Tools for Project Evaluation

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Basic Compounding

- Suppose we invest \$x in a bank offering interest rate i
- If interest is compounded annually, asset will be worth
  - $\$x(1+i)$ after 1 years
  - $\$x(1+i)^2$ after 2 years
  - $\$x(1+i)^3$ after 3 years ….
  - $\$x(1+i)^n$ after n years

Opportunity Cost & The Time Value of Money

- If we assume
  - money can always be invested in the bank (or some other reliable source) now to gain a return with interest later
  - That as rational actors, we will never make an investment which we know to offer less money than we could get in the bank
- Then
  - money in the **present** can be thought as of "equal worth" to a larger amount of money in the future
  - Money in the **future** can be thought of as having an equal worth to a lesser "present value" of money

Notation

- Cost

- Revenue

- Simple investment

Equivalence of Present Values

- Given a source of reliable investments, we are indifferent between any cash flows with the same present value – they have "equal worth"
- This indifferences arises b/c we can convert one to the other with no extra expense

Future to Present Revenue

- Given Future Revenue in year t…

- Borrow a smaller amt now from reliable source

- Transforms future revenue to equivalent present revenue, at no additional cost burden

Future to Present Cost

- Given Future Cost in year t…

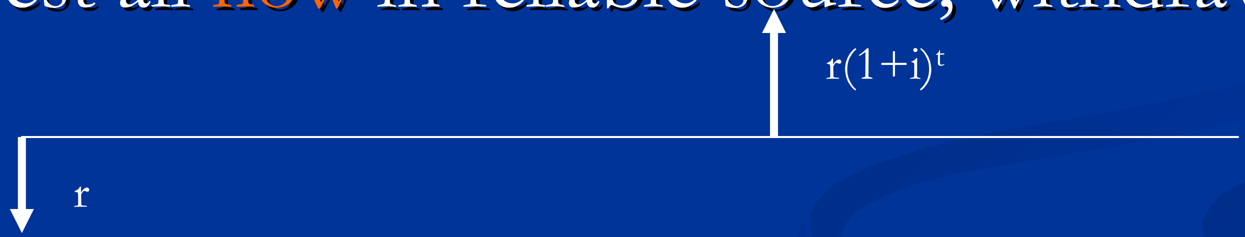
- Invest a smaller amt now in reliable source

- Transforms future cost to equivalent present cost, with no additional cost burden

Present to Future Revenue

- Given Present Revenue…

- Invest all now in reliable source; withdraw at t



- Transforms present revenue to equivalent future revenue, at no additional cost burden

Present to Future Cost

- Given a present cost…

- Borrow = amt now from source; pay back at t

- Transforms current cost to equivalent future, with no additional cost burden

Summary

- Given a reliable source offering annual return i we can shift costlessly between cash flow v at time 0 and $v(1+i)^t$ at time t
- Because we can flexibly switch from one such value to another without cost, we can view these values as equivalent
- The present value of a cash flow *v* at time t is just $v/(1+i)^t$

Notion of Net Present Value

- Suppose we had
  - A collection (or *stream*) of costs and revenues in the future
  - A certain source of borrowing/saving (at same rate)
- The net present value (NPV) is the sum of the present values for all of these costs and revenues
  - Treat revenues as positive and costs as negative

Understanding Net Present Value

- NPV (and PV) is *relative to a discount rate*
  - In our case, this is the rate for the "reliable source"
- NPV specifies the
  - Value of the cash stream *beyond* what could be gained if the revenues were returns from investing the costs (at the appropriate times) in the "reliable source"
    - The "reliable source" captures the *opportunity cost* against which gains are measured
- Key point: NPV of "reliable source" is 0
  - PV(revenue from investment)=PV(investment cost)

Example: High-Yield Investment

- Assume reliable source with 10% annual interest
- Invest $100 in high-risk venture at year 0
- Receive $121 back at year 1
- What is the net present value of this investment?
- What is the net future value of this investment?
- What does this mean?

Example: Money in Mattress

- Assume reliable source with 10% annual interest
- Place $100 in mattress at year 0
- Retrieve $100 from mattress at year 1
- What is the net present value of this investment?
- What does this mean?

