Recitation 4

Aaron Goodman, Alex Olssen, Pierre-Luc Vautrey[1]

[1]These slides are partially based on notes from Drew Fudenberg. All errors are our own.

Outline

1. Rabin (2000)
2. Example problem on risk preferences

Outline

1. Rabin (2000)
2. Example problem on risk preferences

Recap: Expected Utility Theory

In recitation last week and lecture this week, we introduced expected utility theory:

- States of the world $i = \{1, .., n\}$, probabilities p_i, payoffs x_i
- Utility function $u(\cdot)$
- Expected utility is given by

$$EU = \sum_i p_i u(x_i) \tag{1}$$

- We generally assume that $u(\cdot)$ is concave, so agents are risk averse and

$$\sum_i p_i u(x_i) < u\left(\sum_i p_i x_i\right) \tag{2}$$

Rabin (2000)

- Rabin's paper is a very influential critique of expected utility theory
- Main idea: concavity of the utility function cannot be the only source of risk aversion. If it is, then we obtain some absurd results.
- Helpful to understand Rabin's argument, especially as we begin to consider deviations from expected utility theory (loss aversion, reference dependence, etc.) that address his critique
- The discussion today is only meant to be instructive - we won't ask you to prove Rabin's result!

Setup

- Consider an agent with utility function $u(\cdot)$ defined over wealth w
- Assume that at all wealth levels, the agent rejects a 50-50, lose \$100, gain \$110 gamble:

$$\frac{1}{2}u(w-100)+\frac{1}{2}u(w+110) \leq u(w) \tag{3}$$

$$\implies u(w+110)-u(w) \leq u(w)-u(w-100) \tag{4}$$

- Sounds like a reasonable assumption, but will see that it leads to unreasonable results!

First Step

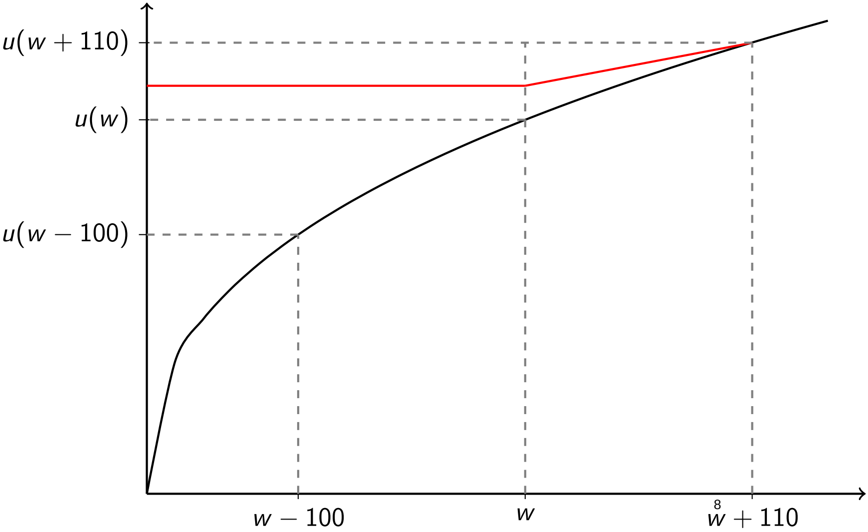
- First, observe that:

$$110u'(w+110) \leq u(w+110) - u(w) \quad (5)$$
$$\leq u(w) - u(w-100) \quad (6)$$
$$\leq 100u'(w-100) \quad (7)$$

