**1.022 Introduction to Network Models** **Fall 2018**

# Homework 6

## 6.1 Romeo Juliet

***[50 points]*** Romeo and Juliet are in love. Romeo positively reacts to Juliet; he loves her more if she shows him more love and he loves her less when she shows less. Juliet is a fickle lover; she loves Romeo more when he loves her less and visa versa. We want to model their love affair as a dynamical system in order to predict what will happen to them in the future. To do so, let $x(k)$ be the amount of love Romeo has for Juliet (measured in love units!), and let $y(k)$ be the amount of love Juliet has for Romeo. A simple dynamical system representing their interactions is as follows: for some real numbers $a$ and $b$, the love at time $k+1$ is given by

$$x(k+1) = x(k) + ay(k) \qquad y(k+1) = bx(k) + y(k).$$

Assume that, initially $x(0), y(0) > 0$. Answer the following questions:

1. Determine the signs of $a$ and $b$ to reflect the behavior of Romeo and Juliet.
2. For what ranges of parameters $a$ and $b$ will Romeo's and Juliet's love fizzle away regardless of where they start?
3. For what ranges of parameters $a$ and $b$ will Romeo and Juliet be forever caught in a cycle of love and hate?
4. Both Romeo and Juliet were burnt before from loving someone else that does not love them. As a result, their love tomorrow discounts their own love today by a factor of 0.5. Rewrite the model and answer questions 1 and 2.
5. What happens if both Romeo's and Juliet's love increases by one unit every single time regardless of the actions of the other? Answer questions 1 and 2.

## 6.2 Markov Chains

There are $n$ fish in a lake, some of which are green and the rest blue. Each day, Helen catches 1 fish. She is equally likely to catch any one of the $n$ fish in the lake. She throws back all the fish, but paints each green fish blue before throwing it back in. Let $G_i$ denote the event that there are $i$ green fish left in the lake.

1. Show how to model this fishing exercise as a Markov chain, where $G_i$ are the states (Explain why your model satisfies the Markov property; how many states does this Markov chain have?)
2. Find the transition probabilities $p_{ij}$.
3. Is $P = [p_{ij}]$ irreducible? Is it aperiodic?
4. Does this Markov chain have a stationary distribution? If yes, what is the distribution?