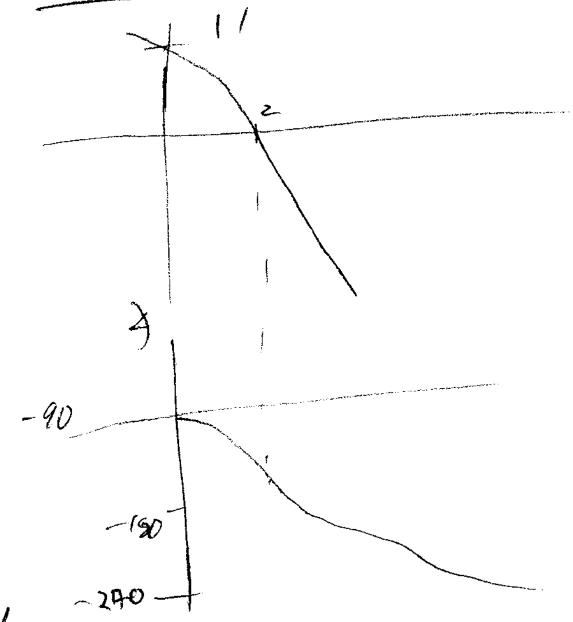
PSET 5

1.) $G(s) = \dfrac{5}{s(s/1.4+1)(s/3+1)}$



Keep DC gain constant

$-140 = -90 - \tan^{-1}(\omega/1.4) - \tan^{-1}(\omega/3) + \sin^{-1}\left(\dfrac{\alpha-1}{\alpha+1}\right)$

α has max value of 10 to keep noise rejection reasonable

$-140 = -90 - \tan^{-1}(\omega/1.4) - \tan^{-1}(\omega/3) + 55^\circ$

$\omega_c = 2.7$ max crossover

$\text{lead} = \left(\dfrac{\alpha\tau s+1}{\tau s+1}\right)$

Need geometric mean of lead to be equal to ω_c.

$\omega_c = \dfrac{1}{\sqrt{\alpha}\,\tau} \Rightarrow \tau = \dfrac{1}{\sqrt{10}\,(2.7)}$ $\quad \tau = 0.1164$

$1 = K \dfrac{((10\cdot 0.1164\cdot 2.7)^2+1)^{1/2}}{((0.1164\cdot 2.7)^2+1)^{1/2}} \cdot \dfrac{5}{2.7\left(\left(\frac{2.7}{1.4}\right)^2+1\right)^{1/2}\left(\left(\frac{2.7}{3}\right)^2+1\right)^{1/2}}$

3.14633 $\quad$ .633613

$K = 1/2$

$\text{lead} = \dfrac{1}{2}\left(\dfrac{10\cdot 0.1164s+1}{0.1164s+1}\right)$

ar

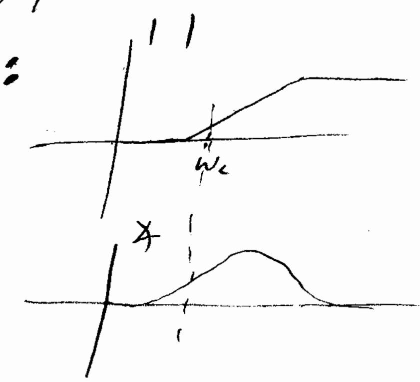
PSET 5

1.) $G(s) = \dfrac{5}{s(s/1.4 + 1)(s/3 + 1)}$

lead $= \dfrac{s/a + 1}{s/b + 1}$ $\quad b/a = 100$ gives $78°$ of phase

Placing zero at current crossover will keep crossover somewhat the same. However the additional increase in gain, due to lead, will increase ω_c slightly, and the phase margin becomes negative. To compensate, we need a large b/a to make up for that lost phase and push up to $\approx 40°$ phase margin. thus, choose $b/a = 100$ placing zero at crossover, $\omega \approx 2$.

