16.13 Aerodynamics of Viscous Fluids Handed out: 3 Oct 01
Problem Set 4 Due: 17 Oct 01

Write a program to solve the Falkner-Skan equation system by finite differences using the Newton-Raphson method. Attached is a program "template" with the matrix solver and input/output in place which will minimize pointless debugging. MATLAB or GNUPLOT are suggested for graphics. You can verify your solutions against the Falkner-Skan $U(\eta)$ plots and parameter table handed out in class.

Your program is to have two basic modes: 1) Specified β_u, 2) Specified H.

1) Calculate a Falkner-Skan solutions for $\beta_u = 1.0 \ldots \beta_u =$ as low as possible $(\simeq -.09)$. Using your computed profiles $U(\eta; \beta_u)$, compute and plot the resulting $H(\beta_u)$ and compare with the H values from the class handout.

2) Calculate Falkner-Skan solutions for $2.2 \geq H \geq 12$, and plot H vs β_u again. Make sure you have enough points to adequately define this curve. Referring to this plot, explain the behavior of the Newton solution algorithm when a β_u value less than the minimum was specified in Question 1. Hint: It is a good idea to use the solution for one H as the starting guess for the next H value.

To suppress boundary layer separation, both suction ($v_w < 0$) and a "moving wall" ($u_w > 0$) have been tried experimentally. You are to examine the relative merits of each approach. Modify the boundary conditions and initial-guess profiles in the program to allow calculation of solutions with specified nonzero $U_w = U(0)$ and nonzero $V_w = V(0)$. Note that you will have to define a transformed vertical velocity V, which is a similarity variable corresponding to v.

3a) With $V_w = 0$, determine the magnitude of U_w required to double the maximum sustainable adverse pressure gradient (minimum $\beta_u \simeq -0.18$). Plot your result.

3b) With $U_w = 0$, determine the magnitude of V_w (negative) to achieve a minimum $\beta_u \simeq -0.18$. Plot your result.

3c) How practical would each approach be on an actual aircraft wing at $Re_x = 10^6$, say? Compare the physical u_w and v_w relative to $u_e \sim$ flight speed.

Disclaimer: In reality the BL on an airfoil would be turbulent, but the comparison is valid on a relative basis.

In problem set 3, you calculated the drag of a thin airfoil, where the surface velocity was approximately described by a power law ($u_e(x) \simeq u_\infty (x/c)^{\pm a}$). Boundary layer suction ($u_w = 0, v_w < 0$) is now applied on the upper surface of the airfoil to suppress separation.

4a) Determine what, if any, restrictions must be placed on $v_w(x)$ so that a similar boundary layer can still result.

4b) Using a value of $\beta_u = -0.09$ and $Re_c = 10^6$, estimate the variation in profile drag of the airfoil as the magnitude of suction is increased. In reality, additional power is required to drive a suction system which would discharge the suction flow to freestream conditions. Assuming an ideal suction system, how should this additional power be included to provide a more meaningful estimate of the drag coefficient c_d of the airfoil ? Is there an optimum suction flow that will result in a minimum overall c_d ?