Discounted Cash Flow

- Computing Present Value (PV) of costs & benefits involves successively discounting members of a cash flow stream
  - This is because the value of borrowing or investment to/from the "reliable source" rises exponentially
- This notion is formalized through
  - Choice of a **discount rate r**
    - In the absence of risk or inflation, this is just the interest rate of the "reliable source" (gain through opportunity costs)
  - Applying discount factor $1/(1+r)^t$ to values at time t

Outline

- ✓ Session Objective
- ✓ Project Financing
  - ✓ Public
  - ✓ Private
  - ✓ Project
  - ✓ Contractor
  - ✓ Additional issues
- Financial Evaluation
  - ✓ Time value of money
  - ✓ Present value
  - ✓ NPV & Discounted cash flow
  - Simple Examples
  - Formulae
  - IRR
- Missing factors

Develop or not Develop

- Is any individual project worthwhile?
- Given a list of feasible projects, which one is the best?
- How does each project rank compared to the others on the list?
  - "Objective: Strive to secure the highest net dollar return on capital investments which is compatible with the risks incurred"

We can EITHER Use NPV to

- Evaluate a project against some opportunity cost
  - Use this opportunity to set the discount rate r

	>		Accept the project
NPV	=	0	Indifferent to the project
	<		Reject the project

  - Use NPV to choose the best among a set of (mutually exclusive) alternative projects

Rates

- Discount Rate:
  - Worth of Money + risk
- Minimum Attractive Rate of Return (MARR)
  - Minimum discount rate accepted by the market corresponding to the risks of a project

Choice of Discount Rate

$r = r_f + r_i + r_r$

<u>Where:</u>

r - is the discount rate.
r_f - the risk free interest rate. Normally government bond.
r_i - Rate of inflation. It is measured by either by consumer price index or GDP deflator.
r_r - Risk factor consisting of market risk, industry risk, firm specific risk and project risk.

$r_r =$
- Market Risk
- Industry Risk
- Firm Specific Risk
- Project Risk

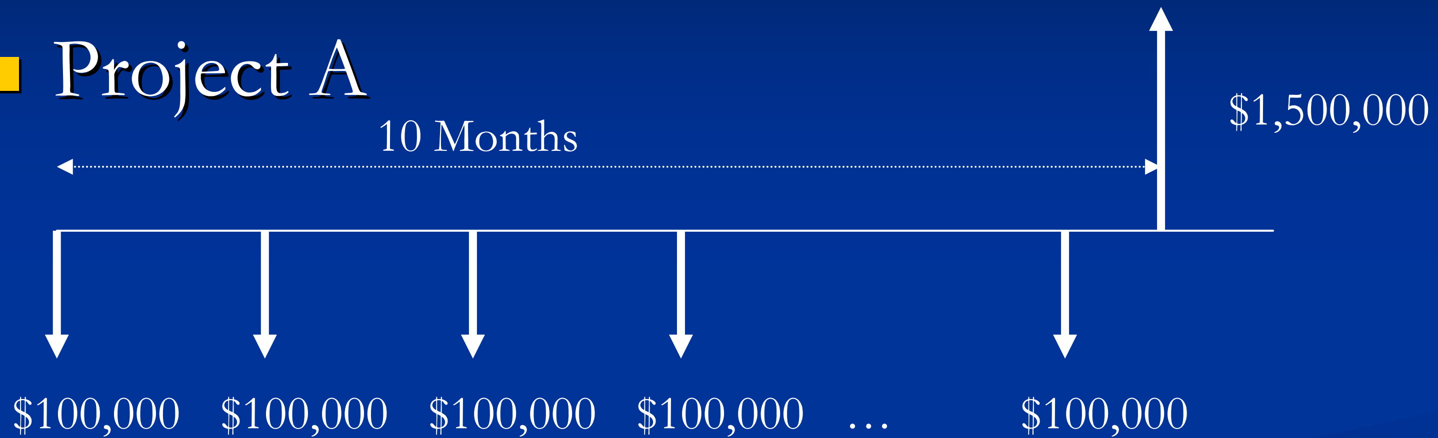
Project Evaluation Example

- Warehouse A
- Construction=10 months
- Cost = $100,000/month
- Sale Value=$1.5M
- Total Cost?
- Profit?
- Better than B?

