14.123 Microeconomics III—Problem Set 1

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Instructions. Each question is 33 points. Make the necessary technical assumptions as you need them. Good Luck!

1. Consider the following game

	w	x	y	z
a	3,2	0,0	0,0	1,1
b	0,0	2,3	0, 0	1,1
c	0,0	0,0	0,0	-1,-1
d	1,1	1,1	-1,-1	0,0

   (a) Compute the set of rationalizable strategies.

   (b) Compute the set of correlated equilibrium distributions.

   (c) Identify a correlated equilibrium that is not a Nash equilibrium.

2. This question asks you to establish the formal link between correlated equilibrium and Bayesian Nash equilibrium. Assuming everything is finite, consider a game $G = (N, S, u)$.

   (a) For any given (common-prior) information structure (Ω, I, p), find a type space (T, p') where the types do not affect the the payoffs in G and a one-to-one mapping τ_i between the information cells $I_i(\omega)$ and types $\tau_i(I_i(\omega)) \in T_i$ (for all $i \in N$), such that an adapted strategy profile $\mathbf{s} = (\mathbf{s}_1, \ldots, \mathbf{s}_n)$ w.r.t. (Ω, I, p) is a correlated equilibrium if and only if $\mathbf{s} \circ \tau^{-1}$ is a Bayesian Nash equilibrium of (G, T, p'). [Here, $\mathbf{s} \circ \tau^{-1} = \left(\mathbf{s}_1 \circ \tau_1^{-1}, \ldots, \mathbf{s}_n \circ \tau_n^{-1}\right)$ is such that, for every type t_i, $\mathbf{s}_i \circ \tau_i^{-1}(t_i) = \mathbf{s}_i(\omega)$ for some ω with $\tau_i(I_i(\omega)) = t_i$.]

   (b) For any type space (T, p') where the types do not affect the the payoffs in G, find a information structure (Ω, I, p) and a one-to-one mapping $w : T \to \Omega$ such that $\mathbf{s} = (\mathbf{s}_1, \ldots, \mathbf{s}_n)$ is a Bayesian Nash equilibrium of (G, T, p') if and only if $\mathbf{s} \circ w^{-1}$ is a correlated equilibrium.

3. For any given game $G = (N, S, u)$, a set $Z = Z_1 \times \cdots \times Z_n \subseteq S$ is said to be *closed under rational behavior* if for every $i \in N$, $z_i \in Z_i$, there exists $\mu \in \Delta(Z_{-i})$ such that $z_i \in \arg\max_{s_i} u_i(s_i, \mu)$.

   (a) Show that if Z is closed under rational behavior, then $Z \subseteq S^\infty$.

   (b) Show that for any family of sets Z^α that are closed under rational behavior, the set $Z = (\cup_\alpha Z_1^\alpha) \times \cdots \times (\cup_\alpha Z_1^\alpha)$ is closed under rational behavior. Conclude that the largest set Z^* that is closed under rational behavior exists.

   (c) Show that $Z^* = S^\infty$.