

12.102 Problem Set #1
POPULATION GROWTH
Due 9/20/05

Part I: Do the following problems:

Exponential growth is a mathematical model commonly applied to populations with constant birth and death rates, and which is reasonably accurate for many populations under ideal environmental conditions for limited periods of time. The differential equation describing exponential growth is:

$$\frac{dP}{dt} = kP$$

which solves to:

$$P(t) = P_0 \exp(kt)$$

where P is the size of the population, t is time, and k is a rate constant based on the birth and death rates.

Go to www.census.gov. There is a wealth of information on this site, much of it difficult to find. One thing that is easy to find is the data for US population, delineated by state, from 1790-2000. You will need that information handy for that following sections.

- 1) Using the 1790 U.S. population as P_0 , find the value for k which gives an exponential model accurately describing the U.S. population statistics from 1790 to 1840. Graph this exponential model along with the U.S. census data. Predict what the U.S. population would have reached by 1930 according to this model and hypothesize on the contrast between your result and the actual U.S. population in 1930. (15 pts)

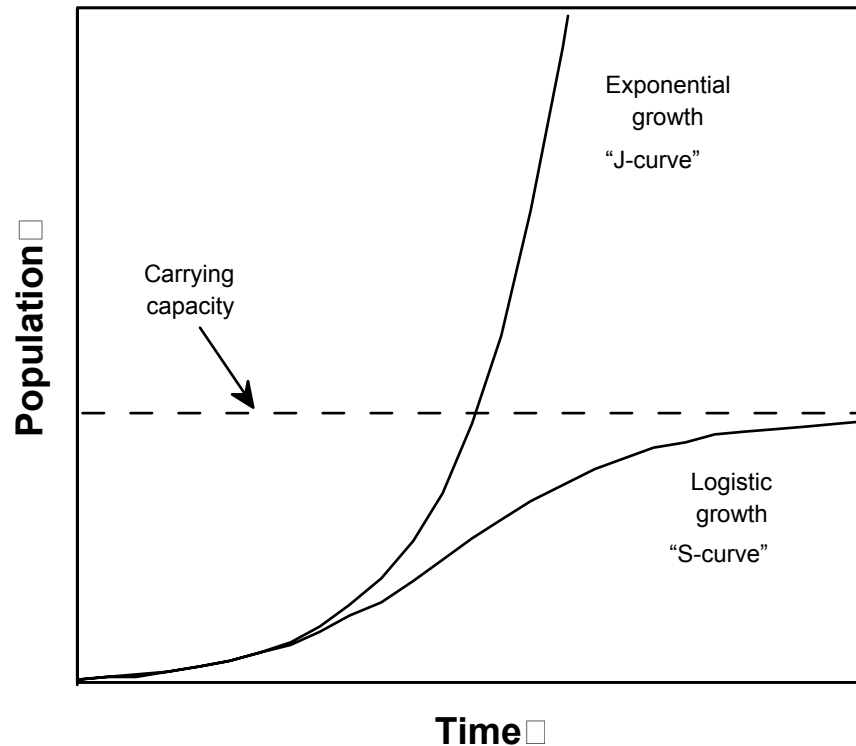
Obviously, there are limiting agents to the growth of natural populations. A simple adjustment to our exponential model is that the birth rate of the population is not constant, but rather is a function of the size of the population. The most famous mathematical expression of this hypothesis is the *logistic* equation:

$$\frac{dP}{dt} = kP(M - P)$$

where k is now what is termed the *intrinsic growth rate*, that is the growth rate in the absence of any limiting factors; and M is the *saturation population*, or the *carrying capacity* of the supporting environment. One solution to the logistic equation can be formulated as:

$$P(t) = \frac{MP_0}{P_0 + (M - P_0)\exp(-kMt)}$$

The form of the solutions to the logistic equation produce what is termed the characteristic S-curve for limiting populations, as compared to the exponential J-curve.



In 1845 the Belgian demographer Verhulst used the logistic equation to model the available U.S. census data and predict the growth of the U.S. population through 1930.

- 2) a) With $P = 3.9$ (in millions), $M = 197.3$ (in millions), and $k = 0.0001589$, generate the curve for U.S. population growth from 1790 to 2000 which Verhulst predicted back in 1845 based on the logistic equation- plot this curve with your exponential function and the U.S. population data set. Comment on the accuracy of his prediction through 1930 and also through the present. (15 pts)

- 3) a) The U.S. population has obviously exceeded the carrying capacity used by Verhulst in his projection. Does the topology of the curve suggest that model is valid? Hypothesize on the factors which determine the carrying capacity of the United States. (15 pts)
 - b) Now plot up, in some fashion you think is reasonable, all the available data for the state of Nevada and compare them with the U.S. results (this is particularly revealing if you observe the *percentage* increase). Consider what you know about the resources required for population growth, and hypothesize about the causes for the trends observed in the U.S. versus the Nevada data. (15 pts)

- 4) Go to <http://www.census.gov/ipc/www/world.html>
 Look at the graphs of:
 - World population: 1950 to 2050
 - World population growth rates: 1950 to 2050
 - Annual world population change: 1950 to 2050

What type of model do you think the projections are based on? Do you think it is accurate?
(15 pts)

Part II:

1. Read the following article:

Liu, J., Daily, G.C., Ehrlich, P.R., and Luck, G.W., 2003, Effects of household dynamics on resource consumption and biodiversity: *Nature*, v. 421, p. 530-533.

Briefly (1-2 paragraphs) discuss the importance of household size on resource consumption, and consider its relevance for the United States' population. (20 pts)

2. Read the following articles:

"The World's Worst Population Problem", by Albert A. Bartlett (Wild Earth, 1998)

"Population and the Environment", (Population Connection, www.populationconnection.org)

"Population Sense and Nonsense", by Nicholas Eherstadt (AEI online, 2002)

Write a ~1 page discussion that evaluates the arguments of the authors for or against a population problem, the data presented to support the argument, and the assumptions made. Conclude by discussing which argument you find more convincing and why. (60 pts)