Recitation 3

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Outline

1. Quasi–hyperbolic Savings
2. Risk Aversion

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1. Quasi–hyperbolic Savings

2. Risk Aversion (also Autor's notes on Stellar: Review notes 3/3)

Solving Problems with (Quasi-)Hyperbolic Discounting

- **Fully naïve** decision-makers ($\hat{\beta} = 1$):
  1. Start at the beginning.
  2. Solve for the optimal plan, assuming future selves will follow the plan.
  3. The person takes the first step in that plan.
  4. Go to the next period, and keep doing the same.
- **Fully sophisticated** decision-makers ($\hat{\beta} = \beta$):
  1. Start at the end.
  2. Solve for optimal action.
  3. Go back to the previous period.
  4. Solve for the optimal action, taking into account what happens in the next period.
  5. Go back to the previous period, and keep doing the same.
- **Partially naïve** decision-makers ($\beta < \hat{\beta} < 1$):
  1. Start at the end. Solve for what the person *thinks* she will do (using $\hat{\beta}$).
     [This is like solving for a fully sophisticated decision maker with a true β of $\hat{\beta}$.]
  2. Work your way to the first period using backward induction until period 2 (using $\hat{\beta}$).
  3. Then solve for the optimal action in period 1 (using the true β and the already derived prediction on future behavior).
  4. Then move to the next period, repeat steps (1) to (3).

The Model: Illiquid savings, credit card debt, commitment

- Alex is a fully naive hyperbolic discounter with $\beta = 0.5$ and $\delta = 1$ and $\hat{\beta} = 1$
- Alex lives for three periods $t = 0$, 1, and 2
- His instantaneous utility from consuming an amount $c_t > 0$ at time t is

$$u(c_t) = ln(c_t) \text{ for } t = 0, 1, 2$$

Alex's discounted lifetime utility from the perspective of period 0 is given by

$$U_0(c_0, c_1, c_2) = ln(c_0) + \beta(ln(c_1) + ln(c_2))$$

Moving money across periods (Q1.1)

- Alex starts with wealth of \$60 at $t = 0$
- Several ways to move money across periods
  - Checking account: put \$$x$ in at time t, can withdraw up to \$$x$ at $t + 1$
  - Retirement account: deposit s at $t = 0$, can withdraw $(1 + r^r)s$ at $t = 2$ $(r^r = .2)$
  - Credit card for $t = 1$: borrow b at $t = 1$, must repay $(1 + r^c)b$ at $t = 2$ $(r^c = .5)$
- How will Alex move money to $t = 1$? How about $t = 2$? Why?
  - To move money to $t = 1$, use checking account because alternative (credit card paid off at $t = 2$) is expensive
  - To move money to $t = 2$, use retirement savings because get a good return!

Optimal plan at $t = 0$ (Q1.2)

- Show that the consumption plan Alex makes at $t = 0$ involves $c_1 = \beta c_0$
- Given the previous answer, interest rate of 0 between $t = 0$ and $t = 1$
- Accordingly, he will equalize marginal utilities at $t = 0$ and $t = 1$
- Direct implication $c_1 = \beta c_0$ (let's work through the FOCs)

Optimal plan at $t = 0$ (Q1.3)

- Use (1) and (2), write Alex's maximization problem in period 0 and solve for planned c_0, c_1, and c_2
- Part (2) means $c_1 = \beta c_0$ at the optimum. Part (1) means we can ignore b. Thus

$$\max_{c_0, c_1, c_2} \; u(c_0) + \beta u(c_1) + \beta u(c_2)$$
$$\text{s.t. } \; c_1 = \beta c_0 \text{ and } c_2 = (60 - c_0 - c_1)(1 + r^r)$$

- Solution: $c_0^* = 30$, $\hat{c}_1 = 15$, and $\hat{c}_2 = 18$ (Let's work through FOCs)

Present Bias (Q1.4)

- What does Alex end up doing at $t = 1$?
- Being naive, at $t = 1$ Alex solves

$$\max_{c_1, c_2} \; u(c_1) + \beta u(c_2) \text{ s.t. } c_2 = \hat{c}_2 - (c_1 - \hat{c}_1)(1 + r^c)$$

- Taking the FOC and simplifying gives

$$\frac{1}{c_1} = \frac{\beta(1+r^c)}{c_2}$$
$$c_2 = \beta(1 + r^c)c_1$$

- Solution: $c_1^* = 18$, $b^* = 3$, and $c_2^* == 13.5$

Full Sophistication (Q1.9)

- Suppose Alex becomes fully sophisticated.
  Argue that at $t = 0$, Alex anticipates that at $t = 1$ he will choose c_1 and c_2 such that $c_2 = \beta(1 + r^c)c_1$.
- Being sophisticated, Alex understands that he will solve his consumption-savings decision in exactly the same way as already determined in (Q1.4)
- Recall that (Q1.4) was $c_2 = \beta(1 + r^c)c_1$

Full Sophistication (Q1.10)

- Write down Alex's maximization problem at $t = 0$. Explain what is different from Alex's maximization problem in part (3) and why
- Alex solves the following maximization problem:

$$\max_{c_0, c_1, c_2} \ u(c_0) + \beta u(c_1) + \beta u(c_2)$$
$$\text{s.t. } \ c_2 = \beta(1 + r^c)c_1 \text{ and } c_2 = (60 - c_0 - c_1)(1 + r^r)$$

- Fully sophisticated Alex knows he lacks time consistency
- Thus he solves his $t = 0$ problem with constraints that reflect his knowledge that he will re-optimize in the future

