Crew Scheduling

Outline

- Crew Scheduling
- Work Rules and Policies
- Manual Scheduling Process
- Model Formulation
- Automated Scheduling

Crew Scheduling Problem

Input

- A set of vehicle blocks each starting with a pull-out and ending with a pull-in at the depot
- Crew work rule constraints and pay provisions

Objective

- Define crew duties (i.e. runs, days, or shifts) covering all vehicle block time so as to minimize crew costs

Crew Scheduling Problem

Constraints

- Work rules (hard constraints)
- Policies (preferences or soft constraints)
- Crews available
  - in the short run, the number of crews available are known

Variations

- different crew types
  - full-time
  - part-time
- mix restrictions
  - constraints on maximum number of part-timers

Typical Crew Scheduling Approach

Three-stage sequential approach

1. Cutting long vehicle blocks into pieces of work
2. Combining pieces to form alternative runs
   - with meal breaks, etc.
3. Selection of minimum cost set of runs

- manual process includes only steps 1 and 2
  - also accomplished with automatic heuristics
- optimization process also involves step 3

Typical Crew Scheduling Approach

1 - Cutting Blocks

- each block consists of a sequence of vehicle revenue trips and non-revenue activities
- blocks can be cut only at relief points where one crew can replace another.
- relief points are typically at terminals which are accessible
  - but en-route timing points are an option
- avoid cuts within peak period
  - don’t complicate an already stressed part of operations
- resulting pieces
  - have minimum and maximum lengths
  - should be combinable to form legal runs
    - meal break
    - maximum spread

Vehicle Block Partitions

Definition a *partition* of a block is the selection of a set of cuts each representing a relief.

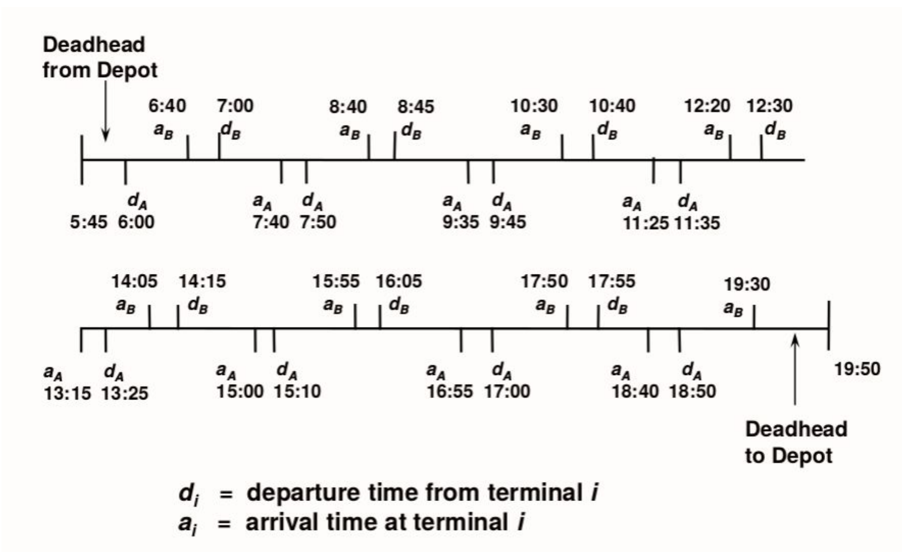
Key problems

- very hard to evaluate a partition before forming runs
- many partitions are possible for any vehicle block

Possible Approaches

- generate only one partition for each vehicle block
- generate multiple partitions for each vehicle block
- generate all possible partitions for each vehicle block

A Vehicle Block on Route AB



Combining Pieces of Work to Form Runs

2 - Combining Pieces

- Large number of feasible runs by combining pieces of work
- Work rules are complex and constraining
  - maximum work hours: e.g. 8 hrs 15 min
  - minimum paid hours (guarantee time): e.g. 8 hrs
  - overtime constraints and pay premiums: e.g. 50% pay premium
- Spread constraints and pay premiums: time between first report and last release for duty, e.g.,

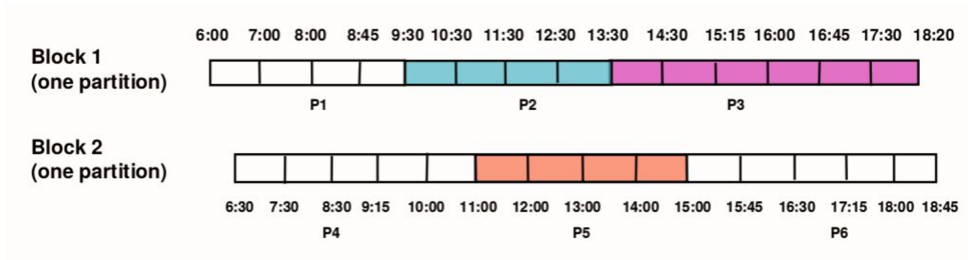
has a spread of 12 hours.

