PROFESSOR: Good. So let us start. So let me first quickly remind you what we did in the last lecture. So we first talked about the-- so we talked general observables and gauge series. So we're interested in correlation functions of gauge invariant operators.

In particular, we will be focused on local operators. So we say, for example-- so the local operator can also be separated into, say, the single trace operator. Yeah, the single trace operator. So n just labels different operators. Or say that the double trace operator-- sometimes, in order to avoid confusion, we put some thoughts that means these two operators should be considered grouped together. And then you evaluate it at some spacetime point. Say this would be a double trace operator. Or maybe the higher ones. OK?

And also, we found that a typical correlation function-- say if I look at endpoint function, endpoint connect equaling function of single trace operators, then the scale, with n as [INAUDIBLE], again, you sum over all the h, which is the topology of your diagrams. And then to the power n to the minus n-- n is the number of the endpoint functions-- the minus 2h. Than say some function. Fn, which can depend on the coupling constants and also will depend on x1, x2, xn, et cetera. OK?

So this will start us leading order with N 2 minus n, which come from the planar diagram. And then the next order, just N to the minus n. So this will be the torus diagram. The diagram you can put on the torus Or genus 2 diagrams. OK? Et cetera.

So we also discussed that this behavior has important physical implications. We talked about two implications. We talked about two implications. You say that kind of independence means that, in the larger limit-- say in the N go to infinity limit-- we can essentially treat the action of the single particle-- a single trace operator on the vacuum-- as generating a single particle state.

We often call this as glueballs state. Glueballs, OK? So this single particle state, they don't have to appear really as, say, a symptotic state in your scattering, et cetera. It just say the behavior of such state behaves like single particles.

And then, if you act on the double trace operator, on the 0, then that gives you the two particle state, and et cetera. And the triple trace operator will give you the three particle state, et cetera. OK?

So this is one aspect. And the second aspect is that if-- say if there is some single trace operator whose expectation value is non-zero, then the fluctuations are suppressed. OK?

So for example, if you look at the ratio of the variance, so O squared, the connected part of the O squared, is essentially the variance of this guy. And divided by expectation value itself, the scale is 1 over N because of that behavior, and this goes to 0. OK? In the margin, as N goes to infinity. So that means the fluctuations are suppressed. So any questions regarding this?

Good? OK. So now let's talk about the third aspect of it, the third implication. So now, suppose we interpret this endpoint function, so this connected endpoint function, say we check it symbolically as, say, some kind of blob. Then you have [INAUDIBLE] x. Say 1, 2, 3-- n minus 1, n. OK?

So each line means an external operator insertions. So if we interpret this as some kind of scattering amplitude, so as generates some kind of scattering amplitude over the n glueballs, then you can show, because of this behavior, you can show, because of this N scaling. And then, again, to leading order, in the N goes to infinity limit, the scatterings are classical.

Classical means-- you said it only involves tree level scatterings. OK?

So now I'm going to show this. This is the conclusion. So now I'm going to justify this statement. So before I start justifying this statement, do you have any questions?

Good. So let me just make a few remarks. So first-- so let's look at the simplest case, which is the three point function. So the three point function can be considered just as effective vertex. It can be considered as an effective vertex of three particles intact with each other. Say one glueball is split into two glueballs, et cetera. OK?

So this, from that definition, scales as 1 over n. OK? This scales as 1 over n. So if we treat it as a basic vertex-- so suppose we treat it. Suppose we treat it as a basic vertex with coupling g. So g is 1 over N. OK.

The from simple, then to easy to see, then tree-level amplitudes for n-particle scattering, n-particle scatterings are precisely scaled as g to the n minus 2, which is n to the power minus 2 minus N. And then this is precisely-- we see for the leading scaling here. OK?

So is this fact obvious to you, that the tree-level scattering for n-particles with basic vertex given by g will scale like this?

AUDIENCE: Sorry. Why is it proportional to g? Because each-- they start in a double line notation. Each line contributes to like a g square. So how that g comes from--

PROFESSOR: Now, which g you are talking about? Oh, no, no, no. Maybe let me call it-- it's a different g. It's a different-- yeah, let me call it kappa. Or let me call it g tilde. Yeah, it's not the same. It's not the same g as before. This is just something I-- [INAUDIBLE] there. I just call it some kind effective coupling. Yeah?

Yeah. Just imagine you have a process whose basic interaction vertex is g tilde, and g tilde is order 1 over N. And then the tree-level scattering for n particle will depend on this coupling to be g tilde to the n minus 2. OK? So this clear to you? Yeah. You can just draw a tree diagram. You can see the immediate tree. OK?

So this is consistent. So this behavior is consistent to interpret that as a tree. But you don't actually have to assume the basic three-point vertex. So you can also say-- you can also include-- higher order vertices. But your vertices which include should be consistent with this scaling. OK? Suppose if you're want to inclue four-particle vertices.

And then this should scale as g tilde squared, which is N to the minus 2, because the four-point connective function should scale as minus 2. And similarly, if you have a five-point vertex, then it should scale as g tilde to the cube, which is N to the minus 3. The five-point function should scale as N minus 3. OK?

So you can also add such higher order vertices as far as you are consistent with this large N scaling. So then you can easily convince yourself that even including such higher order vertices does not change this conclusion, that the N-point tree-level amplitude, again for n-particles, will scale like this. OK? Will not change that conclusion.

