

8.821/8.871 Holographic duality

MIT OpenCourseWare Lecture Notes

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Lecture 5

Reminder from last lecture

The vacuum of Minkowski space can be viewed as an entangled state of left Rindler patch and right Rindler patch

$$|0\rangle_M \propto \sum_n e^{-\pi E_n} |n\rangle_{Rind} \otimes |n\rangle_{\widetilde{Rind}}$$

with $|n\rangle_R$ and E_n the energy eigenvectors and eigenvalues of H_R in the right patch; $|n\rangle_L$ the eigenvectors for the left patch (with opposite time direction).

Then after tracing over the opposite time direction Rindler space, we obtain the reduced density matrix for the normal Rindler space

$$\text{Tr}_{\widetilde{Rind}}(|0\rangle_M \langle 0|) = \rho_{Rind}$$

And this density matrix itself can also be viewed as a thermal density matrix $\rho_{Rind}^T = \frac{1}{Z_{Rind}} e^{-2\pi H_R}$ with the inverse temperature $\beta = 2\pi$.

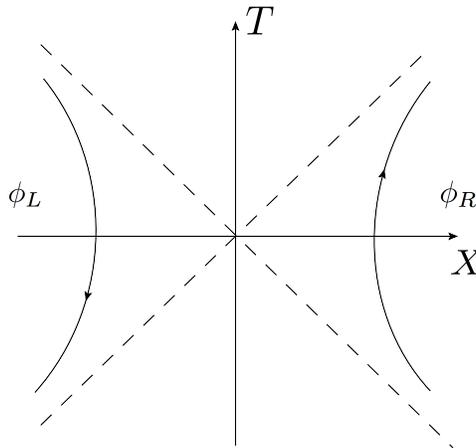


Figure 1: Minkowski Hilbert space as the direct product of left Rindler Hilbert space and right Rindler Hilbert space (but with opposite time direction).

The nature of a Rindler observer in the Minkowski vacuum can now be understood

1. $|0\rangle_M$ is an (specific) entangled state between left and right patches.
2. Tracing out the left patch leaves a thermal density matrix for the right patch.

Further remarks:

- $|0\rangle_M$ is invariant under $H_{Rind}^{(R)} - H_{Rind}^{(L)}$ (here we have a minus sign, because the time flows oppositely in the left patch).

$$e^{i\eta(H_{Rind}^{(R)} - H_{Rind}^{(L)})}|0\rangle_M = |0\rangle_M$$

This can also be seen geometrically: η translation is a boost in (X, T) , *i.e.* H_{Rind} generates a boost. $|0\rangle_M$ is clearly invariant under a boost. Yet boosts act oppositely in the right and left quadrants as indicated in Fig. 1.

- If we expand ϕ_R (ϕ_L) in terms of modes in the right (left) quadrant:

$$\phi_R = \sum_j (a_j^R u_j + a_j^{R\dagger} u_j^*), \quad a_j^R |0\rangle_R = 0$$

then

$$|0\rangle_M = \frac{1}{\sqrt{Z_{Rind}}} \prod_j \exp[-\pi\omega_j a_j^{R\dagger} a_j^L] |0\rangle_R \otimes |0\rangle_L$$

And the usual Minkowski creation and annihilation operators are related to a_j^R, a_j^L by the Bogoliubov transformations just as the earlier harmonic oscillator example.

- All the discussions can be generalized immediately to the Schwarzschild spacetime (Fig. 2).

$|0\rangle_{HH}$ = path integral over the lower half plane of the Euclidean continuation of black hole spacetime

where $|0\rangle_{HH}$ refers to the ‘‘Hartle-Hawking vacuum’’.

- As shown in Fig. 2 black holes formed by the gravitational collapse only have R and F region. Our discussion does not directly apply. Nevertheless, all the conclusion apply, including T_{BH}, S_{BK} , etc.

