

Lecture 22: Liquid Motors: Combustion Instability (Low Frequency)

Combustion Stability

1. General Discussion

Elimination of instabilities has been historically one of the largest components of all new liquid rocket development programs. This is because there has been little reliable methodology to ensure stability through design, and also because of the potentially catastrophic consequences of instability. The situation has improved to some extent because of the vastly enlarged simulation capabilities existing now. As we will see, this area is intimately related to that of spray combustion, where a similar situation has prevailed. As in the combustion area, the advances in recent years are promising, but not yet sufficient to provide reliable tools for a priori design, at least against the high-frequency instability problem.

In a system with the very large energy density of a rocket combustor, there are bound to be many mechanisms by which a small fraction of this energy can be channeled into undesirable oscillations. The resulting instabilities are usually categorized into "low frequency" and "high frequency" types. The former involve pressure oscillations which are slow enough compare to the acoustic passage time that the whole chamber participates in phase, while the latter exhibits acoustic behavior, with different parts of the combustor oscillating with different phase or amplitude. In either case, the instability arises when energy (or sometimes mass) can be added to the gas at, or near, peaks in its pressure oscillation (this is the classical Rayleigh criterion for instability, as explained) for example in Refs. (21) and (26)*. This implies a synchronization of two mechanisms, one which generates the energy or mass in some unsteady manner, and another one which allows the gas pressure and other thermodynamic quantities to oscillate. These two mechanisms must have comparable time constants for the mutual feedback to develop.

The acoustic modes of typical rocket combustors have wavelengths which are some fraction of their linear size, and a wave speed (the speed of sound) of the order of 2000 m/sec. Thus their frequencies are in the KHz range, and hence the oscillation time constant is 10^{-4} to 10^{-3} sec, and potential couplings are to be sought to energy or mass release mechanisms with time constants of that order. To be more precise, it must be noted that a time lag in the release of combustion energy (such as, for example atomization, evaporation, mixing or reaction delays) will not per se provide coupling with the acoustic field. There must also be a sensitivity of the heat release rate, and hence of the time delay, to the pressure or other wave quantity. Since, in general, only some of the additive time lags show such a sensitivity, and then maybe only for a fraction of the effective lag, it is found that the excited acoustic modes have time constants which are a fraction of the time lags themselves. As an example, Eq. (22 of lecture 19) gives for a droplet with $\rho_l = 1000$ kg/m³, $\ln(I + B) = 5$ and $\rho_g D \cong 3 \times 10^{-5}$ kg/m/sec a vaporization time of 8.2 msec. Yet, this size droplets could de-stabilize acoustic modes of about 1KHz frequency, as reported, for instance in Ref. 27.

* See Ref. List and end of Lecture 19

Other excitation mechanisms for high frequency waves have been reported. Gas-phase propellant mixing delays in rapidly evaporating cryogenics may be of the required order and show sensitivity to acoustic fields. Conversion of vortex energy to acoustic dipole radiation upon impingement of a vortex street or a turbulent shear layer on an obstacle can also be a destabilizing factor [28], although this mechanism may also work as a damper, if the phase is arranged appropriately. On the other hand acoustic frequencies are normally too high to allow involvement of elements outside the chamber itself, such as the liquid-handling equipment or the overall rocket structure. Also, chemical reaction times in well-mixed gases tend to be shorter than 0.1-1 msec, and are not usually involved either.

The low frequency types of instability, by contrast, usually results from coupling of oscillations of pressure in the whole chamber to elements outside of it. These can be the injectors, the liquid lines (or the liquid in them), the turbopumps, and, in the instability called POGO, the whole rocket structure. There is substantially more understanding of these than of the fast instabilities, and analytical techniques were developed early on [26] to help avoid them in the design process. Many of the low-frequency instabilities can be described in terms of a single, non-sensitive, combustion delay; as discussed, this delay can be several times longer than its sensitive fraction. In others cases, the mechanism may be more complex, perhaps involving different delays for the two propellant streams.

The literature also mentions an intermediate and benign type of instability, which is a hybrid sometimes described as "entropy wave". Acoustic oscillations may modulate differently the oxidizer and fuel injection rates, creating patches of varying stoichiometry and temperature. These convect with the mean flow, and, upon impingement on the converging nozzle, may synchronize with and reinforce the acoustic wave, which then propagates back to the injector region. Since part of the mechanism is acoustic and part convective, the frequency falls between the first acoustic longitudinal mode and the flow passage time.

