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HONG LIU:

And let us start. So let me again start by reminding you what we did before. So consider, say, we have a scalar field in AdS. So let me just summarize what we did before.

So consider, say, a scalar field, ϕ , in AdS and with mass times square, so with mass squared given by m^2 , then we know, when you go to the boundary of AdS-- so let's take z goes to 0-- then ϕ will have the following asymptotic behavior.

So these are the two independent modes of ϕ [$z \rightarrow 0$ to $z \rightarrow \infty$] infinity. Of course, there are higher order corrections in z . And this δ is given by $\frac{1}{2} d + \nu$. And ν is given by this square divided by $4 + m^2 r^2$.

So this is the behavior. And so this ϕ is assumed to be dual to some boundary, scalar operator. And then all of those quantities, on the gravity side, they all have counterparts on field theory side.

For example, the δ , which, essentially, is a function of mass, then, essentially, it's the scaling dimension of O . It's a scaling dimension of O . And this A_x essentially translates into the source for ϕ . Or you can consider the momentum space version, say, you can do Fourier transform, A_k , then go to the $\phi(k)$.

And then we also showed, last time, that the 2ν times B_x is actually related to the expectation value of O . And again, there's a momentum space version of it. So you can just Fourier transform, then become $O(k)$ and $B(k)$.

And in particular, in the example, we can see the scalar field example. In particular, in the example, you can see that.

So we derived this relation from a free scalar field. but you can actually show this relation is actually generally true at non-linear level. So if you also include the high orders, et cetera, so including non-linear dependence on ϕ , actually this relation remains true. But due to the time, we will not go into that. The proof is actually not very difficult.

So you can see that the free scalar examples-- so, in the example, we can see that actually the B -- so in momentum space, the B is actually proportional to A . If you remember what we did last time.

So in other words, this one-point function-- so B is related to the one-point function. So in other words, the one-point function it's proportional to the source. So this is the standard story. Say all x equal to 0 when ϕ equal to 0. So if you don't have the source, then this one-point function is 0. So you don't have a spontaneous expectation value for the O .

And now the fact that B is proportional to A , so this is just reflects that, in the presence of the source, then, of course, the expectation value-- so let me write it in momentum space for simplicity. Then we're no longer [INAUDIBLE].

Assume the source is small, then you can expand the expectation value in power series of ϕ . So to the 0, so that would be ϕ , and then ϕ^2 , et cetera.

So at linearized level, say we just find O_k is proportional to ϕ_k . We just find that O_k is proportional to ϕ_k . And the proportional constant is, in fact, the two-point function.

So at linear level, so O_k would be just proportional to ϕ_k , but the proportional constant is just the two-point function.

So the reason is very simple. First, let me write down the definition of this two-point function. So G_{Ex} in coordinate space is defined to be $O(x)$ of 0 , because of translational symmetry, so I can put the other point to be 0 . So this is only the function of one variable, x . And then you can Fourier transform, and then you get G_{Ek} . So this is how we define the G_{Ek} .

So to see that this is true, you need to remember that the G_{Ek} is given from taking the generating functional, $\delta \phi_k$ and $\delta \phi_{-k}$, then you set ϕ equal to 0 . So this is the definition of the two-point function.

And then next, you can see that the [? one derivative ?] of ϕ_{-k} -- so you can see that this takes 1 derivative on ϕ_{-k} . That just gives you the one-point function in the presence of ϕ and then $\delta \phi_k$. And take ϕ equal to 0 .

So I just evaluated this derivative. So if you take this generating functional, take one derivative, we just get one-point function. And then this is the other point function.

Then at the linear level-- so if O_k is linear in ϕ_k , of course, taking the derivative is just the same as dividing it. So this is the same. So continue over. This is the same just as to the ϕ_k divided by ϕ_k at the linearized level.

And then so this tells you that this pre-factor is just the two-point function. And so we can now write a two-point function explicitly in terms of B and A , so this is just given, because this is $2 \nu B_k$. So this is just given by that.

So this is just a quick summary of what we did at the end of last lecture. So that's how you can find the two-point function. So you work out what B is, what the A is, and then the rest of them is the two-point function.

