Problem Set #2

Issued: September 25, 2006
Due: October 4, 2006

Problem 1

Mendel arrives at a random time to ride the Lemon Line, on which the intervals between trains are equally likely to be 4, 5 or 6 minutes.

a) What is the probability that he waits between 4 and 5 minutes for the next train?

b) Would the answer to the previous question increase or decrease if the Lemon Line became less variable and all intervals between trains were exactly 5 minutes? Intuitively, why does the answer move in that direction?

Problem 2

In an urban region with the triangular shape shown in Figure 1, demand for a certain service is distributed uniformly throughout the region. Travel is according to the right-angle metric, with directions of travel parallel to two of the sides of the triangle. Distances are given in miles. Each point of the urban region is described by its coordinates along the x and the y axis. For example, A is (1,0).
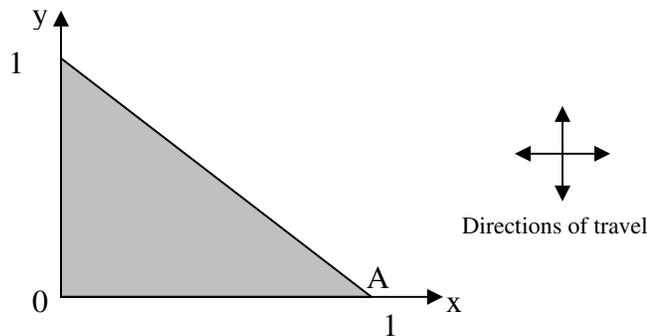

Fig. 1: Urban Area

a) If a single facility providing the service in question is located at point A, find the expected travel distance from a random demand in the region to the facility A.

b) If two identical facilities are provided, one at A and one at B=(0,0), find the expected travel distance from a random demand to its closest facility. (In other words, assume that each demand travels to the facility which is closest to the demand location.)

c) Assume now that there is a single facility located at B. Find the variance $\sigma^2(D)$ of the (right-angle) distance from a random demand to the facility at B.

d) Consider now the same case as in part b), involving two facilities, one at A and one at B. However, a vertical barrier of height 0.25 miles has been placed 0.25 miles from B. Carefully repeat part b), i.e. compute the expected right-angle travel distance between a random demand in the region and the facility closest to it.

Problem 3

A certain rectangular city is five miles long and two miles wide. Mendel lives at (0,0), the geometric center of the city. The city is served by five emergency helicopters, the positions of which at any given time are all independent and uniformly distributed over the city. Any helicopter can travel the straight line path from its present location to the scene of an emergency.

a) Let random variable X be the number of helicopters within one mile of Mendel's home at a random time. What is the probability distribution of x?

b) What is the probability that the THIRD nearest helicopter to Mendel is within one-half mile of his house?

c) A point in the city is said to be covered if there is at least one emergency helicopter within one mile of it. What is the probability that Mendel's house is covered at a random time?

d) At what point(s) in the city is the coverage probability the lowest?

e) Find the ratio of the highest coverage probability for a point in the city to the lowest coverage probability. (The latter quantity is associated with the points identified in d).)

Suppose that the city also has four land-based emergency vehicles, which must travel on a grid of perpendicular streets. Like the helicopters, these vehicles are uniformly and independently distributed over the city. The distance they must travel to the scene of an emergency is based on the Manhattan (horizontal/vertical) metric.

f) What is the probability distribution of z, the number of land-based vehicles within one Manhattan mile of Mendel's house?

g) What is the probability that there are no land-based vehicles within three Manhattan miles of Mendel's house?

Problem 4

Suppose that vehicles are distributed over a large city under a spatial Poisson process with parameter λ, and let v_i be the straight-line distance to a given point from the ith nearest vehicle to that point. Find the joint probability that $v_4 \succ 4v_1$ AND $v_4 \succ 2v_2$.