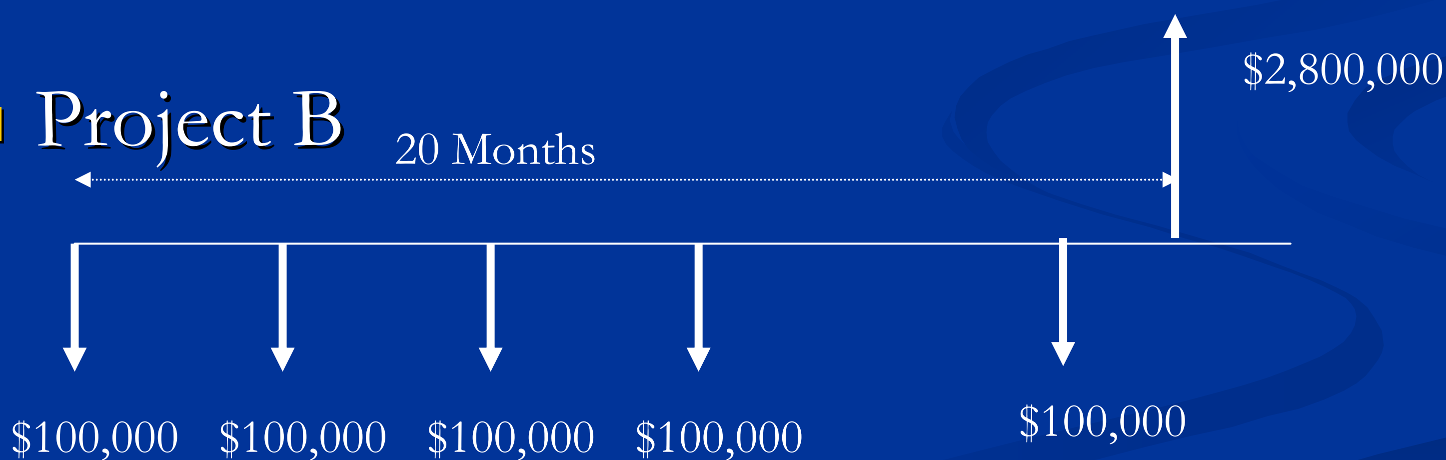
- Warehouse B
- Construction=20 months
- Cost=$100,000/month
- Sale Value=$2.8M
- Total Cost?
- Profit?
- Better than A?

Drawing out the examples

- Project A



- Project B



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Interest Formulas

- i = Effective interest rate per interest period (discount rate of MARR)
- t = Number of compounding periods
- PV = Present Value
- NPV = Net Present Value
- FV = Future Value
- A = Annuity

Interest Formulas: Payments

- Single Payment Compound Amount Factor
  - $(F/P, i\%, n) = (1 + i)^n$
- Single Payment Present Worth Factor
  - $(P/F, i\%, n) = 1/ (1 + i)^n = 1/ (F/P, i\%, n)$
- Uniform Series Compound amount Factor
  - $(F/A, i\%, n) = (1 + i)^n - 1 / i$
- Uniform Series Sinking Fund Factor
  - $(A/F, i\%, n) = i / (1 + i)^n - 1 = 1 / (F/A, i\%, n)$

Interest Formulas: Series

- Uniform Series Present Worth Factor
  - $(P/A, i\%, n) = 1/ i - 1/ i (1 + i)^n$
- Uniform Series Capital Recovery Factor
  - $(A/P, i\%, n) = [i (1 + i)^{n}] / [(1 + i)^n – 1] = 1 / (P/A, i\%, n)$

Note on Continuous Compounding

- To this point, we have considered annual compounding of interest
- Consider more frequent compounding
  - Interest is in %/year
  - Fraction of interest gained over time Δt (measured in years)=$i\Delta t$
  - For n compounding periods/year, effective rate for entire year is