The plant phase rolls off quicker than the phase lead compensation gives you. So you want to push crossover as little as possible. therefore, you want ω_c to occur early in the lead compensator:



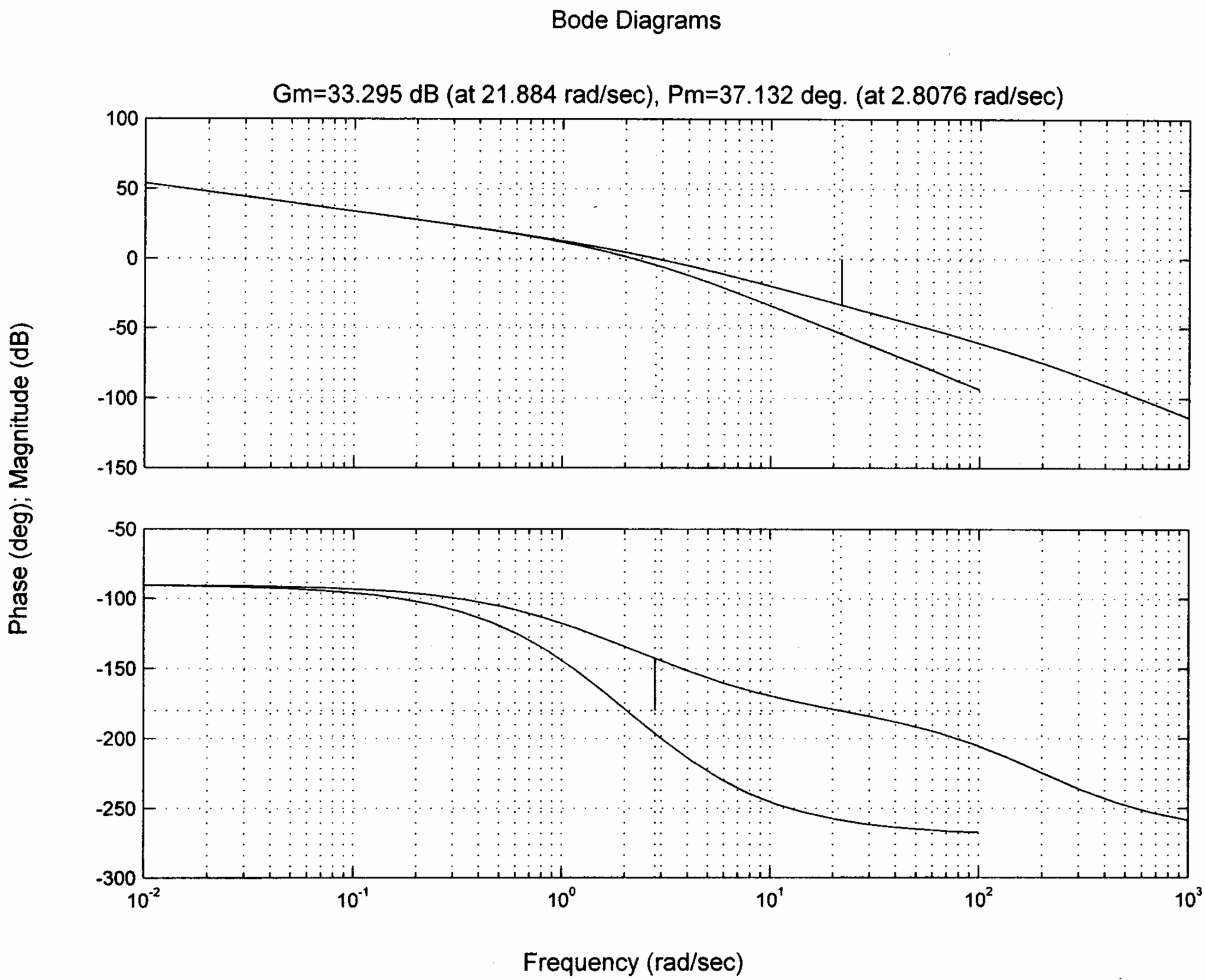
This will reduce the bad effect of the system phase at higher frequencies.

$G_{lead} = \dfrac{s/2 + 1}{s/200 + 1}$ $\quad$ Try this compensator and iterate if necessary

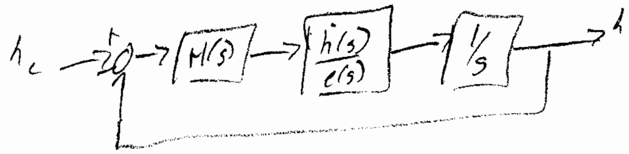
check bode plots .... $\quad$ Approx. bandwidth is open loop ω_c.

