do use laptops in class.

Present bias, cont'd

Utilities from the two choices are:

$$\begin{aligned} U(\text{laptop}) &= L_i + \beta\delta(V - D) \\ U(\text{nolaptop}) &= 0 + \beta\delta V \end{aligned}$$

For students that are indifferent, $U(laptop) = U(nolaptop)$. This gives:

$$\begin{aligned} L_{i^*} + \beta\delta(V - D) &= 0 + \beta\delta V \\ L_{i^*} &= \beta\delta D \end{aligned}$$

Students that choose not to use laptops will have low valuations of using laptops, while students that choose to use laptops will have higher valuations. Given the indifference condition and $\delta = 1$,

- Students i that do not use laptops: $L_i < \beta D$
- Students i that use laptops: $L_i > \beta D$

Present bias, cont'd

1(b). Now consider the policy that allows students to use laptops only if they sign up **in advance** to sit in a laptop section. Why is $L_i \geq D$, not $L_i \geq \beta D$, the threshold for opting into the laptop section?

Present bias, cont'd

Solution to 1(b)

Considered in advance, students evaluate:

$$\begin{aligned} U(laptop) &= 0 + \beta(\delta L_i + \delta^2(V - D)) \\ U(nolaptop) &= 0 + \beta\delta^2 V \end{aligned}$$

The threshold for opting in is defined by $U(laptop) \geq U(nolaptop)$. Using $\delta = 1$, this gives:

$$\begin{aligned} 0 + \beta(L_i + V - D) &\geq 0 + \beta V \\ L_i &\geq D \end{aligned}$$

The threshold changes from βD to D because when laptop use can only happen in the future, all benefits and costs are discounted at the same rate, β.

Present bias, cont'd

1(c). Assume there is no laptop policy. Show that if $\beta D < L_i < D$, the student i engages in preference reversals: she prefers not to use the laptop in future classes, but changes her mind when she's actually sitting in those future classes.

- When thinking about future laptop use, the student's problem is identical to the problem in part (b). Why?
  - Because she discounts time both one and two periods in advance by β
- We know from part (b) that if $L_i < D$, she would like to not use the laptop
- But from part (a), we know that if $\beta D < L_i$, she will end up using the laptop when she's actually sitting in the future class
- This implies a preference reversal! she prefers not to use the laptop in future classes, but switches her mind when she's actually sitting in those future classes.

Present bias, cont'd

1(d). Explain why fraction $1 - \beta D$ of the class uses a laptop in part 1, but fraction $1 - D$ of the class uses a laptop in part 2. Why does a smaller share of the class use their laptops in part 2?

Present bias, cont'd

Solution to 1(d)

In part 1, a student uses a laptop if $L_i > \beta D$. Define $F(\cdot)$ as the CDF of L_i. Given the uniform distribution:

$$\begin{aligned} P(L_i > \beta D) &= 1 - F(\beta D) \\ &= 1 - \beta D \end{aligned}$$

Likewise, in part 2, a student uses a laptop if $L_i > D$. We have:

$$\begin{aligned} P(L_i > D) &= 1 - F(D) \\ &= 1 - D \end{aligned}$$

A smaller share will use laptops in part 2 because the benefit of using a laptop is delayed and hence discounted by β.

Present bias, cont'd

1(e). Why would the social planner prefer the opt-in policy to both the policy of allowing students to choose whether to use their laptops and to banning laptops altogether?

- The planner is not present biased so would want only students with $L_i > D$ to use laptops; the opt-in policy achieves this
- Under the free choice policy, students with $\beta D < L_i < D$ will suboptimally use their laptops
- On the other hand, banning laptops altogether is suboptimal because welfare is gained by allowing the students with the highest valuations, $L_i > D$, use laptops

Long Question, Example 2

Reference dependence

Frank has reference-dependent preferences over donuts d and coffee k, which cost \$1 each. MIT gives him \$13 to spend at the coffee shop. His utility takes the form

$$u(d,k) = u_1(d-6) + u_2(k-2)$$

where

$$u_1(x) = \begin{cases} 2\sqrt{x} & \text{if } x \geq 0 \\ -4\sqrt{|x|} & \text{if } x < 0 \end{cases} \tag{1}$$

and

$$u_2(x) = \begin{cases} \sqrt{x} & \text{if } x \geq 0 \\ -2\sqrt{|x|} & \text{if } x < 0. \end{cases} \tag{2}$$

2(a). If Frank has six donuts, is Frank loss averse to changes in his donut supply?
Yes!

Reference dependence

2(b). Frank buys a positive number of donuts and a positive number of coffees. How many donuts and coffee should Frank buy?

Answer: the Lagrangian is

$$\mathcal{L}(d, k, \lambda) = u_1(d-6) + u_2(k-2) + \lambda \cdot (13 - d - k)$$

When $d, k > 0$, then

$$\frac{\partial u_1}{\partial d} = (d-6)^{-1/2} = \lambda$$

and

$$\frac{\partial u_2}{\partial k} = \frac{1}{2}(k-2)^{-1/2} = \lambda.$$

Then $d - 6 = \lambda^{-2}$ and $k - 2 = 2^{-2}\lambda^{-2}$, so $4(k-2) = d - 6$. And $k + d = 13$. So

$$4k - 8 = d - 6 = 13 - k - 6$$

so that $5k = 21 - 6$ or $\boxed{k = 3}$ and $\boxed{d = 10}$.

Frank's utility is $u(10, 3) = 2\sqrt{4} + \sqrt{1} = 5$.

Reference dependence, cont'd

2(c). Someone tells Frank that they eat fewer than six donuts per day; specifically, they eat two donuts. Frank decides he should cut back his reference point to two donuts, as a benchmark. His new preferences are

$$u(d, k) = u_1(d - 2) + u_2(k - 2).$$

Is Frank happier?

Yes! $u_1(d - 2) > u_1(d - 6)$ for all d.

Reference dependence, cont'd

2(d). Frank has bought his donuts and returned to his office. A doctor arrives from MIT Medical. Frank has a suspicion that the doctor will prescribe any desired level of donuts, $\overline{d} \geq 0$, that he asks. Frank's preferences then will become

$$u(d,k) = u_1(d - \overline{d}) + u_2(k-2).$$

What does Frank ask the doctor to prescribe?

Frank's utility is always diminishing in $\overline{d}$, his reference level for donuts! He asks the doctor to prescribe $\overline{d} = 0$.

2(e). Now the doctor demands payment for his medical wisdom. How much is Frank's maximum willingness to pay the doctor for these new preferences?

Frank's utility rises to $2\sqrt{10}$ from $2\sqrt{10-6}$, so he is willing to pay $2(\sqrt{10} - \sqrt{4})$.

Reference dependence, cont'd

2(f). Suppose that the doctor is receiving payments from the donut industry and can only prescribe $\overline{d} = 1$, but will now also give Frank a machine that allows him to costlessly transform donuts into coffee and vice-versa. How much is Frank now willing to pay the doctor (in utils)?

If Frank can revise his consumption, his first-order conditions become

$$4(k-2) = d-1$$

or

$$4k-8 = 13-k-1$$

so that $5k = 20$, or $k = 4$ and $d = 9$.

$\therefore$ With the time machine and $\overline{d} = 1$, Frank will obtain $2\sqrt{9-1} + \sqrt{4-2} = 2\sqrt{8} + \sqrt{2} = 5\sqrt{2}$.

$\therefore$ Frank's WTP $\leq 5\sqrt{2} - 5$.