And the way we work out the B and A is the following. So at the linearized level, when you solve the equation motion for ϕ , then you impose the boundary conditions at infinity, A should be equal to ϕ .

But then there's another condition being proposed to make sure the ϕ is actually regular in the interior. So the regularity condition, there's still a relation between A and B . And then that leads to this two-point function.

Any questions on this? Is everything clear? Good. So now let's consider the higher-point functions.

So this is, in principle, straightforward to do. Because, recall, that the log CFT ϕ , this generating functional, in the field theory, is just given by the classical action on the gravity side, which is the boundary condition given by ϕ .

You're solving the classical equation motion, the initial action. Then you just, as we did earlier, you just solve the equation motion for ϕ to the [? linear ?] order. And then you validated the initial action.

Then you can express, for example, this SE in power series of ϕ . And then the quotient for the power series of this ϕ then just gives you correlation functions. So the principle of this is straightforward to do.

We know how to solve the classical equation motion. We know how to plug back into the action, and then you are done.

So just to be able to be more specific, let's consider an example. Let's just consider two examples. Again, let we just consider the scalar field.

Now, I add the linear term. For illustration, let me give you two examples. And so a very important thing is that in our set-up, the lambda, this cubic quotient is proportional to kappa. And then if you translate into the field series side, this is $1/n$.

So remember, for anything in the gravity action, you always have the 1 over kappa squared, 1 over G Newton before [INAUDIBLE] action. And then when we try to change phi into canonical normalized, and then you rescale phi.

And the consequence is that this cubic term or quartic term, then they will have quotients proportionate to kappa. And according to this rule, if you have a quartic term, then that will be proportional to kappa squared, et cetera.

And so the key is that as far as [ϕ is ϕ] order 1 , then you can actually treat those higher-order terms by perturbation because the quotient is small.

And of course, you can also consider the situation in which phi is large, and then you have to solve the full nonlinear problem. Then you cannot do a perturbative expansion. You have to do the problem.

So for our purpose, if you want to find the higher-point function for phi, then you need to solve this equation. So box phi is just a Laplace operator. I just save time.

So you need to solve this equation. And with the boundary condition still the boundary condition, the boundary value of capital phi to equal to this phi x. And let me call this equation star.

So as I said, because this lambda is small, you can actually try to solve this equation just perturbatively in lambda. And essentially, you're solving it perturbatively in this phi, because everything, in the end, will be expressed in this phi.

So one can solve star perturbatively in phi, which is the boundary value of the field. And say phi c, you can expand it as phi 1 plus phi 2 , et cetera, so the classical solution expanded. And this linear in phi. And this is quadratic in phi-- and et cetera.

And then when you plug back this into the action, then you find on the initial action can also be expanded in power series of this phi. So you will have S_2 phi plus-- started with quadratic order-- and 3 phi, et cetera.

And then the quotient here gives you the two-point function. And then quotient here gives you the three-point function. And the quotient here gives a four-point function, et cetera.

I hope the procedure is clear. Any questions regarding this? So even though this is very straightforward to do, conceptually, but in practice that it not what we would do, because this is a very tedious. It's very tedious.

And our knowledge of quantum field theory gives us something much simpler to deal with. Because this, essentially, if you think about it, is not very different from doing a correlation function calculation, just in the flat-space quantum field theory.

And there, when you calculate the Green function in flat-space-- in flat-space quantum field theory, what do you do? Do we try to solve the classical equation of motion and you iterate the equation? What do you do?

AUDIENCE: Draw a diagram.

HONG LIU: Yeah, you use Feynman diagrams. So here, it's exactly the same thing. So here it's much easier if you just do the Feynman diagrams. So let me first remind you what we do in the flat-space.

So let's now go to the field theory one, standard, ordinary flat-space QFT. So now let's consider the $\lambda \phi^4$ theory in flat-space. Just the same theory now using the flat-space. Let me call this star-star.

So essentially, let's just consider star-star in flat-space, now in flat Euclidean space. So if we are given a field theory like that, so how would you calculate the following correlation function?