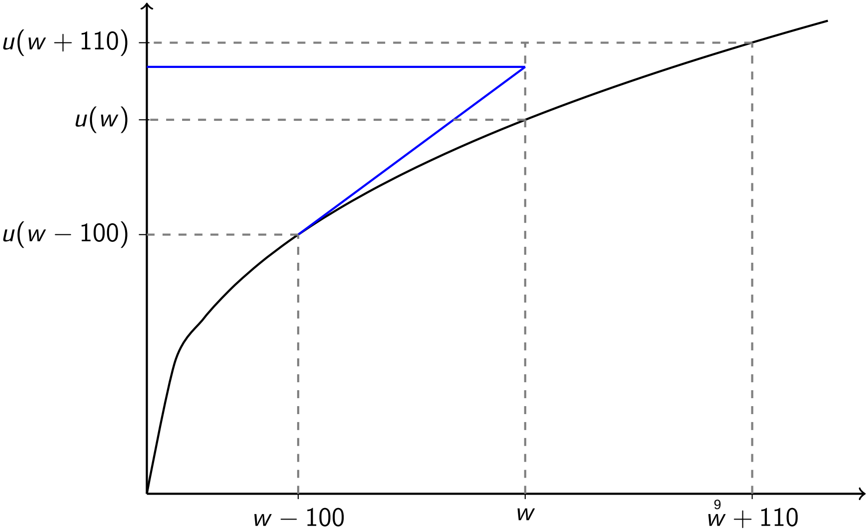
- How do we justify each of these inequalities?
- Rearranging, we obtain

$$110u'(w+110) \leq 100u'(w-100) \quad (8)$$
$$\frac{u'(w+110)}{u'(w-100)} \leq \frac{10}{11} \quad (9)$$

Concavity



Concavity



Iterating Forward

- Under our assumption, the agent also rejects the gamble when his wealth is $w + 210$. Applying the same logic, we obtain:

$$\frac{u'(w+210+110)}{u'(w+210-100)} = \frac{u'(w+320)}{u'(w+110)} \leq \frac{10}{11} \tag{10}$$

- This implies:

$$\frac{u'(w+320)}{u'(w-100)} = \frac{u'(w+320)u'(w+110)}{u'(w+110)u'(w-100)} \leq \left(\frac{10}{11}\right)^2 \tag{11}$$

- We can do this again:

$$\frac{u'(w+530)}{u'(w-100)} = \frac{u'(w+530)u'(w+320)}{u'(w+320)u'(w-100)} \leq \left(\frac{10}{11}\right)^3 \tag{12}$$

Keep Iterating Forward

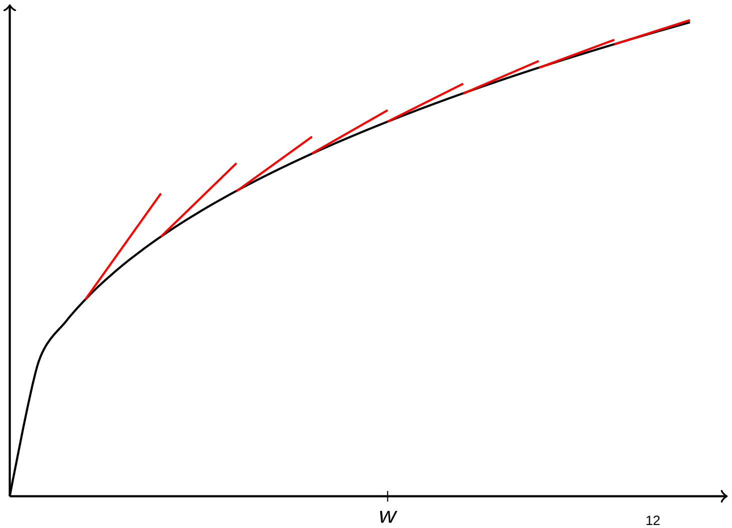
- We can do this as many times as we want. In general:

$$\frac{u'(w+210k+110)}{u'(w-100)} \leq \left(\frac{10}{11}\right)^{k+1} \quad k=1,2,\ldots \tag{13}$$

- Takeaway message: to justify seemingly reasonable risk aversion over small gambles (e.g., our lose \$100, gain \$110 bet), marginal utility must be diminishing very fast. If we iterate forward 100 times, then:

$$\frac{u'(w+210(100)+110)}{u'(w-100)} = \frac{u'(w+21110)}{u'(w-100)} \leq \left(\frac{10}{11}\right)^{101} \approx 0.00007 \tag{14}$$

