1a) At separation $\frac{dH^*}{dH} = 0$. The IBLT matrix becomes

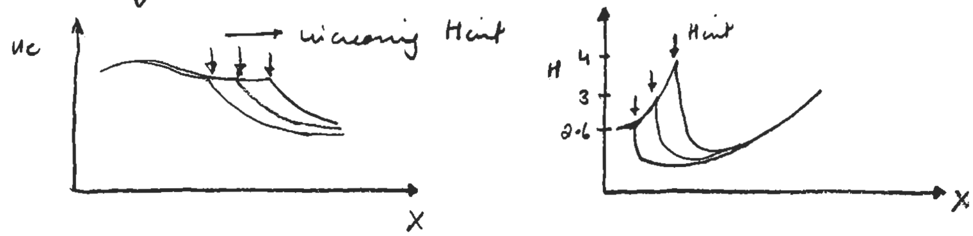
$$\begin{bmatrix} 1 & 0 & H+2 \\ 0 & 0 & 1-H \\ 0 & a & 1 \end{bmatrix}$$

In the classical case, $a = 0$, therefore the matrix is singular.
In the IBLT case the matrix is invertible since $a = -\delta^*/h-\delta^* \neq 0$. This viscous displacement effect $(h-\delta^*)$ modifies $u_e(x)$

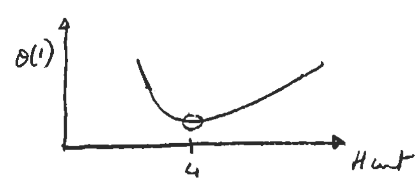
1b) Classical case separates at $x \approx 0.382$. For a given $h(x)$ or $u_e(x)$, a laminar BL has the same $H(x)$ for any Re, so separation, where $H=4$, is always at the same x.

In IBLT, $u_e(x)$ is modified by δ^* which depends on Re. $H(x)$ also depends on Re, so x_{sep} moves upstream toward the classical $x_{sep} \approx 0.382$ as Re is increased.

2) x_{trans} moves gradually downstream as H_{crit} is increased



3) $\theta(1)$ is minimum when $H_{crit} \approx 4$
Minimum loss occurs when extent of laminar flow is maximized as long as separation does not occur



$$\frac{d\theta}{dx} = \frac{C_f}{2} - (H+2)\frac{\theta}{u_e}\frac{du_e}{dx}$$

If $H_{crit} < 4$, $\int_0^1 \frac{C_f}{2}\,dx$ is dominant term (penalty)

$H_{crit} > 4$ $\int_0^1 -(H+2)\frac{\theta}{u_e}\frac{du_e}{dx}$ is dominant term. This describes mixing losses which occur during reattachment. Form drag for airfoils.

4) To allow design calculation, simplest approach is to introduce a new variable

$$\beta_h = \frac{x}{h}\frac{dh}{dx},$$

and augment system into a 4x4.

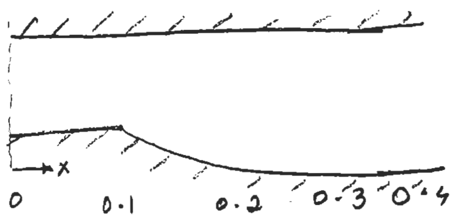
$$\begin{bmatrix} 1 & 0 & H+2 & 0 \\ -\frac{H}{H^*}\frac{dH^*}{dH} & \frac{H}{H^*}\frac{dH^*}{dH} & 1-H & 0 \\ 0 & -\delta^*/h-\delta^* & 1 & h/h-\delta^* \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \beta_\theta \\ \beta_{\delta^*} \\ \beta_u \\ \beta_h \end{Bmatrix} = \begin{Bmatrix} x/\theta \; C_f/2 \\ x/\theta \left(\frac{2C_D}{H^*} - C_f/2\right) \\ 0 \\ \beta_{H spec} \end{Bmatrix}$$

For design condition of $(u_e/u_0) = 0.5$

- Minimum length $\approx$ 0.35 at $H_{spec} \approx 2.35$
- Minimum $\theta(1)$ $\approx$ 0.0043 at $H_{spec} \approx 2.0$

See attached plots.