There are MANY answers! $\quad \omega_c = 2.9076$

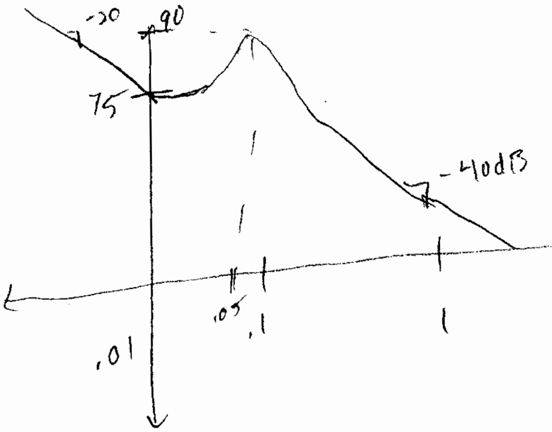
Gt

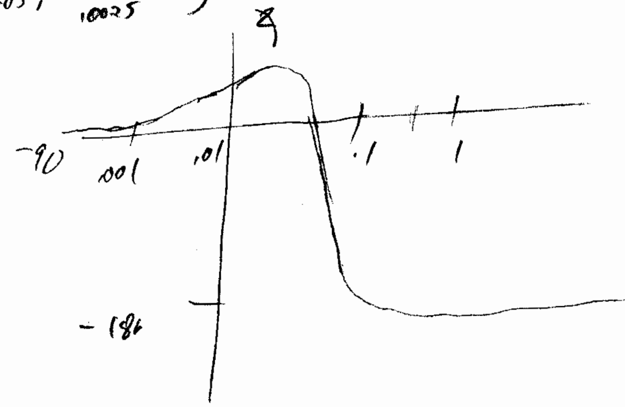

Bode Diagrams
Gm=33.295 dB (at 21.884 rad/sec), Pm=37.132 deg. (at 2.8076 rad/sec)
Phase (deg); Magnitude (dB)
100
50
0
-50
-100
-150
-50
-100
-150
-200
-250
-300
10^-2
10^-1
10^0
10^1
10^2
10^3
Frequency (rad/sec)

2.) a.) $\frac{h'(s)}{e(s)} = \frac{15(s+0.01)}{s^2+0.01s+0.0025}$



$$\frac{h(s)}{e(s)} = \frac{15(s+0.01)}{s(s^2+0.01s+0.0025)} = \frac{60\left(\frac{s}{0.01}+1\right)}{s\left(\left(\frac{s}{.05}\right)^2+\frac{0.01}{.0025}s+1\right)}$$





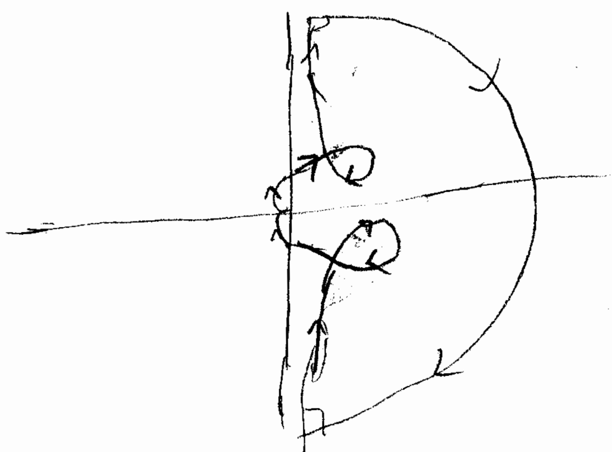
b.) $\left|\frac{h(s)}{e(s)}\right| = 1 = \frac{K\,60\left(\left(\frac{.16}{.01}\right)^2+1\right)^{1/2}}{.16\left(\left(\frac{0.01\cdot.16}{0.0025}\right)^2+\left(1-\frac{.16}{.05}\right)^2\right)^{1/2}}$ $\quad K = \frac{1}{649} \boxed{= .0015}$

c.) Yes, the phase never goes below -180

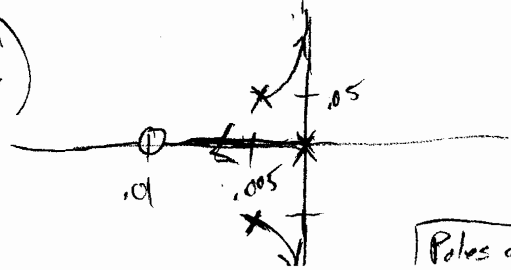
d.) $\angle G(s) = -90 - \tan^{-1}\left(\frac{.16}{.01}\right) - \tan^{-1}\left(\frac{.01\cdot.16}{.0025-.16^2}\right) = -179.615$

$\boxed{\phi_m = 0.385}$

e.)



f.)