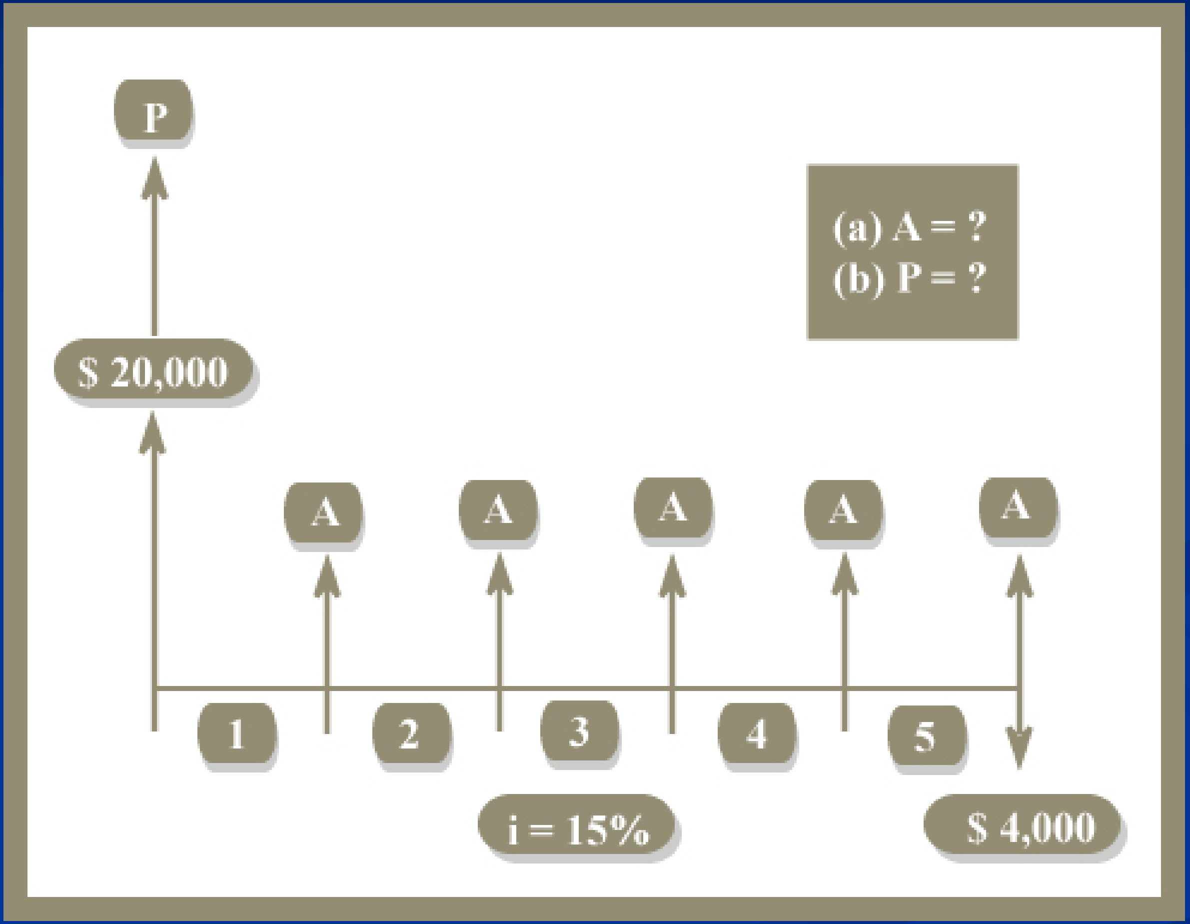
$$\left(1+\frac{i}{n}\right)^{n}$$

  - As $n\to\infty$ we approach continuous compounding and quantity approaches e^{i}
  - Over t years, we have e^{it}

Equipment Example

- $ 20,000 equipment expected to last 5 years
- $ 4,000 salvage value
- Minimum attractive rate of return 15%
- What are the?
  - A - Annual Equivalent
  - B - Present Equivalent

Equipment Example



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Internal Rate of Return (IRR)

- Identifies the rate of return on an investment
  - Example: Geometrically rising series of values
- Typical means of computing: Identify the discount rate that sets NPV to 0

IRR Investment Rule

	$>$		Accept
r^-	$=$	r^*	Indifferent
	$<$		Reject

"Accept a project with IRR larger than the discount rate."

Alternatively,

"Maximize IRR across mutually exclusive projects."