Diffuser shape:



5a) Integral Momentum Eqn:

$$\frac{d\theta}{dx} = C_f/2 - (2+H)\frac{\theta}{u_e}\frac{du_e}{dx} + \left(\frac{V_w}{u_e}\right)$$

Kinetic Energy Shape Parameter Eqn:

$$\frac{\theta}{H^*}\frac{dH^*}{dx} = \frac{2C_D}{H^*} - C_f/2 - \frac{V_w}{u_e}\left(1 - 1/H^*\right) + (H-1)\frac{\theta}{u_e}\frac{du_e}{dx}$$

The equations are valid for laminar and turbulent flows with suction. However, turbulent closure rel. have to be modified to account for the effect of suction on the shear stress near the wall (turbulent C_f and C_D have to be modified)

Modified interaction law

$$\dot{m}(x) = m_0 + \int_0^x \rho v_w \, dx - \rho u_e (h - \delta^*)$$

$$\Rightarrow \quad \frac{du_e}{dx} = \frac{u_e}{h-\delta^*}\left[\frac{d\delta^*}{dx} - \frac{dh}{dx} + \frac{v_w}{u_e}\right]$$

5b) $h_1 = 0.07$, $H_{crit} = 500$, $Re = 10^6$

$C_Q \approx -0.013$ (1.3%) required to suppress separation

There is no minimum in loss with increasing suction

5c) Introduce (v_w/u_e) as a design variable. Augment system similar to part 4

$$\begin{bmatrix} 1 & 0 & (2+H) & -(x/\theta) \\ .. & .. & (1-H) & (x/\theta)(1 - 1/H^*) \\ 0 & -\delta^*/(\delta^*-h) & 1 & -x/(h-\delta^*) \\ -1 & 1 & 6 & 0 \end{bmatrix} \begin{Bmatrix} \beta_\theta \\ \beta_{\delta^*} \\ \beta_u \\ v_w/u_e \end{Bmatrix} = \begin{Bmatrix} \cdot \\ \cdot \\ \cdot \\ \beta_{H_{spec}} \end{Bmatrix}$$

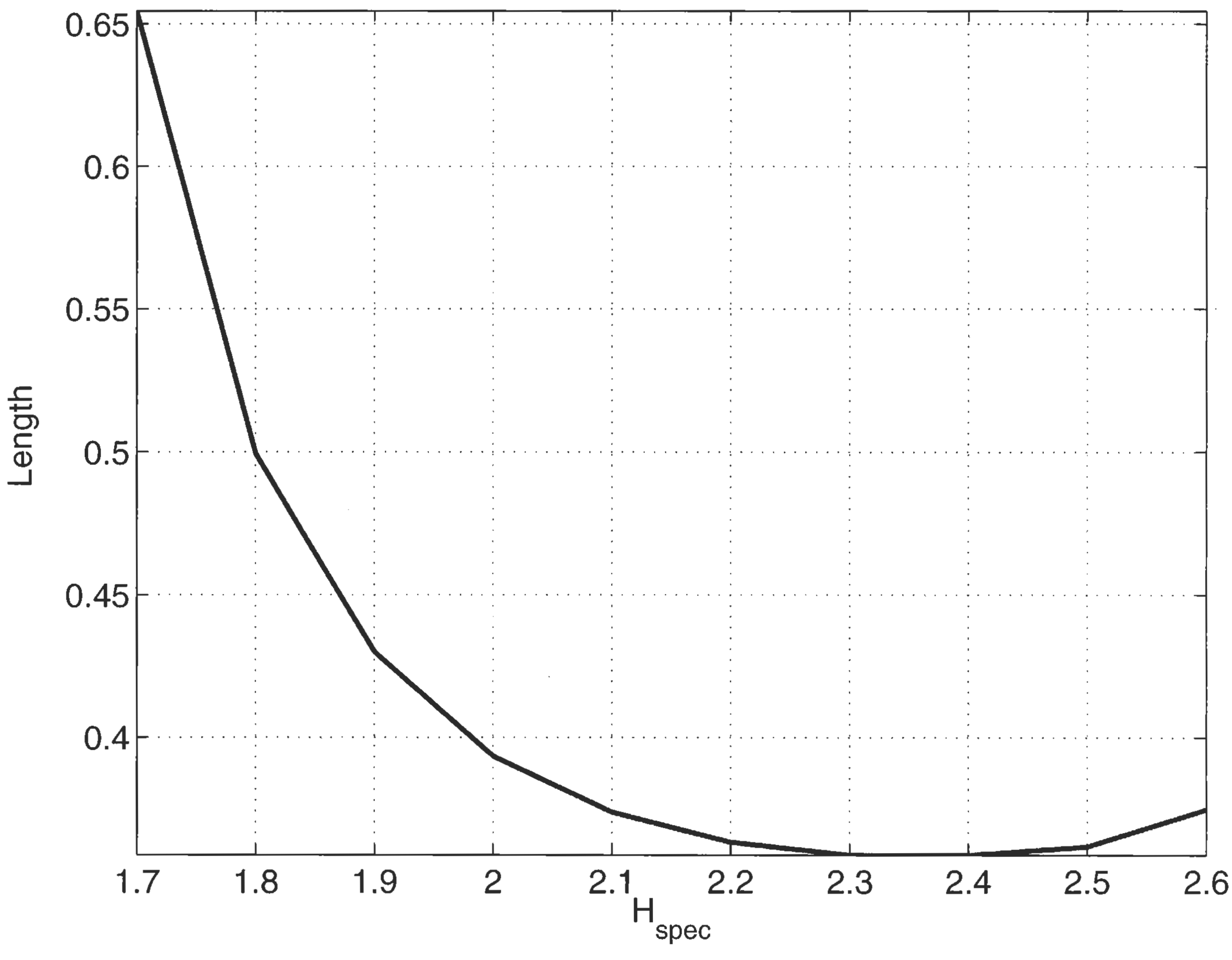
There is no minimum C_{pt} as in (5b) for laminar flow. Attached plot shows $v_w(x)$ for $H_{spec} = 4$ and $H_{spec} = 2.0$ (min)