Say, suppose I want to calculate $\phi(x_1) \phi(x_n)$. So now this x_1, x_n , they're just flat-space coordinates. To distinguish them, let me just call y_1, y_n , just not to confuse about the AdS coordinate. So in flat-space with y, y_μ , OK? In flat-space with coordinate y^μ .

So now suppose you want to calculate the correlation function for this. What do you do? So what do you do? It's easy. What you do is that this theory has a propagator, G , [with η] [with η] propagator. And also, this theory has an interaction vertex, which is controlled by λ .

And then, when you calculate such endpoint functions, you just fix your endpoint. So now remember, this is slightly different than what you would normally do in field theory one, because, here, we're doing coordinate space rather than momentum space. In field theory one, you're more used to doing momentum space.

So here what you do is that you fix $y_1, y_2, y_3, \dots, y_n$. So this is essentially the location of your insertion, of your field insertion.

So essentially, imagine there's a source there. And then you just connect all the extended points by propagators and with these kind of interaction vertices. So for example, you can have something like this, et cetera.

And this connects to some boundary point, some other point, et cetera-- say y 's. And this also has lines. We just draw all possible diagrams, and you calculate that diagram.

You calculate those diagrams, and they give you this thing. And essentially, the diagrams automatically give you the iteration procedure.

So now, back to AdS, essentially, we can just do exactly the same thing. Just remind yourself of the procedure of calculating this and what we are going to do here, and you easily convince yourself.

So it requires a couple of minutes' thinking, but we'll leave it to yourself. Actually, that procedure is no different from just doing the calculation like this.

Of course, the difference is that now we are in the curved spacetime rather than flat spacetime. But there's another major difference.

So here, the source all lies in the interior of your spacetime, where you inserted the operator. But here, all the source, $\phi(x)$, they lie at the boundary. That's, essentially, the only difference.

So if I just schematically-- so this is AdS. This is the boundary. And essentially, all the points are lying on the boundary, like we have x_1, x_2 . So let's consider four-point function. Because each one of them is labeled by the boundary point.

And then for the four-point function, you just connect. So you can connect the diagram like this. Again, you have the vertices, which is the [? cube ?] of vertices. But your endpoint is all lying on the boundary.

So this structure also tells you, actually, there's two types of propagators. Because one type of propagator connects the two bulk points. And then there's this kind of propagator which connects the bulk point into the boundary points.

So this is the difference, one major difference, from the flat-space case. So here, all the propagators are the same. So here, the difference is now, you also have propagators which come from boundary to the bulk.

So in this case, there are two kind of propagators. Here, you have two type of propagators. So one type of propagator is what we call the bulk-to-bulk propagator.

Somehow this is going up and up. OK, I lost some blackboard space.

So this the complete analog, this is the precise counterpart of the flat-space propagator. So this connects two points, z, x, z', x' . So I still use the notation that z is the bulk [INAUDIBLE] direction. And the x is along the boundary direction.

And this bulk-to-bulk will satisfy the standard Laplace equation just as in flat-space. So this gives you the delta function. Let me just express it, $z - z' \delta(x - x')$.

So this is a complete analog of the standard flat-space propagator. It's just now in Anti-de Sitter space. So this is a counterpart in AdS standard flat-space propagator.

So in particular, so as a propagator, this should be normalizable in either z or z' , so when you take them to the boundary. So more explicitly, for example, the z at z', x' , should scale as z' to the power Δ , when you take z' goes to 0.

And of course, this should also be regular. We should not have singularities as z goes to infinity. They should also be regular. So essentially, these condition will precisely define those propagators.

But the idea is we also, because the source lies on the boundary, because of this boundary condition, we also have so-called boundary-to-bulk propagators.

Actually, the standard in flat-space one, it propagates the field from one point to the other point. So that's why there's a delta function here. So the standard story, there's a delta function here. And you propagate the field starting from this point to that point.

So this boundary-to-bulk propagator normally is written as $k(z, x)$, but the second index only has x prime, because this is when you take the boundary point to a bulk point. A boundary point, of course, there's no z anymore. So z prime is 0 here. You don't need to write it.