Diminishing Marginal Utility



- Each slope is at most $\frac{10}{11}$ of the last

Implications

- Because marginal utility is diminishing so quickly, our agent turns down gambles with enormous upside
- In fact, there is no number x such that our agent will accept a 50-50, lose \$1,000, gain \$$x$ gamble. He refuses this offer even if $x = \infty$!
- The marginal utility of wealth becomes infinitesimally small at large dollar values, so the upside of any such gamble is outweighed by the downside:

$$u(w + x) - u(w) \leq u(w) - u(w - 1000) \ \ \forall x \tag{15}$$

Rabin's Corollary

TABLE I

IF AVERSE TO 50-50 LOSE \$100 / GAIN g BETS FOR ALL WEALTH LEVELS, WILL TURN DOWN 50-50 LOSE L / GAIN G BETS; G'S ENTERED IN TABLE.

		g		
L	\$101	\$105	\$110	\$125
\$400	400	420	550	1,250
\$600	600	730	990	∞
\$800	800	1,050	2,090	∞
\$1,000	1,010	1,570	∞	∞
\$2,000	2,320	∞	∞	∞
\$4,000	5,750	∞	∞	∞
\$6,000	11,810	∞	∞	∞
\$8,000	34,940	∞	∞	∞
\$10,000	∞	∞	∞	∞
\$20,000	∞	∞	∞	∞

Outline

Setup

From problem set 2 in 2017 (on course website):

- Alex is buying home insurance
- His current wealth is $w = \$100{,}000$
- He has CRRA utility with coefficient of relative risk aversion γ
- Damage occurs to his house next year with probability $\pi = .05$

Plan Choices

Alex is offered four plans by his insurance company

- Assume that not buying insurance is not an option
- Assume that if damage occurs, it always exceeds the deductible

Option	Deductible	Premium
1	1,000	757
2	500	885
3	250	999
4	100	1,171

Plan Choices

We can also represent the plans in terms of Alex's terminal wealth in each state of the world:

Option	Damage	No Damage
1	$w - 1{,}757$	$w - 757$
2	$w - 1{,}385$	$w - 885$
3	$w - 1{,}249$	$w - 999$
4	$w - 1{,}271$	$w - 1{,}171$

Is there a plan that Alex will *never* choose, regardless of his risk preferences?

Bounding Risk Aversion

Suppose Alex chooses plan 2. Calculate bounds on his risk aversion parameter γ.

What's the first step in answering this question?

Write down the expected utility of choosing plan j, with premium p_j and deductible d_j:

$$V_j = \pi u(w - p_j - d_j) + (1 - \pi)u(w - p_j) \tag{16}$$

$$= \pi \frac{(w - p_j - d_j)^{1-\gamma}}{1 - \gamma} + (1 - \pi)\frac{(w - p_j)^{1-\gamma}}{1 - \gamma} \tag{17}$$

Alex chooses the plan that maximizes his expected utility:

$$j^* = \underset{j \in \{1,2,3\}}{\operatorname{argmax}} \; V_j \tag{18}$$

Bounding Risk Aversion

Since Alex chose plan 2, we have, for $k \in \{1, 3\}$:

$$V_2 \geq V_k \tag{19}$$

How do we use this to bound γ?

$$\pi u(w - p_2 - d_2) + (1 - \pi)u(w - p_2) \geq \pi u(w - p_k - d_k) + (1 - \pi)u(w - p_k) \tag{20}$$

Bounding Risk Aversion

We thus have:

$$0.05 \cdot (w - 1,385)^{1-\gamma} + 0.95 \cdot (w - 885)^{1-\gamma} \geq 0.05 \cdot (w - 1,757)^{1-\gamma} + 0.95 \cdot (w - 757)^{1-\gamma}$$
$$0.05 \cdot (w - 1,385)^{1-\gamma} + 0.95 \cdot (w - 885)^{1-\gamma} \geq 0.05 \cdot (w - 1,249)^{1-\gamma} + 0.95 \cdot (w - 999)^{1-\gamma}$$

Using a computer, we find that the first inequality implies

$$\gamma \geq 243.26$$

and the second inequality implies

$$\gamma \leq 726.50$$

Why does the first inequality place a lower bound on γ? Why does the second inequality place an upper bound on γ?

Note: these are implausibly high values for risk aversion!