Combining Pieces of Work to Form Runs

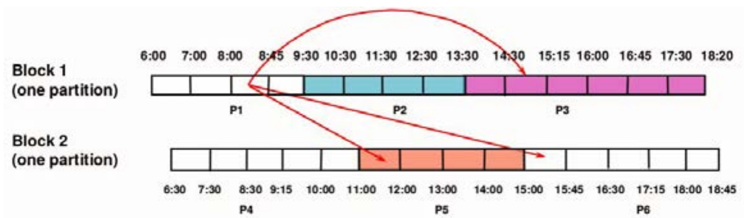
- Swing pay premiums associated with runs with pieces which start and end at different locations, e.g.,

- Different types of duties
  - straight: a continuous run
  - split: a two-piece run
  - trippers: a short run, usually worked on overtime
- Approach: generate and cost out each feasible run
  - infeasible runs are not generated

Combining Pieces of Work to Form Runs



Combining Pieces of Work to Form Runs



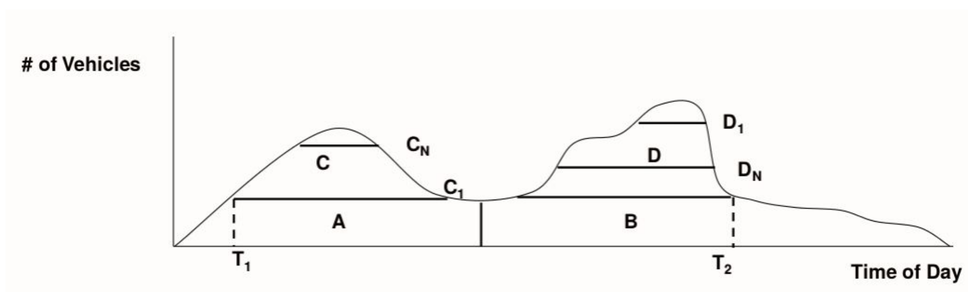
Run #	1st piece	2nd piece	Spread Time	Work Time	Cost
1	P1	P2	07:30	07:30	C1
2	P1	P3	12:20	08:20	C2
3	P1	P5	09:00	07:30	C3
4	P1	P6	12:45	07:15	C4
5	P2	P3	08:50	08:50	C5
6	P2	P6	09:15	07:45	C6
7	P4	P3	11:50	09:20	C7
8	P4	P5	08:30	08:30	C8
9	P4	P6	12:15	08:15	C9
10	P5	P6	07:45	07:45	C10

Possible Runs from pieces P1-P6

Illegal run: Max work time violation

Crew Scheduling: Manual Techniques

3- Minimizing Cost



- T1 is earliest AM pullout which can still serve PM peak
- T2 is latest PM pullback which can still serve AM peak
- A are AM straights (or short split runs)
- B are PM straights (or short split runs)
- C and D are long split or part time runs

Typical Sequence

1. Based on total vehicle hours, estimate total operators required
2. Determine # operators required in AM and PM peaks
3. Determine B based on: # of pull-ins after time T_2.
4. Determine # split runs: (# of PM Peak Vehicles - B)
5. Determine A based on: # of AM Peak Vehicles - split runs
6. Combine earliest pullouts in C with earliest pull-ins in D to produce minimum spread split runs C_1D_1. Iterate until all split runs are matched C_ND_N.

Examples (1/2)

Time Period	# Vehicles	Period Length	# Vehicle Hours
AM Peak	8	3	24
Base	4	6	24
PM Peak	8	3	24
Evening	4	6	24
			96 12 FTOs

Examples (1/2)

Time Period	# Vehicles	Period Length	# Vehicle Hours
AM Peak	8	3	24
Base	4	6	24
PM Peak	8	3	24
			PM + Evening runs = 4 (Straight)
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Examples (2/2)

Time Period	# Vehicles	Period Length	# Vehicle Hours
AM Peak	8	3	24
Base	6	6	36
PM Peak	8	3	24
Evening	3	6	18
			102 13 FTOs

Examples (2/2)

Time Period	# Vehicles	Period Length	# Vehicle Hours
AM Peak	8	3	24
Base	6	6	36
PM Peak	8	3	24
			PM + Evening runs = 3 (Straight)
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Examples (2/2)

Time Period	# Vehicles	Period Length	# Vehicle Hours
AM Peak	8	3	24
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PM Peak	8	3	24
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Selection of Minimum Cost Set of Runs