And this boundary-to-bulk propagator, again, should satisfy the Laplace equation. But now the key point is that, on the right-hand side, because there's no bulk source, so the right-hand side should be 0 rather than the delta function. So that delta function is due to the bulk source.

But we have to make sure this k have the right non-normalizable boundary conditions, so you have the right boundary condition to the boundary. So this when it approach to the boundary, as z goes to 0, should be like $z^d \delta(x - x')$.

So the k satisfies the Laplace equation without a source. But when you take this propagator to the boundary, then it should give you the non-normalizable boundary conditions, because you should approach a source.

And the delta function is put here. So if you write, say, a bulk field, which is sourced by some boundary source, say $k(z, x, x')$ times $\phi(x')$, and then this will have right boundary condition that the $\phi(x)$, z goes to z^d minus $\delta(x - x')$.

So this boundary-to-bulk propagator does not have a source in the bulk but does have a source in the boundary, non-normalizable off in the boundary.

So now with this two propagator, then the story is just actually as in the standard way you obtain the Green functions. It's just really no different.

So let me just summarize what we have discussed.

The boundary endpoint function can be obtained by, essentially, endpoint function of the ϕ field in the gravity side-- let me write it here-- just related to the endpoint function of the gravity field on the gravity side, with all these points, x_1, x_n , they're lying on the boundary.

This side is treated as ordinary field theory. Then compute this endpoint function of your field but just put the field on the boundary. And then that would just give you the AdS correlation functions.

Yeah, and then you just need to distinguish two types of propagators, a bulk-to-bulk propagator and bulk-to-boundary propagator. So any questions regarding this?

AUDIENCE: Now, here, [INAUDIBLE] z^d minus δ . Right there, the G goes to $[z^d \delta]$ ϕ in the δ .

HONG LIU: Yeah.

AUDIENCE: But why aren't they different?

HONG LIU: It's because this must have a source. So the boundary-to-bulk propagator-- you propagate the source, from the boundary to the bulk, so that means that field must have the right boundary condition. So that's the reason for this. We must have non-normalizable boundary conditions.

And that propagator is just the standard flat-space propagator. Of course, it's always normalizable. And in the flat-space, when you construct a propagator, of course, it's always used in a normalizable wave function.

AUDIENCE: This is that's non-normalizable?

HONG LIU: No, this is non-normalizable. This is designed so that you have the right boundary condition. So this is designed so that you can propagate just as in flat-space. If you have ϕ at this point, and then you can propagate to the other point by convolution.

And of course, when you write down the bulk field ϕ , in general, you only construct them out of the normalizable modes. Any questions? Good.

So now let me make some remarks. So here, this procedure of iterating the classical equation of motion, solving the classical equation of motion, then iterating the action, of course, again, from our experience with quantum field theory, that only gives you the tree-level diagrams.

So if I really write the bulk path integral, so $Z_{\text{CFT}}[\phi]$, then, in the same classical limit, it can be written as Z_{gravity} , which is ϕ with the right boundary condition. And then on the gravity side, at semi-classical level, we can actually write down the path integral for this gravity partition function.

So that's what we discussed before. And then at the leading order, in performing this path integral, of course, you just get what we have been doing so far.

You just evaluate it at the classical solutions. But in principle, you can also see the fluctuations. So this is the fluctuations around, say, the classical solution, ϕ_c . And then the action for the fluctuations is just given by $\phi_c + \chi$. The minus, of course, is the classical action. So we just expand around the ϕ_c .

So if we expand this, the leading order would be quadratic in small χ . I should now quote small χ . Maybe let me quote χ , because that is the same as that ϕ . So this χ is the small fluctuations around the ϕ_c .

So if you expanded this in χ , the linear order is 0, because ϕ_c satisfies the equation of motion. So this will start in quadratic order and then cubic order, et cetera. So you validated the quadratic order as a Gaussian.

And then that's the one-loop diagram. And then if you go to higher orders, then it'll give you higher-order diagrams. So if you now include the fluctuations, so the SE ϕ_c , which is a classical action. This encodes only tree-level diagrams.