So this is second statement. And these two statements, just to give you a heuristic way to see that this kind of scaling can be generated-- indeed, generated by tree-level scatterings.

So now I'm going to prove that in the large N limit to leading order, in N, in any such kind of correlation functions, there are no more than one-- there's no more-- there are only, maybe. Let me just say it easier. There are only one particle intermediate states.

OK? So I'm going to show this. So let's consider, say, a three part-- so by one particle, I always mean one particle in this sense. OK? So one particle in this sense.

So I read through it a little bit heuristically. It's not very precise, but I think it's rigorous, but it's not-- but they will not [? right ?] the precise-- yeah, will not [? right ?] rigorous formulas.

So let's consider, just as an example, consider the three-point scattering, three-point functions. Say I have three operators. And look at this connected [INAUDIBLE] function. OK?

So now, in those places, we can, in principle, insert the complete set of states. OK? And those can be considered as intermediate states-- contribute to this process. So if I consider a scattering process, and then I can insert a complete set of states here, then those states which gives a nontrivial contribution can be considered as a contribution from some intermediate states. OK?

So for example, let me introduce a single particle state here-- say this O1, O2, O3. Suppose I insert a complete set of-- so I insert a 1 here. This 1, we include the sum of all possible single particle states and also include possible two-particle states, et cetera. OK?

And look at the [INAUDIBLE] equaling function. OK? Yeah, just heuristically, when I insert the complete set of states, I can insert [INAUDIBLE] like that.

So now let's look at the scaling. So this scale, according to our general behavior actually scales as 1 over N. OK? So this guy is indeed scale as 1. So two-point function still says 1, N to the power of 0.

And then this guy scales as N to the power minus 1. So this term is OK. So insertion of the single particle state is OK. But now, this will give you a higher order contribution, because this scales as N equal to minus 1. And this scales as N equal to minus 2. So all together, this contributes to N to the minus 3. OK?

So the contribution of the two particle states, they will be suppressed compared to the contribution of the single particle states. And similarly, with higher-- even higher-- even more particle states. OK?

So if I draw a diagram, this process is like the following. It said I can start with 1. And then this 1 can, say, turn into i through a two-point function. And then this i can go to 2 and a 3. OK? So this is a tree-level process.

And this is like the following. So this is a two-particle state. So this, if I draw a diagram, would be like if I started with 1. And then this 1 can split into ij. And then this ij combines into 2, 3. OK?

So this is like a one-loop process. OK? And this, we see, it's N to the minus 3. And this is indeed just N to the minus 1. So this is suppressed compared to that.

So this is a direct way to say that, in this kind of-- to leading order, indeed, can only tree-level processes. You can never have these kind of loops. Because whenever you have a loop, then of course one of you must have a multi-particle. You must have multi-particle intermediate states. And then they're [INAUDIBLE] Is it clear? So this tells you that all loops are suppressed.

And so abc together-- so this abc together, then justify the statement that to leading order. this scattering amplitude involving only tree-level. When you have only a tree-level scattering, then this is like a classical story. Only size you land in quantum field theory in the loops can be considered as quantum fluctuations. And if you only have trees, it's essentially just a classical process.

So when you solve the classical equation motion, and then what do you get? It's the tree-level processes. OK? Yes?

AUDIENCE: What is the number of operators [INAUDIBLE]?

PROFESSOR: Hm?

AUDIENCE: What is the number of operators in the sum?

PROFESSOR: Right. That's a very good question. And essentially, by definition, such operator only scales. Essentially, by definition, such kind of so-called single trace operators and the scale set-- yeah, there's no-- yeah, just the way you can make such single trace operator is not proportional to N. Questions?

AUDIENCE: What is-- each of these terms individually small, but when you take summation over them--

PROFESSOR: Yeah. Yeah, that's the question was just asked. So those operators, if you count them, the number of single trace operators you can construct is independent of N. And so so when you suppress them by N. There is nothing you can compensate them. In principle, yeah, there's nothing can compensate them. Yeah?

AUDIENCE: So how many operators that are Oi, Oj can you construct? What is the magnitude?

PROFESSOR: Oh, it's an infinite number of them. It's an infinite number of them. But the key is the infinite [INAUDIBLE].

AUDIENCE: It's a factor of infinite. We can just--

PROFESSOR: Yeah, it's infinite. It's infinite. But compared to the large N limit, it's still order 1.

[LAUGHS]

It's infinite. It's infinite, but it's order N to the power 0.

AUDIENCE: Oh, my gosh.

AUDIENCE: All right.

PROFESSOR: It's N to the power of 0. And so they will be suppressed, by physical process. So for example, say this operator-- say I have some dimension. Say this operator-- suppose this Particle 1 have some energy. And a particle has some energy. And then the process of this generated this, too. And when their energy becomes very large, then it's suppressed. And you have this kind of physical suppression.

Of course, if you include all energies, there are an infinite number of them. But in fact, if you can-- if you consider, yeah, energy [INAUDIBLE] this kind of thing. Then essentially, there's a finite number of contributes.

AUDIENCE: Finite. OK.

PROFESSOR: Yeah. But the key is that this is order N to the power of 0.

AUDIENCE: Maybe one more thing--

PROFESSOR: Yeah.

AUDIENCE: What if it includes the loop, closed by itself?

PROFESSOR: Yeah?