The effects of these instabilities range from simple vibratory and acoustic disturbances to very rapid burnout of walls. The most severe effects are associated with high-frequency instabilities of the tangential chamber modes. These create sloshing gas motions which completely disrupt the boundary layer near the injector end of the chamber, and can lead to failure in a matter in seconds. Thus, most effort has gone into understanding and eliminating tangential mode instabilities. Low frequency instabilities, if allowed to proceed, can also be structurally damaging, but, as noted, they can often be designed out, and if not, there are reliable countermeasures against them.

2. Methods of Analysis for Low Frequency Instabilities

The compilation by Harrje and Reardon (Ref. 26)* remains the best source of information on this topic. The analysis typically separates into a chamber portion, balancing the gas generation, storage and disposable, plus a series of submodels detailing the response of various other parts of the system, such as injectors, pumps, etc. For preliminary analysis, linear perturbation models are used, and Ref. (26)* shows a variety of subsystem response functions adequate for this purpose.

* - D.T. Harrje and T.H. Reardon (Editors) Liquid Propellant Rocket Combustion Instability, NASA SP-194, 1972.

As a simple prototype, we discuss here the coupling of chamber and injector alone (single stream). It is assumed that vaporization of the liquid happens a time τ_v after injection, which may be a reflection of the time delay for atomizing into droplets, plus the droplet evaporation time given by Eq. 22 (Lecture 19), although this part would be better modeled as a distributed, rather than a lumped lag.

Let P_c and T_c be the chamber pressure and temperature, both of which are taken to be uniform, and V_c the chamber volume. The vaporization rate is:

$$\dot{m}_v(t) = \dot{m}_i(t - \tau_v) \quad (1)$$

while the gas disposal rate through the nozzle is as given by $\dot{m} = P_c A_t / c^*$. We then have

$$\frac{d}{dt} \left(\frac{P_c}{R_g T_c} V_c \right) = \dot{m}_i(t - \tau_v) - \frac{P_c A_t}{c^*} \quad (2)$$

The injection rate through an effective injector orifice area A_i , with a pressure P_0 behind it, is

$$\dot{m}_i = \sqrt{2 \rho_L (P_0 - P_c)} A_i \quad (3)$$

where ρ_L is the gas liquid density. Linearization of these equations, assuming constant R_g , T_c , P_0 and c^* , gives

$$\frac{V_c}{R_g T_c} \frac{dP_c'}{dt} = \bar{m} \left[-\frac{1}{2} \frac{P_c'}{P_0 - \bar{P}_c} - \frac{P_c'}{\bar{P}_c} \right] \quad (4)$$

where $\bar{m} = \frac{\bar{P}_c A_t}{c^*} = \sqrt{2 \rho_L (P_0 - \bar{P}_c)} A_i$ is the mean mass flow rate, \bar{P}_c is the mean chamber pressure, and P_c' its perturbation. We can define a "chamber evacuation time"

$$\tau_c = \frac{P_c}{R_g T_c} \frac{V_c}{\bar{m}} \quad (5)$$

and an injector overpressure parameter

$$\Delta = \frac{P_0 - \bar{P}_c}{\bar{P}_c} \quad (6)$$

and assume pressure variations of the form

$$\frac{P'_c}{P_c} = \hat{c} e^{i\omega t} \quad (7)$$

Substituting into (2.25) yields an equation for the complex ω :

$$i\omega\tau_c = -\frac{e^{-i\omega\tau_v}}{2\Delta} - 1 \quad (8)$$

and an instability will result when the imaginary part of ω as given by this equation is negative. The stability threshold occurs when ω is real. Imposing this and eliminating ω between the real and imaginary part of Eq. (8) yields the threshold condition

$$\frac{\tau_v}{\tau_c} = \frac{2\Delta}{\sqrt{1-4\Delta^2}} [\pi - \cos^{-1}(2\Delta)] \quad (9)$$