H_{spec} =	4,	2
C_Q =	-0.009	-0.038
C_{pt} =	0.025	0.0105

PS 5 Solution

Fall, 03.

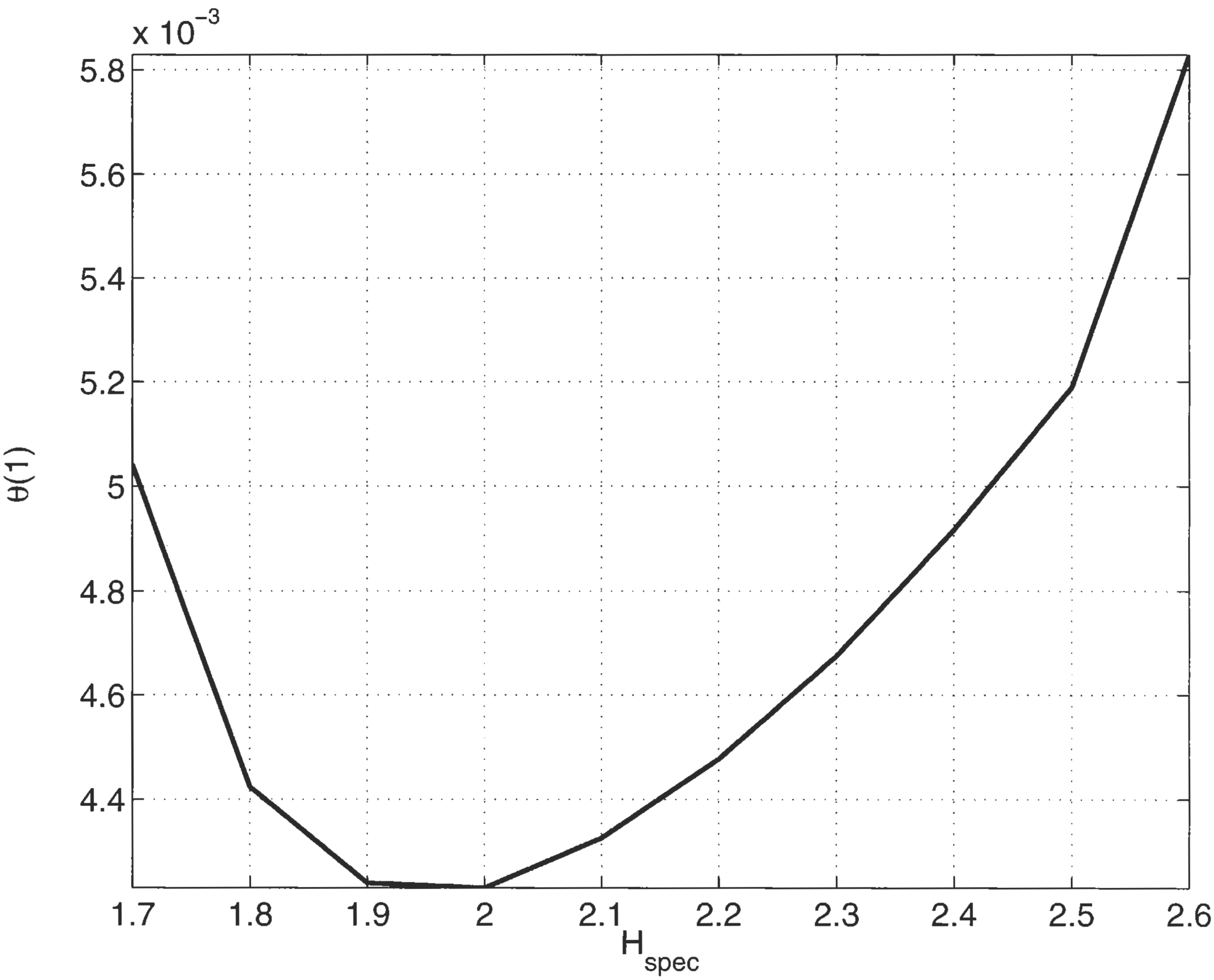
Problem 4 (i)