AUDIENCE: Like a closed fermion loop [INAUDIBLE] I mean, it doesn't need to involve [INAUDIBLE] state. It can just [INAUDIBLE].

PROFESSOR: No, no, no. Loop always involves particle states. So any loop, when you cross it, there's always two states.

AUDIENCE: Don't they need to cross it at the end? I mean, like a [INAUDIBLE].

PROFESSOR: Sorry, I don't quite understand. Any loop, by definition, when you cross it, there's always two points. Then that means this is more than one particle.

AUDIENCE: Just something like this, don't cross any--

PROFESSOR: No, no, no, no, no. You cross here. You still have two particles.

AUDIENCE: Right.

AUDIENCE: Oh, OK.

PROFESSOR: Yeah, you cut at any time. There's still two particles. OK. Let's summarize. So now let's combine 1, 2, 3. So we can combine 1, 2, 3. So this tells us-- so this 1, 2, 3 tells us at leading order in large N expansion, we have a series-- a classical series. We essentially just have a classical series of glueballs.

You just have a bunch of particles, and they interact classically. There's no quantum fluctuations. And we see interactions among glueballs, among them, given by some effective company which I call g tilde, which scales as 1 over N. OK? Scales as 1 over N.

So let me just elaborate this statement a little bit more. So this can be considered as, say, when we take a gauge theory-- say, for [INAUDIBLE] theory. Say QCD. You take N goes to infinity limit. And you take h bar finite. So by definition, when we talk about this theory, h bar is always finite. But in the N equals infinity limit, this is equivalent to a glueball theory where h bar tilde goes to 0. OK.

So you get the classical theory and with effective h bar tilde, which is controlled, essentially, by 1 over N goes to 0. OK? So here, you can do perturbative expansion 1 over N. So this is just a leading order. Here, you can do perturbative expansion in 1 over N. So in this side, it's like some kind of semi-classical expansion in this h tilde, which, of course, should be scaled as 1 over N. OK?

So on this side, when h bar tilde goes to 0, you have a classical theory. But then you can do expansion h tilde. So this is one that we normally call a semi-classical expansion. OK? So the large N expansion has been translated into some semi-classical expansion when you translate into this glueball [INAUDIBLE].

And we're now-- so is this clear? Yeah, this is a little bit small corner. Is it readable? Yeah. This is a very important statement. Maybe I should rewrite it. Yeah, let me just rewrite it. So perturbative expansion-- perturbation in N, in 1 over N. So this is equal to semi-classical expansion in h bar tilde, which scales 1 over N. OK.

And we will now show that these guys, this glueball theory, and this semi-classical expansion, is actually a string theory. OK? No, we will not show that. I say I will motivate that. I say that's a string theory. So before I go that, do you have any questions? Yes?

AUDIENCE: Just to clarify, doesn't the sort of second statement-- the fact that the perturbation 1 over N corresponds with semi-classical expansion in h bar tilde, isn't that just a generalization of the thing you just wrote? It's the same statement.

PROFESSOR: Yeah, it's the same statement. No, no, no. This true statement, they are--

AUDIENCE: Basically the same.

PROFESSOR: They are not equivalent. This is talking about this particular limit.

AUDIENCE: OK.

PROFESSOR: This is talking about this particular limit. And then this tells you what happens when we'll relax this limit by including [INAUDIBLE] fact by 1 over N expansion. And to make this identification-- so this by itself does not imply that.

AUDIENCE: The other way does, right?

PROFESSOR: Yeah. Of course, the other way does. So this is a stronger statement.

AUDIENCE: Right. OK.

PROFESSOR: Under this statement, it's supported by just looking at the leading model behavior.

AUDIENCE: Right.

PROFESSOR: But if you want to look at this behavior, and then you look at-- then you need to look at the sub-reading terms of 1 over N. Say, for example, here. Then you see the fluctuation is controlled by 1 over N. It's like your fluctuation is controlled by h bar. And similarity, here, then when you include 1 over N [INAUDIBLE], then you will see a single particle. And the two particle, multi-particle particle, they can mix together due to fluctuations. And again, that amplitude controlled by 1 over N, et cetera.

Yeah, just say this statement is stronger. You need to look at sub-reading corrections carefully, and you'll see that fits into an h bar pattern.

AUDIENCE: Great.

PROFESSOR: Yeah. Yes?

AUDIENCE: So instead of this [INAUDIBLE], we consider regular creation-- [INAUDIBLE] Do we still come to the conclusion that all the loop diagrams are trace?

PROFESSOR: Yeah. That's a good point. So in essence, you can imagine this Oi, each Oi defines-- this point, 1, is precisely a statement. So you can imagine each Oi defines a creation-- defines an independent creation under [INAUDIBLE] operator. Yeah.

AUDIENCE: [INAUDIBLE] theory [INAUDIBLE] expression of both diagrams.

PROFESSOR: Sorry? No, in the original theory, we are not. Only in the large N limit.

AUDIENCE: So in the original theory, in the large N limit, we do have suppression?

PROFESSOR: No. You don't have a suppression of loop diagram in terms of the standard loops. You only have the suppression of the loop diagram in terms of those loops-- in terms of loops of glueballs.

AUDIENCE: But here we don't specify-- don't put any restrictions on what are the operators O. So why can't we say just Oi is just psi i?