And when you include the fluctuations, ϕ , the fluctuations, χ , and then this corresponding to include loops.

So again, the whole procedure can be captured by the Feynman diagrams. So the advantage of the Feynman diagram is actually there's a natural generalization to the loop diagrams. And then just include the loops here. For example, here, you can do something like this. Now you can also have something like this.

So that's, of course, one and two, including the fluctuations, are under the saddle point, are including the fluctuation around the saddle point. Any questions regarding this point? Good. Yes?

AUDIENCE: So when people draw these like so-called Witten diagrams, where they draw up like this circle.

HONG LIU: Right.

AUDIENCE: Is that basically what we're doing?

HONG LIU: Yeah. Yeah, right. Yeah, so let me just explain one thing. In the Euclidean, I have been drawing the ideas like this. So this is z equal to 0, et cetera. So in the Euclidean signature, as you will do in your PSet, then this z equal to infinity, you can argue is actually a single point.

So let me just write down the metric. So the metric is $R^2 + z^2$. When you go to Euclidean signature, you just have dx^2 . So this is a full Euclidean metric.

So there's something interesting about the difference between the Euclidean and Lorentzian, even though, from this picture, it's roughly the same. So if it's equal to 0, you go to z notch.

And then in the Euclidean case, you can actually show that z equal infinity, even though, [$?$ naively, $?$] you still have a full Euclidean plane, but you can argue, actually, that the whole thing is actually a point.

And you can roughly see, when you go to z equals infinity, the overall factor goes to 0. So essentially, the whole space shrinks to a point. The whole space shrinks to a point.

So in Euclidean, if it's equal to infinity, it's essentially a point. And then topologically, this is equivalent to having a disk. Topologically, the whole space is just disk. And the z equal to infinity is one point on the boundary. And then the other is z equal to 0. And so this is the interior of the space.

Yes?

AUDIENCE: So in other words, the entire perimeter of the circle is identi--

HONG LIU: It's the boundary.

AUDIENCE: [INAUDIBLE].

HONG LIU: Yeah, this is the boundary. Essentially, you can imagine, each point here is R to d . Then you add the point at infinity, and then it becomes a sphere.

AUDIENCE: [INAUDIBLE].

HONG LIU: Yeah, it becomes a sphere. And then the whole thing becomes like a disk. So normally, when people will draw-- so topologically, you're creating a disk, just like this. Then people will just draw diagrams like this. Yeah, I draw diagrams like this.

AUDIENCE: If you add a single point to the boundary, that single point is equal to infinity to the boundary, which is z equals 0?

HONG LIU: Sorry?

AUDIENCE: You mean that you have a point that [INAUDIBLE].

HONG LIU: Yeah. Yeah, this point turns out, it can actually lie on the boundary.

AUDIENCE: But then, on the boundary, the value of t is not continuous.

HONG LIU: Hm? Yeah. So this is due to the coordinate choice. It's just this coordinate becomes singular at that point. You can rewrite it.

Yeah, the simplest thing is that you rewrite these [INAUDIBLE] coordinates to the coordinate which has a parameter that has a boundary really as a sphere. And then you will see it. Because when you write the sphere as a plane, then this one point becomes singular. This is just the standard story. Just that coordinate choice is singular.

Any other questions? Good. So this is the first remark. The secondary remark is that this is a little bit of an unusual correlation function. So this is the bulk correlation functions, but with all points lying on the boundary.

So the counterpart or just the exact analog of standard flat-space correlation functions in AdS-- you can see the end bulk point and look at its correlation functions.

So this then will be really just bring you to the standard QFT one correlation functions. Then you have endpoints, z_1, x_1 , or in the bulk, z_2, x_2 , et cetera. And then you just draw a diagram between them, et cetera.

The area propagator here would be just bulk-to-bulk propagator. There's no boundary-to-bulk propagator. There's no boundary-to-bulk propagator just all bulk-to-bulk propagator.

So this is a complete analog of your ordinary flat-space Green functions. But we can easily imagine that this correlation function must be related to those correlations functions if you just take those points to the boundary.