Commitment devices (Q1.11)

- Aaron offers (fully sophisticated) Alex a commitment device
- Can Alex be worse off (using discounted utility at $t = 0$) by (voluntarily) choosing *any* commitment contract that Aaron offers to him at $t = 0$?
- Solution: No, it is impossible for fully sophisticated Alex to be worse off.
- A fully-sophisticated agent anticipates his/her future behaviors
- At $t = 0$ Alex makes plans that maximize his utility from the perspective of $t = 0$
- If Aaron's commitment contract would make Alex worse off, then he would never (voluntarily) choose it

Commitment devices (Q1.12)

- Suppose Alex is partially naive
- Can Aaron make Alex worse off by offering him a commitment device (using discounted life-time utility at $t = 0$)?
- Yes, partially-sophisticated Alex can be worse off even when (voluntarily) choosing.
- Suppose the commitment device raises r^c at $t = 1$ above 50%.
- Alex might (voluntarily) choose the commitment device, hoping it will help him avoid borrowing.
- However, if β turns out to be (much) lower than anticipated, then he might end up borrowing at high interest rates after all
- This would make him worse off than he would have been borrowing at a 50% interest rate

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1. Quasi–hyperbolic Savings
2. Risk Aversion (also Autor's notes on Stellar: Review notes 3/3)

Expected Utility Theory

- Describes agents' preferences and behavior when faced with uncertainty
- General lottery setup:
  - Agent gets utility from wealth $u(.)$
  - Potential states of the world: $i \in \{1, ..., n\}$
  - Each state has associated probabilities p_i and monetary payout x_i
- Expected value of lottery: $EX = \sum_{i=1}^{n} p_i x_i$
- Expected utility of lottery: $EU = \sum_{i=1}^{n} p_i u(x_i)$
- Utility of the expected value: $UE = u(\sum_{i=1}^{n} p_i x_i)$

Risk Preferences

- Risk loving: $EU > UE$
  - Prefers taking the lottery to receiving the expected value with certainty
- Risk neutral: $EU = UE$
  - Indifferent between taking the lottery and receiving the expected value with certainty
- Risk averse: $EU < UE$
  - Prefers receiving the expected value with certainty to taking the lottery

Curvature of $u(.)$

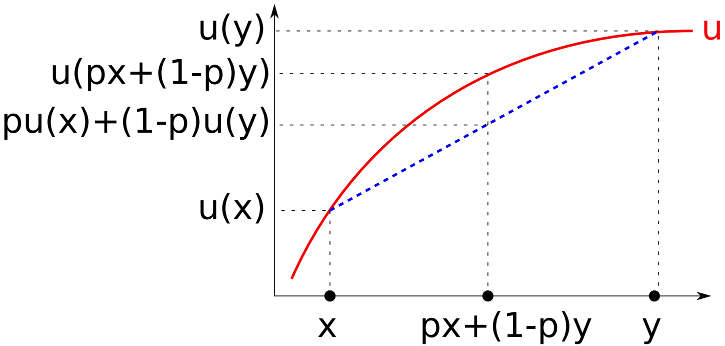
- Jensen's inequality: $f(.)$ is concave iff $f(\sum_{i=1}^{n} w_i y_i) > \sum_{i=1}^{n} w_i f(y_i)$
- Risk preferences involve comparison between:
  - $EU = \sum_{i=1}^{n} p_i u(x_i)$
  - $UE = u(\sum_{i=1}^{n} p_i x_i)$
- This implies:
  - Risk loving ($EU > UE$) iff $u(.)$ is convex
  - Risk neutral ($EU = UE$) iff $u(.)$ is linear
  - Risk averse ($EU < UE$) iff $u(.)$ is concave

Risk Aversion and Certainty Equivalents

- Certainty equivalent: the level of x that would make the agent indifferent between taking x and participating in the lottery
- Formally:
  - $u(CE) = EU = \sum_{i=1}^{n} p_i u(x_i)$
  - $CE = u^{-1}(EU) = u^{-1}(\sum_{i=1}^{n} p_i u(x_i))$
- Equivalent definition of risk preferences:
  - Risk loving if $CE > EX$
  - Risk neutral if $CE = EX$
  - Risk averse if $CE < EX$

Risk Aversion in a Picture

Lottery with 2 outcomes: (1) $x_1 = x$, $p_1 = p$; (2) $x_2 = y$, $p_2 = (1 - p)$



- Where is *EX*? *EU*? *UE*? *CE*?

CARA

- Coefficient of absolute risk aversion: $r = -\frac{u''(x)}{u'(x)}$
  - Normalized by $u'(x)$ (why?)
- Constant absolute risk aversion (CARA) utility: $u(x) = -\frac{e^{-rx}}{r}$
  - Absolute risk aversion is constant in x
- Problem: we typically believe wealthier people are riskier so risk aversion should be decreasing in x

CRRA

- Coefficient of relative risk aversion: $\gamma = -\frac{xu''(x)}{u'(x)}$
- Constant relative risk aversion (CRRA) utility: $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$
  - CRRA utility generates constant relative risk aversion
  - CRRA utility generates absolute risk aversion that is decreasing in wealth

Risk Aversion Takeaways

- Expected utility is (another) work horse model in economics
- Important distinction between the expected value of an uncertain lottery and the expected utility
- Risk aversion explains why people want insurance (some of the biggest markets in the economy are insurance markets)
- CARA and CRRA utility functions are common special cases (worth knowing)
- For further reading, see David Autor's notes on Stellar (Review notes (3/3) risk preferences)

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14.13: Psychology and Economics
Spring 2020

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