PROFESSOR: No, no, no. O is [INAUDIBLE] operator, O is composite operator. O is some composite operator. So for example, in the [INAUDIBLE] series, so one example is O can be TrF squared. So if you look at the two-point function of O, then that's corresponding to two gluon propagators. And you have loops here.

But this is not the loop in terms of the glueballs here when we talk about loops-- yeah, so let me emphasize. All loops of glueballs. Because here-- so the key of this statement is that in a large N limit-- and if you can see-- if our thing is gauge invariant operators, then that's a [INAUDIBLE] that you should consider. And if you're not worried about the original stuff. And those that have loops, et cetera, [INAUDIBLE] the h bar, et cetera.

But in terms of those glueballs, then you have a classical theory. There's no loops. You only have tree-level scattering in the large N limit. Yes?

AUDIENCE: So are we getting a general gauge of varying operators or explicitly single trace operators?

PROFESSOR: No, no. I have talked about everything.

AUDIENCE: So I mean like the single-- a single O operator can be several traces inside?

PROFESSOR: No, no, no. Single O is a single trace. It's a single trace that's defined there.

AUDIENCE: What about non-trace gauge invariance operators?

PROFESSOR: We don't consider them. Oh, you mean non-trace gauge invariant operator? No, every gauging variance operator always involves trace.

AUDIENCE: Always?

PROFESSOR: Yeah, sure. You will be a famous mathematician if you can construct something invariant which does not involve a trace. Yeah, construct something about the matrix. Construct something using matrices which does not involve traces. Yeah. Even if you do determinant, and determinant can still be written as traces. Yeah. Yeah, just everything can be written as traces.

AUDIENCE: So determinant would just have-- if you have a determinant operator, that can just be written as a product of many Os.

PROFESSOR: That's right. That's right. That's right. That's right. Good. Any other questions? Yes?

AUDIENCE: Just to clarify, we've been talking about the-- [INAUDIBLE] [INAUDIBLE].

PROFESSOR: Yeah. Here, we are talking about only-- every field is matrix valued fields. Just every field is matrix valued fields. Yes?

AUDIENCE: I don't know, I just-- I thought there was more invariance that you can construct [INAUDIBLE]. Like I was asking you the other day, basically it's all of the-- each one of the coefficients of the characters [INAUDIBLE] polynomial of a matrix.

PROFESSOR: Yeah, they can be written as traces. Yeah, they can be written as traces.

AUDIENCE: Oh, I didn't know that.

PROFESSOR: Yeah.

AUDIENCE: Usually we don't discuss quarks here.

PROFESSOR: Yeah. Yeah, here we have-- not as quarks. And we're going to mention quarks very briefly at the end. Yeah. Any other questions? Good. Good. OK.

So now let's talk a little about string theory. Maybe I will leave it there. Let me see where they leave that. Yeah, let me leave that. OK.

So first we'll talk a little bit about the strings. So first, let me remind you, QFT, normally when we say a QFT, we say QFT is a series of particles. OK? So the standard approach to the QFT-- say you write down a field theory Lagrangian, and then you quantize the [INAUDIBLE]. So this is normally called the second quantized. So the standard approach is called the second quantized approach. So you can quantize the approach. OK?

But there also exists a first quantized approach. There also exists a first quantized approach. There also exists a first quantized approach. Say you directly quantize-- you just quantize a quantum motion, say, over a particle or particles. Say this is considered one particle-- say a particle in spacetime. OK?

So for example, if you can see the particle propagate in spacetime along some trajectory-- so let's parametrized this trajectory using a parameter by tau, which labeled a point along the trajectory, and then, so this will give you a mapping. Say the trajectory will map out the trajectory-- yeah, so this will map out the trajectory in spacetime, which can be written as a spacetime coordinate as a function of tau. OK?

So this parametrized the [INAUDIBLE] of the particle. So this parametrized the [INAUDIBLE] of the particle. And to quantize the particle, the simplest way is to imagine just sum over all possible paths over the particle. So essentially, we're doing a path integral. Essentially we're doing a path integral.

So you just integrate over all possible paths of a particle, and then this is essentially a quantum motion. Just quantum mechanically, it can fluctuate, anyway. And then this particle-- then this action, for the particle, essentially it's just the lens. So suppose the particle has mass m, it's just in the lens of the particle, dl, along the trajectory. OK?

And if you include interactions, then you have to include the trajectories into which the particle can split. Say a line can split in two lines, et cetera. So this gives you interactions, et cetera. OK? And this we need to add by hand, because the particle in principle can-- you can draw infinite ways of particle split into more particles, et cetera. So depend on your specific theories, you need to specify, say, the interactions. So this, you need add by hand.

OK. So in principle, you can forget about the standard formulation of quantum field theory just to work with this particle theory. So the string theory is a generalization of this first quantized approach.

So string theory, as is currently formulated, is formulated in this first quantized approach. So you just quantize motions of a string in spacetime. OK?

So for example, so particle case, you have a [INAUDIBLE] particle string case, you have a closed loop. So imagine we have a closed string. OK? You have a closed string. A string is a one-dimensional object, so let's consider a closed string. Then the string can be parametrized. So the string itself can be parametrized say by a parameter, sigma, along the string.

But then, then this string can move in the spacetime. Then we'll trace out some trajectory. Trace out some trajectory. So this gives rise to worldsheet. So in the particle case, you have a [INAUDIBLE]. So in the string case, you have a worldsheet, which we normally call sigma.