So it's natural to expect, say, those correlations functions, since they're at the boundary, must be related to those kind of standard correlation functions, in which you would take the point to the boundary.

So those, somehow, in the end, must be the same. So if I take those points to the boundary, I should recover that guy. So indeed, this relation is true. And you will work it out yourself in the PSet.

You see, the only difference is the following. When you take those points to the boundary, what you get is the boundary limit. So now let's imagine you take those points to the boundary.

Because here, it's all bulk propagator. So those propagator, which connect to the boundary, would be the boundary limit of the bulk propagator. And then, it just boils down to, what is the relation between the boundary limit of the bulk propagator and this boundary to the bulk propagator?

So the above relation-- let me just-- I'd rather just call it a number. So this star-star-star just boils down to the relation between the k_z, x, x' , which is our boundary-to-bulk propagator, and limit z' goes to 0, $G(x, z, x', z')$.

So you will find that these two guys are proportional to each other. And once you extract out that proportional factor, then you just relate them. So this is just another way to calculate. Because this is the boundary correlation function.

So this is the boundary correlation function. So this is related to the CFT correlation functions. And then you can also calculate the CFT correlation function just by taking the limit of the bulk correlation functions.

So this you will work out a little bit. Yeah, you will work out the precise relation in your PSet. Maybe some of you have looked at it already. So any questions on this?

AUDIENCE: What is the advantage of maybe using another to express it?

HONG LIU: Yeah, just different ways of calculating things. And of course, normally, I have many different ways to calculate things. Some of them may be convenient this way. So of them may be convenient for that purpose, et cetera. It depends.

Normally, we just directly are using the bulk-to-boundary propagator. If you just do the calculation, the bulk-to-boundary propagator actually is simpler. But sometimes, for certain conceptual questions, this actually becomes simpler.

So now let's look at Wilson loops. So how do you calculate the Wilson loops using gravity? So in the gauge theory-- so I assume you have already done the QFT II and the gauge theory?

In the gauge theory, we also know it's essentially one of the most important observables. The Wilson loop is normally defined in the following way. So now I will explain my notation.

So let me also add in subscript r here. So c is a closed path. So this is defined for closed paths. And A_μ is a matrix. So we can see the [INAUDIBLE] in gauge theory.

A_μ is a matrix, writing the standard away, in terms of the generator, in some representations r . So this is in some representations. So this is just the generators of the gauge group in some representation, which we'll call r .

And P is the path ordering. So in general, you can choose any representations you want. But often, we choose r , say, in fundamental representations. For example, that's normally what we do, in QCD, in fundamental representation. But you can choose it to be any representation.

So the physical meaning of this, so this operator, by definition, is gauge invariant, because of this trace. And need the path ordering, because you have a matrix here. And the matrix, at different points, don't commute. So you need to specify ordering. And so you just specify it by the order of the path.

And so the physical meaning of the Wilson loop, so this is essentially the phase factor associated with transporting an external particle, in a given representation, say, in r representation, along c .

So you transport the particle along some path. And you come back to the same point. Then you find that that phase does not necessarily go back to 0. And so this is a nontrivial phase. It essentially tells you there's a nontrivial gauge field there.

Yeah, so this provides a probe of the gauge fields. Yes?

AUDIENCE: I know that people talk about-- I know that it's somehow and observable. But is it actually observable? Can you actually do an experiment to measure this phase?

HONG LIU: I don't think so. Ha. It depends. Yeah, in some situations, you can. Let me mention something else, then the answer to this question will be seen.

So the simplest case would be the W_c , just the single-point function of W_c , say, in a vacuum. So this is the simplest observable. But of course, you can also consider the generic, say, a large number, several [? routes ?] in some general state, say, for example, [? finite ?] [? telemetry ?], et cetera.

So these are the typical observables. So now let me emphasize this external. So this external is very important. So normally, by calling something external, we mean that this particle has infinite mass. It's infinitely heavy.

And the reason we want it to be infinitely heavy is because only for heavy particles, in principle, you can localize the lower path. Then you can make mathematical sense of the precise path. Otherwise, if it's a fluctuating particle, then you can not specify a path-- yeah, ambiguously.