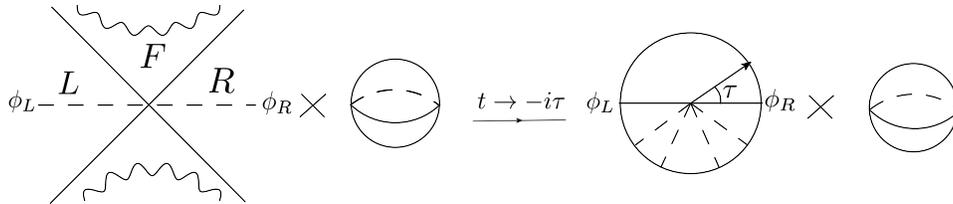


Figure 2: Hartle-Hawking vacuum (left) can be expressed as an path integral over the lower half plane of the Euclidean contention of the black hole spacetime (right).

1.2.4: BLACK HOLE THERMODYNAMICS

From the previous discussion, we know that a black hole has a temperature:

$$T_{BH} = \frac{\hbar}{8\pi G_N m}$$

Thus a black hole is a thermodynamic object, and it must obey thermodynamics. Now recall thermodynamic relations:

$$\frac{dS}{dE} = \frac{1}{T(E)} = \frac{8\pi G_N m}{\hbar}$$

since for a black hole $E = m$

$$S(E) = \int \frac{dE}{T(E)} = \frac{4\pi G_N E^2}{\hbar} + const = \frac{4\pi r_s^2}{4\hbar G_N} = \frac{A_H}{e\hbar G_N}$$

The integral constant can be determined to be 0 since $S(E) = 0$ for $E = 0$, A_H is the area of black hole horizon. So we now have the most important conclusion for black hole

$$T_{BH} = \frac{\hbar K}{2\pi}, \quad S_{BH} = \frac{A_H}{4\hbar G_N} \quad (1)$$

Note that T_{BH} decreases as mass m increases, the system has a negative specific heat:

$$C = T \frac{\partial S}{\partial T} = \frac{\partial E}{\partial T} < 0$$

The relation Eq. 1 is in fact universal, applying to all black holes in Einstein gravity with matter fields (including those from string theory).

General black holes

In this section, we list the mostly quoted results about general black holes.

No hair theorem: a stationary, asymptotically flat black hole is characterized by its

1. mass M
2. angular momentum J
3. conserved gauged charges (*e.g.* electric charge Q).

This means after the process: star \rightarrow black hole, all features of the stars have lost (classically).

Now we summarize four laws of black hole mechanics:

- 0th law: surface gravity K is constant over the horizon.
- 1st law:

$$dM = \frac{K}{8\pi G_N} dA + \Omega dJ + \Phi dQ$$

where Ω is the angular frequency at the horizon, Φ is the electric potential at the horizon (assume that at ∞ the potential is 0).

- 2nd law: horizon area never decreases classically.
- 3rd law: surface gravity of a black hole cannot be reduced to 0 in a finite number of steps.

These laws become the standard laws of thermodynamics with the identification Eq. 1. In particular the 1st law becomes

$$dE = T dS + \Omega dJ + \Phi dQ$$

Historically, before Hawking's discovery of black hole radiation, Bekenstein (1972-1974) has found $S_{BH} \propto A_H$, the motivation is to save the 2nd law of thermodynamics for a system with black holes. if an ordinary system falls into a black hole, the ordinary entropy becomes invisible to an exterior observer, therefore we have the generalized 2nd law (GSL):

$$dS_{tot} \geq 0, \quad S_{tot} = S_{BH} + S_{matter}$$

If we accept a black hole as a thermal object, GSL is of course automatic.

Finally, we give some puzzles/paradoxes

1. Does black hole entropy has a statistical interpretation?
2. Does black hole respect quantum mechanics?

The first question has been answered in the affirmative for many different types of black holes in string theory and holographic duality. That is a black hole has internal states of order:

$$N \sim e^{\frac{A_H}{4\hbar G_N}}$$

The second question is related to Hawking's information loss paradox. The rough description of this paradox is: consider a star in a pure state collapse to form a black hole, which then radiates thermally. If to a good approximation, the radiation is thermal for $m \gg m_p$, so before $m \sim O(m_p)$, very little information about the original state can come out. Once $m \sim O(m_p)$, it will be too late for all the information to go out. Then we start from a pure state and eventually get into a thermal state with density matrix description, *i.e.* information is lost!

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