And then the sigma can be, again, written in terms of the spacetime coordinates as a function of this tau and the sigma. Now you have two coordinates to parametrize this worldsheet. So this is essentially a two-dimensional-- so essentially, you have a two-dimensional surface in the spacetime, embedded in spacetime.

And the string theory just quantized-- you just can see the motion of all such kind of surfaces. Yes?

AUDIENCE: So in the case of a single particle, we have a point. Does this action actually-- what does it reproduce exactly? Is it capable of reproducing all of quantum field theory using this [INAUDIBLE]?

PROFESSOR: Yeah, in principle. But it's not convenient. Yeah, for example, if you have a scalar particle, this can certainly easily reproduce your free scalar field theory. And if you want to reproduce phi-cubed theory, then you need to add this kind of a split in your-- when you do your path integral including the trajectories, you need to include these kind of trajectories.

Yeah, in principle, you can do it. It's not very convenient.

AUDIENCE: I see. OK. Interesting.

PROFESSOR: Yes?

AUDIENCE: How do you add interactions by hand in that particle?

PROFESSOR: Oh, well you just [? recreate. ?] In the path integral, here, you integrate all such kind of paths, right? And then you have prescription. You say, what kind of paths to include. And for example, if I have phi-cubed kind of interaction, then you say we should include this kind of path.

AUDIENCE: But in case it cannot be parametrized as a top, [INAUDIBLE]?

PROFESSOR: Yeah. Yeah. Then you have to-- then there are tactical complications in doing these extensions.

AUDIENCE: So one other question. Is string theory formulated in terms of-- is it possible to perform a second quantization?

PROFESSOR: Not right now. So people have written down classical string field theory. And in principle, you can quantize it. But that's a very complicated thing. But I think nobody have successfully quantized it. Even to write down the theory is very non-trivial.

AUDIENCE: OK. So similarly, to do quantization for string, you just look at the-- you just can see that the path integral-- so in principle, to do the quantize of the string, you can again just imagine your sum over all possible worldsheet configurations. All possible this kind of embedding of this two-dimensional surface in spacetime, then that essentially gives rise to all kinds of string motions. OK?

So weighted by some string action. So the simplest way to write down the string action is a straightforward generalization of this particle case, if I just integrate over its area. So here, it's just the lens of the string. Here, it's the area. But here, we also need to include-- so here is the particle mass. So a generation of the particle mass is the so-called string tension. So t, which is normally written as 2 alpha prime. So you introduce a parameter. You introduce a dimensional parameter alpha prime to parametrize this t. And so this is string tension.

So this is the mass per unit. String tension is just mass per unit length. OK? And this is normally called a Nambu-Goto action, so NG. So this is Nambu-Goto action, who first wrote it down. OK? So the A is essentially just the area of the surface in the spacetime. OK? it's just straight generalization of that length of the trajectory.

And yeah, depend on the worries-- depend on what kind of questions you are interested, then you try to track those kind of physical answers from this-- say this path integral. OK? Any questions so far? Yes?

AUDIENCE: Do we allow paths to contract the string to a point?

PROFESSOR: Yeah. Yeah. I will show you. Yeah?

AUDIENCE: Is that 2 times variable?

PROFESSOR: No, single. No, sigma parametrized the string itself, and the tau, it parametrized the motion of the string along the motion of the string.

AUDIENCE: But there is a time variable in sigma? Or it's just a--

PROFESSOR: No, no. Tau is the time. Tau is the internal time for the string. And then, there's a time in here-- in the x0, in the spacetime. So this can be considered as some kind of proper time for the string-- internal time for the string.

Yeah, just like you parametrized the trajectory of the particle, you have some tau to parametrize the trajectory of the particle. This is the same thing.

AUDIENCE: That means so any proper time-- isn't a variance-- itself is four-dimensional spacetime. Why there is only like one parameter of tau which does not have these four components?

PROFESSOR: No, I don't understand what you-- no, there's only a single time. You have a particle, you have only a single time. Right? Now do you agree here there's only tau here? Now here, I just have a line. I've parametrized this line. I call it tau.

AUDIENCE: OK, yeah.

PROFESSOR: Here, I have a two dimensional-- here, I have a string. And then each point on the string moves like a particle. And then my parametrize by tau. It's the same thing.

AUDIENCE: [INAUDIBLE]. Yeah. Yeah.

PROFESSOR: Yes?

AUDIENCE: You go ahead.

AUDIENCE: What's the metric on the worldsheet? Does that make sense to ask? Because it's like minus plus or--

PROFESSOR: Of course, it makes sense. Of course, on the worldsheet, it should be a Lorentz in-- whatever is the metric on worldsheet will be induced from the spacetime.

AUDIENCE: OK. So it would be a minus plus.

PROFESSOR: Yeah, for example. Yeah. Yeah. It's [INAUDIBLE]. Yeah. Yeah, you can also-- we can talk about-- yes?

AUDIENCE: So one other question. So if that should be interpreted as proper time, if I have a string which is moving in some weird, non-uniform way, how does it make sense to talk about the proper time of the string when you could imagine the different points on the string?