at which the oscillations occur with a frequency given by

$$\omega\tau_v = \pi - \cos^{-1}(2\Delta) \quad (10)$$

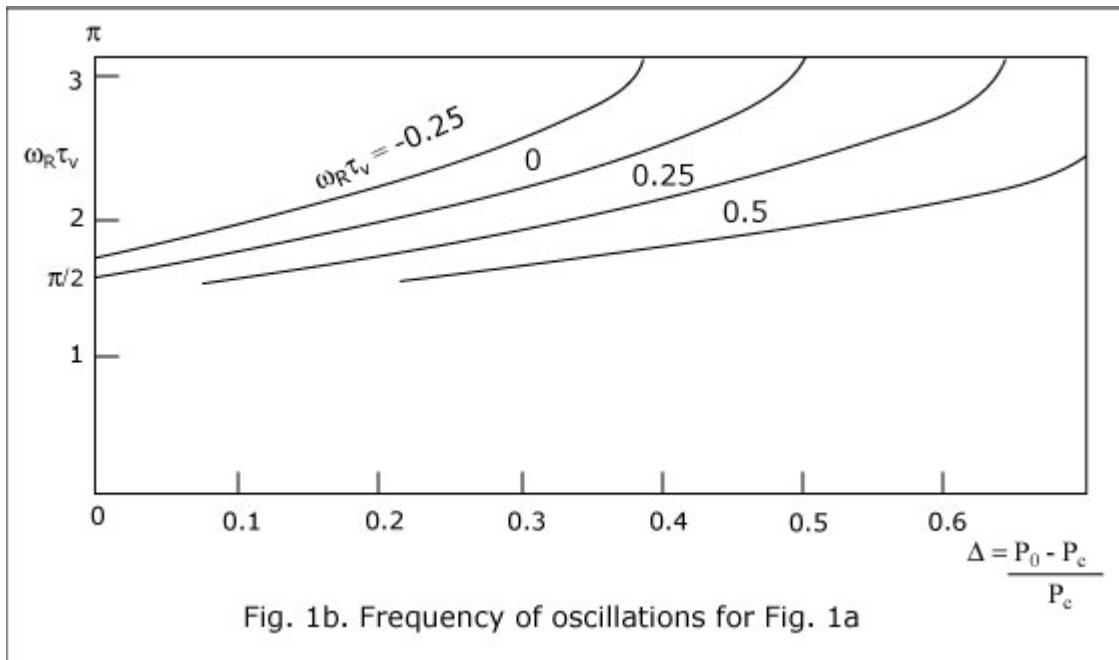
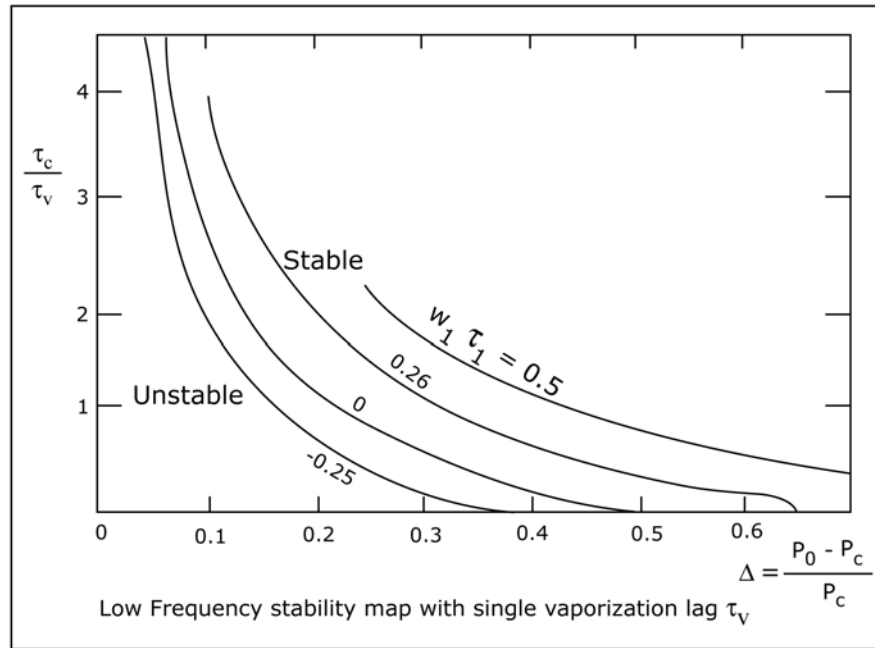
Notice that this instability threshold cannot be reached if $\Delta > \frac{1}{2}$. This is because of the strong energy-dissipating role of the injector, which acts as a damper for the oscillations. Relatively large injector pressure drops are routinely used, of the order of $\Delta \cong 0.2$. Values above $\Delta = \frac{1}{2}$ would lead to excessively and lossy pressurization and injection systems.