So when we say, an external particle, I always say a particle which is assumed to be infinitely massive. And also, I've introduced terminology. When this r is in the fundamental representation-- yeah, maybe not right here.

When this r is in the fundamental representation, we'll call the corresponding particle a quark. If it is in the fundamental representation, I will call it a quark. I will often just consider the fundamental representation. In the phase factor, where we're corresponding to, you transport a quark.

So one of the very often used loops is a rectangular loop. So this is along some spatial direction, so this is the time direction. So let's say the spatial direction is L , and the time direction, using the length, is T . So imagine you have a loop like this.

So normally, we can see that the T , say, the length of the loop in the time direction, is much, much greater than the L . So in such a limit, then, essentially, you can ignore the contribution from this side, from these two short sides. So in the limit, in which this T goes to infinity, to leading order, you can ignore the short side.

And then you can think of this loop when it just has a particle-- so this is a particle moving forward in time. This is a particle going backward in time. Then you can consider this as a particle and an anti-particle moving forward in time.

So in this case, we can argue, on general grounds, that this will just, to leading order, will give you something iET . And this e , it just can be interpreted as a potential energy between a single quark and an anti-quark.

So in this example, this is the same. In this case, you may be able to measure this Wilson loop, because if you can measure-- experimentally, you may be able to measure this energy between the two particles. And then, essentially, you can say, I have measured the loop. And then you can compare with the prediction of the loop.

So again, in order to make this interpretation make sense, this particle having infinite energy is very important. And again, only for the very heavy particle, you can really localize them, and then talk about the potential energy between these two particles. Otherwise, if they fluctuate a lot, then it's hard to make it precise.

So now the question is how do we calculate this quantity in AdS? When I don't put an index here, I just mean that this the fundamental representation. The question is how we do that.

So in order to answer this question, we first need to understand how to introduce a fundamental external particle, external quark in the N equals 4 super Yang-Mills theory.

So how to introduce a fundamental external quark in the N equals 4 super Yang-Mills theory. So we know, as we discussed before, that everything in the N equals 4 super Yang-Mills theory, so all fundamental fields, all the fields in the N equals 4 super Yang-Mills theory, they are in a joint representation. There's nothing in the fundamental representation. Everything is in the joint representation.

So first we need to introduce a fundamental object, an object we transform under fundamental representation. After we have done that, then we need to translate that quantity to the gravity side. And then we understand how the gravity description of such an external particle.

So this is easy to do. This is easy to do. So let's first think about how to do this in the N equals 4 super Yang-Mills theory. And then this is a very intuitive by using this [INAUDIBLE] picture.

So we know the N equals 4 super Yang-Mills theory comes from low-energy theory of N D3-branes. They come from the low-energy theory of N D3-branes.

So how do we introduce an external particles? So now let's imagine, consider N plus 1 of them, and let me separate one from the rest. So this is still N , separate one from the rest.

So then there can be open string connects between them. So suppose I separate them by some distance, r , in the transverse direction to the D3-brane. So as we described before, such a separation breaks the $SU(N+1)$ symmetry of the D3-brane $SU(N)$ and then a single $U(1)$ of this 1-brane.

In particular, such an open string-- so let's look at the endpoint. Then the endpoint of this string can only have indices. Because this is a single stream, one end ending on the other side. There's only one index. And here, they can only have N possible index.

So such a fundamental string transforms in the fundamental representation. So from the point of view of this $SU(N)$ gauge theory, this is a quark. So this endpoint is a quark.

So this is a quark. And the mass of this quark is equal to-- so let me call the mass capital M -- is equal to r , the length between them divided by tension of the string. So this we have derived before.

So this just gives you the mass of the quark. So when you separate one D-brane from the other, so you have introduced this quark, with a mass.

So now let's consider that there's no low-energy limit [INAUDIBLE]. So in the low-energy limit of [INAUDIBLE], so we want to keep this quark, so there, of course, you want to take α' go to 0.