$1+KG(s) = (s^2+0.01s+.0025)s + K(15)(s+0.01) = 0$

$K = 0.0015$

Solve system for s

$\boxed{\text{Poles at: } s = -.009,\ -.0005 \pm 0.158i}$

9.) $\frac{E(s)}{R(s)} = \frac{1}{1+G(s)} = \frac{1}{1+\frac{15k(s+0.01)}{s(s^2+0.01s+.0025)}} = \frac{s(s^2+.01s+.0025)}{s(s^2+.01s+.0025)+15(s+.01)k}$

$$e_{ss} = \lim_{s\to 0}(sE(s)) = \lim_{s\to 0}\left(\cancel{s}\cdot\frac{1}{\cancel{s^2}}\cdot\frac{\cancel{s}(s^2+.01s+.0025)}{s(s^2+.01s+.0025)+15(s+.01)k}\right)$$

$$= \frac{.0025}{15\cdot .01k} = \frac{1}{60k} = \boxed{11.1 = e_{ss}}$$

h) $H(s) = K \dfrac{s/a + 1}{s/b + 1}$ PS

write $H(s) = K' \dfrac{\alpha T s + 1}{Ts + 1}$ because know $\phi_H = \sin^{-1}\left(\dfrac{\alpha - 1}{\alpha + 1}\right)$ and $\omega_c = \dfrac{1}{\sqrt{\alpha}\, T}$

$\measuredangle G(\omega = 0.16) = \tan^{-1}\left(\dfrac{0.16}{0.01}\right) - 90^\circ - \tan^{-1}\left(\dfrac{0.21}{0.005}\right) - \tan^{-1}\left(\dfrac{0.11}{0.005}\right) = -179.6^\circ$

choose $\alpha = 10$ so that $\phi_H \approx 55^\circ$

$T = \dfrac{1}{\sqrt{\alpha}\, \omega_c} = 1.98$

$a = \dfrac{1}{\alpha T} = 0.05$

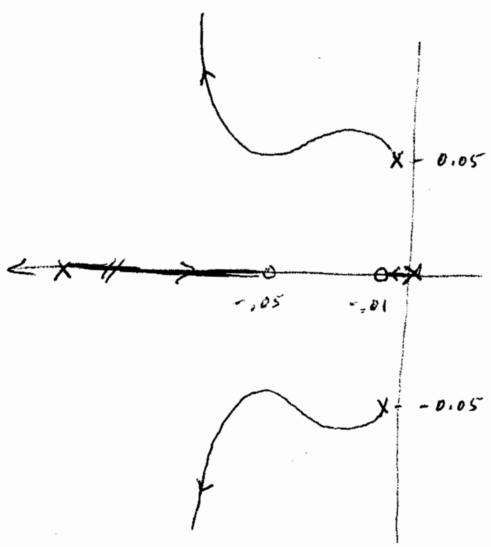
$b = \dfrac{1}{T} = 0.5$

need $K \left| \dfrac{(s/0.05 + 1)}{(s/0.5 + 1)} \right| |G(s)| = 1$ at $\omega_c = 0.16 \longrightarrow K = 4.83 \times 10^{-4}$

Result is $PM = 55.3^\circ$ at $\omega_c = 0.16$

other sol'ns are possible

i)



CL poles for K from part (h)

$-.0072$

$-.31$

$-0.097 \pm 0.083j$

PS
GT

j) for type 1 system

e_{ss} for a ramp input $= \dfrac{1}{K_{bode}} = \dfrac{1}{(60)(4.83 \times 10^{-4})} = \boxed{34.5}$

k) Need to increase the type of the system to type 2 to make error go to zero.

- Add integrator, $1/s$

However, the integrator will give too much negative phase.

- Add low frequency zero at least 1 decade before ω_c

$\tau > 1/0.016$

Compensator of form $\boxed{K \dfrac{\tau s + 1}{s}}$