NOTE:
(at threshold)

$$\dot{m}_v e^{i\omega t} = -\frac{1}{2\Delta} \left(\frac{\hat{P}'_c}{P'_L} \right) e^{i\omega t} e^{-i\omega\tau_c} = \frac{1}{2\Delta} \left(\frac{\hat{P}'_c}{P_c} \right) e^{i(\pi-\omega\tau_c)} = \frac{1}{2\Delta} \left(\frac{\hat{P}'_c}{P_c} \right) e^{i\cos^{-1}(2\Delta)}$$

Hence, mass addition occurs at an advance phase angle $\cos^{-1}(2\Delta)$ w.r.t. pressure peaks. If Δ is small, nearly on top.

Allowing a non zero (positive or negative) imaginary part of ω , we can obtain the results shown in Fig. 1. In general, stability is enhanced by increasing injector drop or chamber residence time (in relation to the evaporation lag). As shown in Fig. 2.3b, the stable or neutral oscillations have frequencies of the order of $f_r \approx (1 - 2) / 4\tau_v$. As an example of the kinds of delays encountered in LOX-LH rockets, we reproduce Fig. 2 (from Ref. 26).



So, three cases:

$$1. \quad \Delta = 0.5 \rightarrow \frac{\dot{m}_v}{\dot{m}} = \frac{1}{2\Delta} \left(\frac{P'_c}{P_c} \right)$$

Gas mass production in phase with gas expulsion (as with P'_c).

$$2. \quad \Delta > 0.5 \rightarrow \frac{\dot{m}_v}{\dot{m}} = \frac{1}{2\Delta} \left(\frac{P'_c}{P_c} \right) e^{-i(-)}$$

\dot{m}_v lags \dot{m}_{ont} , chamber begins emptying before new gas is introduced \rightarrow Stable.

$$3. \quad \Delta < 0.5 \rightarrow \frac{\dot{m}_v}{\dot{m}} = \frac{1}{2\Delta} \left(\frac{P'_c}{P_c} \right) e^{+i()}$$

\dot{m}_v leads \dot{m}_{ont} , New gas starts accumulating before it can be expelled. This can be unstable, if the τ_c is small enough compared to τ_v .

Notice that the "sensitive" (to pressure) part of this time lag is 0.1 to 0.2 of the total lag. According to the results in Fig. 1, the marginally stable oscillations with the total delay of Fig. 2, (and with $\Delta \approx 0.2$) would be at approximately a frequency

$$f = \frac{1}{2\pi} \frac{1.7}{10^{-3}} \approx 270 \text{ Hz}$$

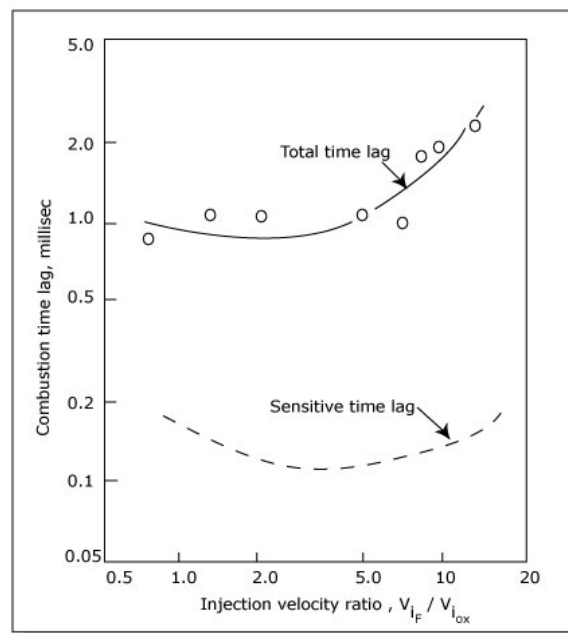


Fig. 2 Experimental values of combustion time lag for supercritical chamber pressure