But we want to keep this quark in the low-energy spectrum, because we want this to remain in N equals 4 super Yang-Mills theory, because we want to introduce this particle. So in this low-energy limit, when we take off α' , go to 0 limit, we also want to take, at the same time, r goes to 0 limit, so that this mass term, r divided by α' , will remain finite.

Then that means this will remain, this particle, such kind of fundamental representation, will remain in the N equals 4 super Yang-Mills theory. Because other modes are massive. And then when α' goes to 0 limit, it goes to infinity. And under those modes, we will remain with the N equals 4 super Yang-Mills theory.

So now let's see what happens on the gravity side. So here, you will need to fill in some details yourself. I'll only tell you the answer.

On the gravity side, you have this N D3-brane. They have one brane which [r is r] [r r] separated from this N D3-brane. And they're separation is such that, when you take off α' and go to 0 limit, their ratio is finite.

Now, in the gravity side, you have to take this so-called near-horizon limit, with taking into account. So what you find in the gravity side is that, as before, that N D3-brane now disappeared into r equal to 0.

Remember, r equal to 0 is that infinite proper distance away. And now we are using this r coordinate. And when we take this low limit, we're using this r coordinate rather than this z coordinate.

So in that case, you have i equal infinity. Then the brane, essentially, goes to r equal to 0. which has infinite proper distance away from any finite r .

So now this is an exercise for yourself just to repeat our previous argument. Then you will find that in this regime, when you take α' go to 0, this r finite, this D3-brane does not disappear. The single D3-brane will remain in the AdS.

So the difference, from our previous story, is that, in addition to the AdS, now you have an additional D3-brane at some point in S^5 . Because D3-brane is in some transverse direction, which now become S^5 .

So you have a D3-brane, which is parallel to the boundary coordinates, but sitting at one point in S^5 . And I think this picture is reasonable, but you should check yourself. Because in this regime, this D3-brane does not go away.

And a remarkable thing you will check yourself is that when you take the [INAUDIBLE] limit, in this picture, this is flat-space. Then you have this formula. And now, when you take this limit, this now become AdS. Now, this AdS at some radius r , controlled by this r .

And now you can check one nice thing. In AdS, if you consider a string, straight string, from this D3-brane all the way to r equal to 0, which is this string, this string has exactly the same mass.

So the mass is equal to r divided by $2\pi\alpha'$ -- also in AdS5. So this is a consistency check. Because when you take the low-energy limits, of course, you should not change the mass.

The mass of the string will not change. The only question is whether it will stay in the spectrum or will not stay in the low-energy spectrum. The mass will not change. And you will see that when you take that limit, you find this D3-brane remains here.

And this string, which now will connect some r to r equal to 0, which is now just some AdS. This is just some interior of AdS, infinite proper distance away.

And then you can check that, in the AdS metric, the image of this brane, viewed from the boundary time-- you have to this ratio factor, et cetera-- remains exactly equal to this. So this is an important self-consistency check, which I will not do here.

So now, the story becomes simple. Now we generate such a string, such a fundamental particle with such a mass. But in order to introduce an external particle-- in order to have an external particle take a mass to go to infinity, so that means we need to take r goes to infinity.

So that means we want to this D3-brane to lie on the boundary of AdS. So to summarize, now we want to move this D3-brane-- in order for the external particle, we want to take M goes to infinity, then we want to take r goes to infinity. So we want this D3-brane to actually lie on the boundary of AdS.

So let us just conclude, summarize. An external quark with infinite mass, in N equals 4 super Yang-Mills theory, is described by a string ending on the boundary of AdS.

So this is r equal to infinity. Because now, we have put in this D3-brane to the boundary, and then we'll, of course, run into a string ending on the boundary of AdS. A external quark with [INAUDIBLE] AdS, of course, run into a string ending at boundary of AdS.

And in particular, the location of the string can be mapped to the location of the quark. So then endpoint of the string is the location of the quark.

So now we have found a very nice picture that introduces a fundamental quark, in N equals 4 super Yang-Mills theory, because [? what you ?] [? need to ?] [? do with ?] such a string.

And now, when you do a Wilson loop, then corresponding to transport this string around some path, then endpoint of the strings runs on a path in the field theory.

I thought we would stop here.