PROFESSOR: Yeah. Yeah. Good point. So often, you cannot talk about that. And this just gives you a heuristic picture. Because in principle, string can do a highly non-classical motion. And then that's what's encoded, say integrating over all such kind of surfaces. And some of them, you cannot interpret classically or semi-classically into this kind of picture. You visualize it. So I will draw some pictures later. You will see. Yes?

AUDIENCE: This might be a question without a good answer, but what is the-- why does the action take that form? What is that accomplishing physically? Or I guess maybe an easier example would be up there in the first quantization approach. How does that reproduce [INAUDIBLE]?

PROFESSOR: You say why that produce gravity? Yeah. How that reproduces gravity, of course, is a technical question which you have to calculate explicitly. But the question, you said, why do we choose that? The reason we choose that is it's based on various principles.

So first, that thing, whatever it is, should not depend on how you choose this tau. Because you can parametrize your trajectory anywhere you want so that the l-- so whatever this action should not depend on the parametrization. And also, this action should be Lorentz invariant, and the Lorentz boost should be translating [INAUDIBLE], et cetera. And with those conditions, then essentially uniquely determined to be this form. It's [INAUDIBLE] unique [INAUDIBLE] Yes?

AUDIENCE: So I guess if you were doing a quantization of the bosonic string, like if you were deriving the field equations, would you use Nambu-Goto or would you use Polyakov?

PROFESSOR: Yeah. Yeah. That's right. If you really quantize it, you actually don't use Nambu-Goto. Nambu-Goto, you use it for the classical description. Yeah. I did not mention the other forms. It's just for our current discussion, it's not essential. Yeah. Indeed, when you quantize it, you actually use a different form of this action. Any other questions? Good.

So now, for example, say how you-- so it depends on what kind of questions you are interested, and you try to extract them, say, from this kind of path integral. And of course, this is a highly non-trivial questions. And they require, say, some kind of trial and error, et cetera.

So let me just tell you-- say if I want to calculate the vacuum process. Say if I want to calculate the vacuum energy-- OK? So let me call this Z string. So remember, when we talk about large N gauge series, and if you want to calculate the vacuum energy, then you calculate the-- you sum over all possible vacuum diagrams, et cetera. OK? You sum over all possible vacuum diagrams.

But suppose we want to calculate the vacuum energy in the string theory, and then you do the similar thing. You just do the, what do you want to do for the string? You say you sum over-- all closed 2D surfaces. OK?

So the "closed" here is essential. So this is based on the intuition that when we-- in the quantum field theory, when you sum over fermion diagrams, and the fermion diagrams heuristically can be considered as a particle trajectories, et cetera. And for the vacuum process, none of-- there's no external lags. So everything coming out of the vacuum then come back to the vacuum. So that's why-- so if we want to calculate the vacuum energy in the string theory, and again, corresponding to the virtual string motion.

And again, the virtual string motion should correspond into the sum of all such kind of services without any external lags. So it should be closed surfaces. OK? It should be closed surfaces. So it's the analog of the particle case.

So exactly as in the quantum field theory case, it's easier to do this using the Euclidian-- by going to the Euclidian signature, and say go to the Euclidian signature. It's going to Euclidian signature. So essentially, analytic-- so instead of considering a surface in the Lorentzian spacetime, you consider-- your analytics continue your spacetime, the full spacetime to the Euclidian signature. And then this just becomes some surface in some Euclidean space.

And then this is just a much easier mathematical object to deal with. OK? So that's what you get. And if you have a string, and then you have this. OK? OK.

So now, when you sum over all surfaces, now you have a choice. First, you can sum over the topology. So we talked about before-- or two-dimensional surfaces, topologies classified by this genus. And then, you can sum over surfaces with a given-- so you can separate the sum into sum over topology and then some surfaces of a given topology, given h. OK? So this h is the genus. So essentially, we are just summing of all possible two-dimensional closed surfaces. OK? Is this point clear? This is absolutely key point. And with this, minus SNG. OK?

So now there's a very important mathematical trick. And this mathematical trick can be justified vigorously, but I will not just put it here. I will just add it here. It says now, just in physic,s whenever you see a discrete sum like this, what do you do? Or in mathematics, whenever you see a discrete sum like this, what do we do? Hm?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Sorry?

[INTERPOSING VOICES]

[LAUGHTER]

PROFESSOR: Exactly. You add weight. So here, I will add a weight here. So here, I am summing of O topology. But now I will add by hand a weight to weigh the different topology. So lambda is just some parameter. So lambda is just a parameter I introduced by hand myself.

OK? So this is just the weight for a different topology. So chi is the Euler number. So lambda is just some-- so chi is the Euler number, 2 minus 2h. And the lambda just can be considered as some kind of chemical potential-- some kind of chemical potential, say, for topologies. OK? Yeah. Yeah, just like in the system, say you have a conservative charge, then when you add particles, different particle number, it's convenient to add a chemical potential. So lambda is like that.

So even though I added by hand now-- even though I added it by hand here, but in string theory, actually this is-- arises completely natural from the string theory. There's a rigorous way one can show how that arises, but I will not do it here. It's not important for our purpose. And I will also introduce a notation-- say gs is equal to exponential lambda. OK?

So now, let's look at the summation. So here, you sum over-- so at h equal to 0, you sum over all surface with the topology of a sphere. And the weight is given by gs minus 2, which is expansion minus 2 lambda. OK? Now because the chi-- when h equal to 0, chi equal to 2. So this is just minus 2 lambda. And so this is one over gs squared. OK?

