Homework 4

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1. Consider the plant

$$G(s) = \frac{1}{(s^2 + 4s + 4)(s + 5)(s + 10)}$$

We are considering proportional compensation.

   (a) Plot the root locus of the system?

   (b) For what value of K is the system at the stability boundary?

   (c) What are gain and phase margins of the system if the gain is set to the value determined in the preceding question?

   (d) Pick a value of K for which the damping ratio is 0.707. COnsider those poles closest to the origin.

   (e) What are the gain and phase margins now?

2. Let

$$G(s) = \frac{(s + 2)(s + 4)}{s^2(s + 3)(s^2 + 2s + 25)}.$$

We are considering using proportional control for this plant. Use a Bode plot and root-locus to determine the gain and frequency at which instability occurs. What gain (or gains) give a phase margin of 20 degrees? What is the Gain Margin for a phase margin of 20 degrees?

3. An uncertain plant model is described by

$$G_{\text{true}} = G(s)(1 + L(s)),$$

with

$$|L(j\omega)| < l(j\omega),$$

and

$$G(s) = \frac{100}{s(s/10+1)}$$

and

$$l(s) = \left|\frac{s+10}{300}\right|.$$

We are interested in controlling this system using proportional control only.

(a) For what range of gains is the nominal, closed loop system stable? (the nominal system corresponds to $L = 0$.

(b) For what range of gains is the true closed loop system *guaranteed* to be stable? Support your argument with appropriate Bode plots.

(c) What maximum bandwidth do you think proportional control can reasonably achieve?