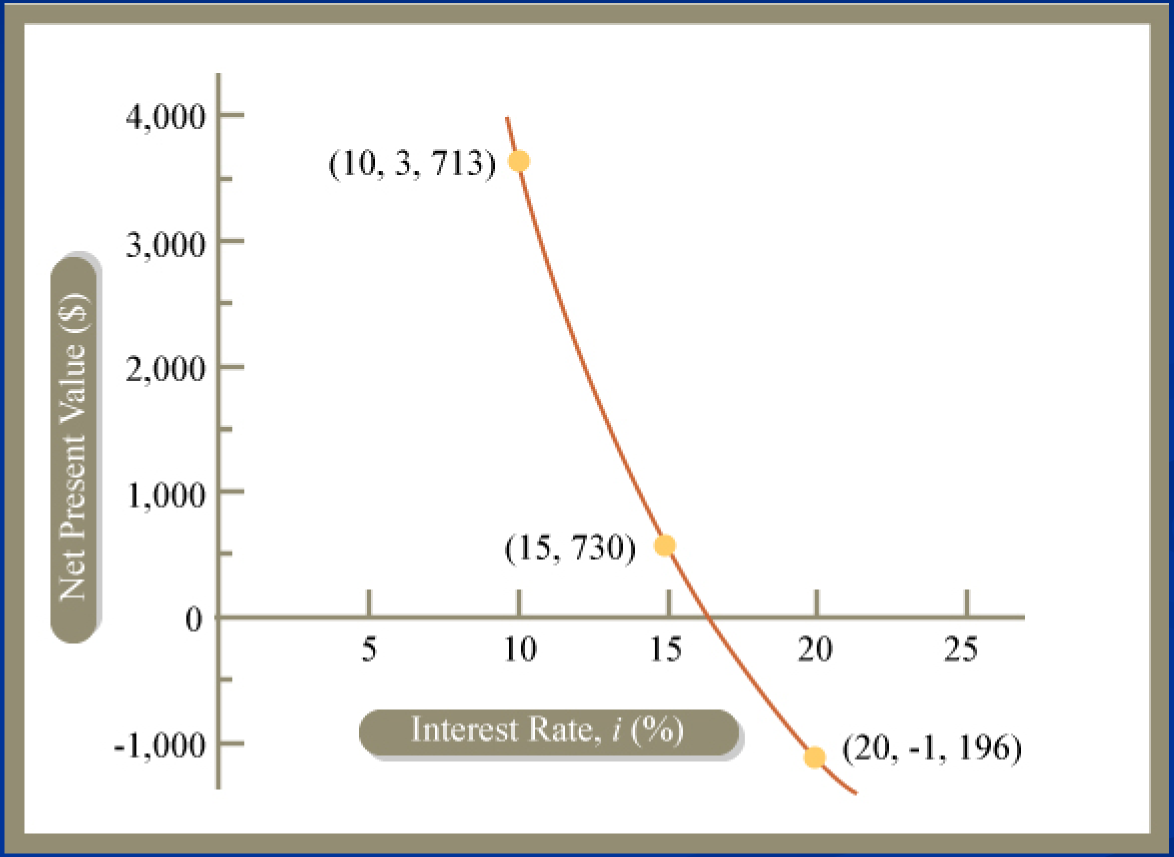
Pena-Mora 2003

Internal Rate-of-Return Method (IRR) Example

	MACHINE A
Initial cost	$ 20,000
Life	5 years
Salvage value	$ 4,000
Annual receipts	$ 10,000
Annual disbursements	$ 4,400

- $0(r\%) = -20{,}000 + 5{,}600\ (P/A, r\%, 5) + 4{,}000\ (P/F, r\%, 5)$
- i = +/- 16.9% > 15% then the project is justified

Internal Rate-of-Return Method (IRR) Graph



IRR vs. NPV

- Most times, IRR and NPV give the same decision / ranking among projects.
- IRR does not require to assume (or compute) discount rate.
- IRR only looks at *rate* of gain – not *size* of gain
- IRR ignores capacity to reinvest
- IRR may not be unique (payments in lifecycle):

Trust NPV: It is the *only* criterion that ensures wealth maximization

Pena-Mora 2003

Other Methods I

- Benefit-Cost ratio (benefits/costs) or reciprocal
  - Discounting still generally applied
  - Accept if <1
  - Common for public projects
  - Does not consider the absolute *size* of the benefits
  - Can be difficult to determine whether something counts as a "benefit" or a "negative cost"
- Cost-effectiveness
  - Looking at non-economic factors
  - Discounting still often applied for non-economic
    - $/Life saved
    - $/QALY

Other Methods II

- Payback period ("Time to return")
  - Minimal length of time over which benefits repay costs
  - Typically only used as secondary assessment
  - Drawbacks
    - Ignores what happens after payback period
    - Does not take discounting into account
  - Discounted version called "capital recovery period"
- Adjusted internal rate of return (AIRR)

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What are we Assuming Here?

- That only quantifiable monetary benefits matter
- Certain knowledge of future cash flows
- Present value (discounting) using equal rates of borrowing/lending

Money is not Everything

- Social Benefits
  - Hospital
  - School
  - Employment opportunities
- Intangible Benefits
  - New cafeteria
- Strategic benefits
  - Partnering with firm for long-term relationship

We are missing critical uncertainties

- Revenue
  - Level of occupancy
  - Elasticity and Level of cost
  - Duration of project
  - Post-construction revenue
    - *Sale of building*
- Costs
  - Construction costs
    - *Environmental conditions*
    - *Labor costs*
    - *Size of lowest bid*
    - *Variable interest rates*
  - Maintenance costs
    - *Energy costs*
    - *How quickly items wear out*
    - *Labor costs*