And then the torus is h equal to 1. So this is just the gs to the power 0. So order 1. If you have genus 2, so this would be gh equal to 2, and it would be gs squared. OK?

So here is a remarkable fact. So here is a remarkable fact. Here is a remarkable fact. It said summing over topology automatically includes interaction of strings. OK? In fact, this fully specifies string interactions. OK?

So let me just elaborate on this statement. So we can see this statement from these sums here. OK? So now let's look at the physical meaning of those diagrams. So let's first look at the sphere diagrams. So suppose we draw a sphere. OK?

So what I'm describing to you is heuristic, from physical perspective. So if you think about the sphere embedded in the space-- so this is [INAUDIBLE]. So if you think from, say, imagine your time going up. OK? Imagine your time going up. So this [INAUDIBLE] you nucleate an infinitesimal string at the tip of the sphere, and then this stream propagates. So then each time, you have a string. And then, at some future time, then go back to the vacuum. OK? And so you just have a string come in and come out-- a string come out from the vacuum, and then go back. And the sphere just described this kind of virtual process. So you look at string, and then disappear. OK?

Say nucleate string-- string nucleation, and it then disappears. OK? Yes?

AUDIENCE: Why did we only talk about close strings?

PROFESSOR: Because it's enough for my purpose now.

[LAUGHTER]

Yeah. I'm not trying to give you the full string theory at the moment. I'm just trying to give you the bit of string theory that's enough to make that point. Say large N expansion is like string theory. Yeah. Yeah, because otherwise, take the whole semester to reach the point.

So now, let's look at this torus diagram. Because if you want the sum of all surfaces, then you have to sum over different topologies, then you have to sum over the torus. Then let's look at what torus diagram means. So let me show the torus in the slightly different way. OK?

So the torus, again, if you try to view the time going up, then from here, again, you do create the single string. But now, at this time, then you actually have two strings. And then, here, you have one string back again. OK? And then the torus actually describes such a virtual process, is that you nucleate a string from the vacuum. And then this string splits into two strings. And then these two strings come back to join, again, into a single string and then go to the vacuum. OK? So this is the nucleation out of vacuum.

And then here, it splits. So here, a single string splits into two string. And then here, they join together, join back into a single string. And then here, disappear into the vacuum. OK? So we find, by including the torus diagram you automatically include this kind of interaction, that the string can split into two strings, and two strings can join into single string. And essentially, you don't have choice. This is just determined by the topology. OK? You don't have choice. Just once you write down this path integral, then this is fully determined. OK? It's fully determined.

And similarly, if you look at the [INAUDIBLE] process-- so you look at the string split, join, splits and join again. So the [INAUDIBLE] string again can see that it has such in the acting process. So now the key-- so now if we compare the gs dependence of these diagrams. So this one-- so each diagram, compared to the earlier one, you increase by gs to the power squared.

But whenever you include a genus, essentially [INAUDIBLE] include such a splitting under the joining. So that means you can actually associate with each process to be a factor of g string. OK? So for the sphere, you have-- say from here. Say if you normalize a sphere to be gs minus 2, then now if you include-- so essentially, this weight which we added here is to wait for such a splitting and the joining process.

So each splitting and joining essentially give you a weight of g string. OK? Weight g string. So in some sense, we concluded that-- so the basic string interaction vertex is just a single string can join into two string, or two string-- or single string can split into two string, or two string can join into a single string. And the weight for each process is given by g string. OK?

So that's effectively what this mathematical trick does. So effectively what this mathematical trick does is to assign a weight for each such splitting process. Yes?

AUDIENCE: So if you start the-- so you drew it like this for the two thing. But if you start the nucleation from the side rather than the bottom, then wouldn't it split into three and then back into one?

PROFESSOR: That's right. Yeah, it's-- because of the topology, you can split in an arbitrary way. And this just tells you, using this basic vertex, then for fixed parameterization-- yeah, this vertex already enough. And you can-- indeed, it's a very good question. You can try to slice the diagram in arbitrary way. Then that may give you some arbitrary other things. Then that's fine. And then you can assign weight for some other vertices. But they will be all consistent with these fundamental vertices.

So essentially this gs, which I called to be-- exponential lambda, it's essentially the coupling for the string. OK? It essentially determines the strength of the string interactions. Yeah?

AUDIENCE: Our choice of weight is not unique, right? [INAUDIBLE] something ugly like [INAUDIBLE] chi squared and then including that.

PROFESSOR: Yeah, that's a good point. Indeed. Indeed. Yeah. So here is the simplest choice. And it's a choice which you can-- yeah. So this is the simplest choice you can put, and then this is a choice which will also arise if you property quantize string theory. And indeed, you can write down some other string theory with a different power. It might be possible. Yeah. I mean just as a mathematical-- say at the mathematical level, it doesn't prevent you to add some arbitrary function of chi. And this is just the simplest way to do it. And this is the way which arises out of string theory.

Yeah. So this process is not arbitrary. It's something which you can derive from string theory. Just here, we will not go through the whole thing. Any other questions?

AUDIENCE: Do we have an extra minus sign in the exponent?

PROFESSOR: No.

AUDIENCE: Oh. Because chi goes-- it's more negative as h increases?

PROFESSOR: Yeah.

AUDIENCE: So the exponent becomes larger and larger?