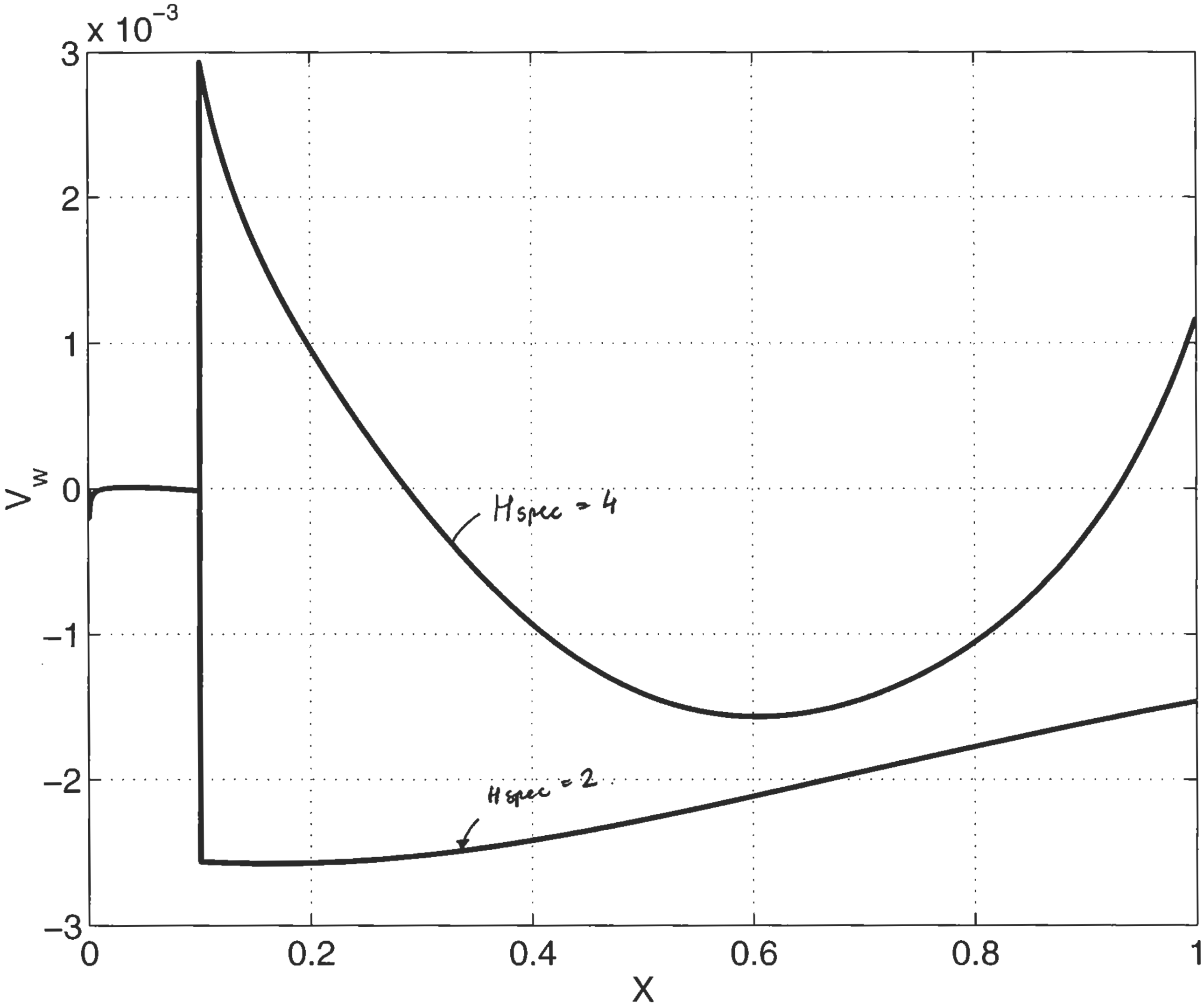


Fall, 03

PS 5 Solution

Problem 4 (ii)





x 10^-3
3
2
1
0
−1
−2
−3
V_w
0
0.2
0.4
0.6
0.8
1
X
Hspec = 4
Hspec = 2