16.30/31 September 25, 2010
Prof. J. P. How and Prof. E. Frazzoli **Due:** October 1, 2010
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16.30/31 Homework Assignment #3

Goals: Classical lead/lag controller design; state space realizations

3. A simplified model of a glider is

$$\begin{aligned}\dot{\gamma} &= -\cos(\gamma)g/v + ng/v \\ \dot{v} &= -\sin(\gamma)g - k_1 n^2 g/v^2 - k_2 v^2 g,\end{aligned}$$

where γ is the flight path angle in radians, v is the airspeed in m/sec, $n = \frac{L}{mg}$ is the load factor, L is the lift in Newtons, m is the mass in kg, and $k_1 = 61.6594$ and $k_2 = 4.8747 \times 10^{-5}$ are constants for the glider.

(a) Given that $\gamma = -0.15$ rad, and the airspeed is 50.8691 m/sec, find the necessary load factor to maintain equilibrium.

(b) Let the state vector be $[\gamma \;\; v]^T$, let the input be n, and let the output of interest be v. Derive the linearized system about the equilibrium point obtained from above.

(c) From the linearized system dynamics, derive the transfer function from input n to output v, and the transfer function from input n to output γ. (*Hint:* Use `tf` and `ss` in Matlab to check your answers.)

(d) Plot the unity gain negative feedback root locus of the linearized system using Matlab. Give the locations of the open loop poles, and the range of gains for which the closed-loop system is stable.

4.

5. (16.31 required/16.30 extra credit) Consider the state-space equations

$$\begin{aligned}\dot{x}_1 &= x_1(u - \beta x_2)\\ \dot{x}_2 &= x_2(-\alpha + \beta x_1)\end{aligned}$$

where $u \in \mathbb{R}$ is the input and $\alpha, \beta > 0$ are positive constants.

(a) Is this system linear or nonlinear, time-varying or time-invariant?

(b) Determine the equilibrium points for this system, assuming a constant input u.

(c) Near the positive equilibrium point from (b), find a linearized state-space model of the system. What can you say about the stability of the nonlinear system at this equilibrium point, as a function of u?

[1] R. C. Nelson, *Flight Stability and Automatic Control*, 2nd ed.

16.30 / 16.31 Feedback Control Systems
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