PROFESSOR: Yeah. Yeah. Yeah. Because I can take g to be small. Yeah. Yeah, the only thing is I want to be consistent with this power. So now, here is the key. So now let's look at the external strings.

So here, at the beginning, I was looking at the-- I was looking at the vacuum process. There was nothing. Just everything come out of the vacuum. But you also consider, say, some process. Say you started with two strings. Then you scatter them together, you get two string back. So you can also consider such kind of strings. So you started with actually two initial strings. And then through some complicated interaction, then you have two strings back. OK?

So for example, this would be such kind of process. Anything can happen between. I have two initial strings, but I have two final strings come out. OK? So this would be some kind of surfaces. So this would-- because when do you have your sum of all possible surfaces which four external string-- say, two coming from minus infinity, and two come from t [INAUDIBLE], two come out at t plus infinity. OK?

And again, you need to sum of all possible topology in between. Say you sum over sphere in between. Yeah, let me just save time, draw it quickly. So you can sum over all spheres in between. Some spheres. Or you sum over torus, et cetera. OK?

But now, if you do this weight, according to this rule-- OK? Now if you do the weight the according to this rule, something changed. Because again, we want you sum of all possible surfaces with the same rule as you do for the vacuum. But now, there's something important that changes. So these are the [INAUDIBLE] with the boundaries. So you want to start with two initial strings and two final strings.

So each initial string could use a boundary. So now you have four different boundaries. So you have four boundaries. And so when you introduce boundaries into two-dimensional surfaces, then the Euler number changes. So sum of you already may know this from high school. 2 minus h, and the minus n, and n is the number of boundaries. OK? Number of boundaries. And which is the same as the number of external strings. Number of external strings. OK?

So now, if you use this rule-- so now for the n-string scattering-- for n-string scattering-- so let me call this n, An. Then you will, again, sum over all genuses. So now this weight will translate into gs, become n minus 2 plus 2h. OK? So now, because of this n, so I have additional power of gs to the power n. Say I can write as Fn h. OK?

So if you look at the sum, so this is gs to the power n minus 2 come from the sphere. The sphere, surface, [INAUDIBLE] surfaces, which can be considered as a tree level from the string point of view, because the string just come out and then nucleate.

And then the next order is gs to the-- gs n. Then this is the torus topology. So this may be considered as some kind of one-loop process, and gs n plus 2. So this is genus 2, et cetera. OK?

So now, let's compare with this. So this also included in the vacuum. For the vacuum, it just says n equal to 0-- [INAUDIBLE] n equal to 0. So now, we see exactly parallel with what we see in the large N gauge series. In both cases, you have summing over topology. And in both cases, we'll have summing-- you have this expansion.

So identical mathematical structure with large N expansion. In particular, g string now corresponds into just 1 over N. And these external strings-- say if you have some-- just goes 1 into the glueballs, what we call the glueballs. So these kind of single trace operators.

And the sum of string-- over string worksheet of, say, topology of genus h is mapped to, say, sum over Feynman diagrams of genus h. Yeah. So you see an exactly parallel mathematical structure between the two. So the question, is this an accident? Or this is something deep? OK? I think I'm running out of time. Yeah. OK. Maybe I will stop here. Oh, you don't want to stop?

AUDIENCE: There's only 1 and 1/2 pages left.

PROFESSOR: OK. Yeah. Let me just say a few words. Let me say a few words. If you look at these two sums-- this sum and this sum. Oh, yeah, let me call this fn. So that's a distinction. Let me call this f. So that's f.

So if you look at these two sums-- so the question is, can you really physically identify these two? OK? Let me just say a couple words. Say fn h, which will appear in the correlation functions, essentially just sum over Feynman diagrams of genus h. And this is just the Feynman diagrams-- the expansion for the Feynman diagrams.

And this one, this fh-- fn h-- So this is sum over-- this is path integral, say minus SNG of genus h surfaces. So now remember one thing we said before. We said each Feynman diagram can be considered as a partition of a 2D surface. OK?

So now, if you sum over all possible Feynman diagrams-- so you can also think of it geometrically as sum of all possible triangulations of a surface. So this kind of partition essentially just a triangulation of the surfaces divided into different parts, and with some amplitude-- with some weight.

So sum of all possible triangulations over the surface, of course this is precisely just a discrete form of summing over all surfaces. So this just a discrete sum of summing over all surfaces. And so this really identified these two are essentially the same object. So when you're summing over diagrams, you're actually summing [INAUDIBLE] actually summing over all possible embeddings of some surfaces in spacetime.

And the precise nature of the surface, of course, depends on the diagram itself. So this really tells you that the larger N expansion is really just a string theory. OK? It's really just a string theory. But from here, you cannot immediately tell what kind of string theory is this. Because from famine Feynman diagram itself-- so this Nambu-Gotu action, which I wrote earlier, goes 1 into your map, from some surface-- embedding of some surface in some spacetime. And that spacetime can be some arbitrary spacetime, et cetera. For a different spacetime, of course, you get a different action.

So the question is, what does this correspond to? Say I give you a field series, and you have those Feynman diagrams. And what Feynman-- and for each Feynman diagram, you can write down expression for them. Just whether this really corresponding to, say, some geometric actions. The Feynman diagrams [INAUDIBLE] really cause some geometric action describing the motion of some surface in some spacetime. Yeah. So let me just stop here.

[APPLAUSE]