- Usually built around mathematical programming formulation
- **Problem** Given a set of *m* trips and a set of *n* feasible driver runs, find a subset of the *n* runs which cover all trips at minimum cost

Mathematical Model for Crew Scheduling

Basic Model Set Partitioning Problem

- Notation
  - P = set of trips to be covered
  - R = set of feasible runs
  - c_j = cost of run j
  - δ_i^j = binary parameter
    - 1 - trip (or set of trips) i is included in run j
    - 0 - not included
  - x_j = binary decision variable
    - 1 - run j is selected
    - 0 - not selected

$$\text{Min} \quad \sum_{j \in R} c_j x_j$$

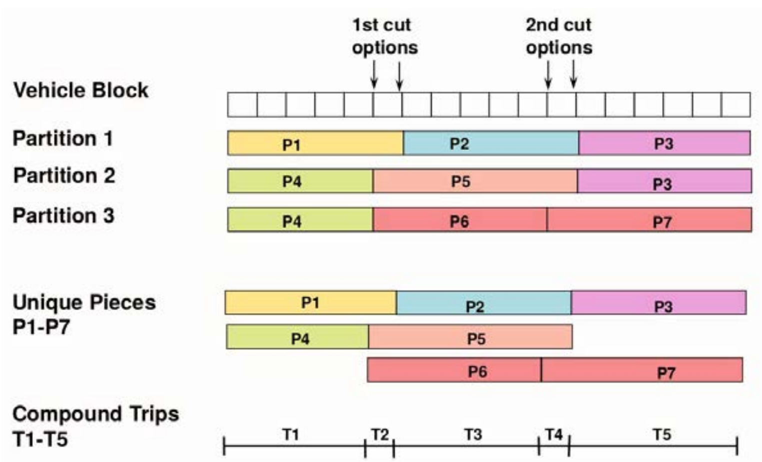
$$\text{Subject to:} \quad \sum_{j \in R} x_j \delta_i^j = 1 \qquad \forall i \in P$$

$$x_j \in \{0,1\}, \qquad \forall j \in R$$

Mathematical Model for Crew Scheduling

- Problem size
  - R decision variables (likely to be in millions)
  - P constraints (likely to be in thousands)
  - much more difficult than the MDVSP
    - complex work rules
    - many valid duties
- Problem size reduction strategy
  - replace individual trips with compound trips consisting of a sequence of vehicle trips which will always be served by a single crew
  - sometimes the first constraint is relaxed to simplify computation
    - allow more than one driver per trip
    - usually leads to few cases of overcovering, which can be eliminated afterwards with heuristics
- Optimization methods
  - column generation
  - branch and price
  - heuristics, e.g. genetic algorithms

Partitions of Vehicle Block, Pieces of Work, and Compound Trip



Variations of Set Partitioning Problem

1. Set R consists of all feasible runs given all feasible partitions for all vehicle blocks
   - size of model explodes with problem size
   - only possible for small problems
2. Set R consists of a subset of all feasible runs
   - not guaranteed to find an optimal solution
   - effectiveness will depend on quantity and quality of runs included
3. Column generation based on starting with a subset of runs and generating additional runs which will improve the solution as part of the model solution process.

Model with Side Constraints

Often the number (or mix) of crew types is constrained in various ways which can be formulated as side constraints

Example: Suppose total tripper hours are constrained to be less than 25% of timetable time.

Let WT = total timetable time

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R^T = set of tripper runs

t_j = work time for tripper run *j*

Then the additional constraint is $\sum_{j \in R^T} t_j x_j \le 0.25 \text{ WT}$

Automated Crew Scheduling Systems

- Virtually universally used in medium and large operators worldwide
- Two most widely used commercial packages are HASTUS (by GIRO Inc in Montreal) and Trapeze (by Trapeze Software Inc in Toronto/Phoenix), each with over 200 customers worldwide
- Typical cost ranges from $100K to $2 M for the software
- Key benefits of automated scheduling are:
  - scheduling process time reductions
  - improved accuracy
  - modest improvements in efficiency (typically 0-3%)
  - provides a key database for many other applications

Automated Crew Scheduling Systems

- Evolution of software has been from “black box” optimization/heuristics to highly interactive and graphical tools
- Current systems allow much greater ability to “shape” the solution to the needs of specific agencies
- One implication however is a profusion of these “soft” parameters which means greater complexity and it is very hard to get full value out of systems.

Rostering

Problem Given duties for a week or a month, combine duties to form rosters

Approaches

- Optimization
  - Minimize required drivers
  - Distribute work evenly - most common in Europe
  - Improve likeability
    - consecutive days off
    - Monday-Friday with weekend off
  - Very large problem: exact OR formulations are seldom used
- Cafeteria pick
  - Most common in North America
  - Pick in order of seniority