

## MATLAB Tutorial

You need a small number of basic commands to start using MATLAB. This short tutorial describes those fundamental commands. You need to *create* vectors and matrices, to *change* them, and to *operate* with them. Those are all short high-level commands, because MATLAB constantly works with matrices. I believe that you will like the power that this software gives, to do linear algebra by a series of short instructions:

<i>create E</i>	<i>create u</i>	<i>change E</i>	<i>multiply Eu</i>
$E = \text{eye}(3)$	$u = E(:, 1)$	$E(3, 1) = 5$	$v = E * u$
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$

The word *eye* stands for the identity matrix. The submatrix  $u = E(:, 1)$  picks out column 1. The instruction  $E(3, 1) = 5$  resets the (3, 1) entry to 5. The command  $E * u$  multiplies the matrices  $E$  and  $u$ . All these commands are repeated in our list below. Here is an example of inverting a matrix and solving a linear system:

<i>create A</i>	<i>create b</i>	<i>invert A</i>	<i>solve Ax = b</i>
$A = \text{ones}(3) + \text{eye}(3)$	$b = A(:, 3)$	$C = \text{inv}(A)$	$x = A \backslash b$ or $x = C * b$
$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$	$\begin{bmatrix} .75 & -.25 & -.25 \\ -.25 & .75 & -.25 \\ -.25 & -.25 & .75 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

The matrix of all ones was added to  $\text{eye}(3)$ , and  $b$  is its third column. Then  $\text{inv}(A)$  produces the inverse matrix (normally in decimals; for fractions use *format rat*). The system  $Ax = b$  is solved by  $x = \text{inv}(A) * b$ , which is the slow way. The backslash command  $x = A \backslash b$  uses Gaussian elimination if  $A$  is square and never computes the inverse matrix. When the right side  $b$  equals the third column of  $A$ , the solution  $x$  must be  $[0 \ 0 \ 1]'$ . (*The transpose symbol ' makes x a column vector.*) Then  $A * x$  picks out the third column of  $A$ , and we have  $Ax = b$ .

Here are a few comments. The comment symbol is `%`:

`%` The symbols  $a$  and  $A$  are *different*: MATLAB is case-sensitive.

`%` Type *help slash* for a description of how to use the backslash symbol. The word *help* can be followed by a MATLAB symbol or command name or M-file name.

Note: The command name is upper case in the description given by help, but must be lower case in actual use. And the backslash  $A \setminus b$  is different when  $A$  is not square.

% To display all 16 digits type *format long*. The normal *format short* gives 4 digits after the decimal.

% A semicolon after a command avoids display of the result.

$A = \text{ones}(3)$ ; will not display the  $3 \times 3$  identity matrix.

% Use the up-arrow cursor to return to previous commands.

## How to input a row or column vector

$u = [2\ 4\ 5]$  has one row with three components (a  $1 \times 3$  matrix)

$v = [2; 4; 5]$  has three rows separated by semicolons (a  $3 \times 1$  matrix)

$v = [2\ 4\ 5]'$  or  $v = u'$  *transposes*  $u$  to produce the same  $v$

$w = 2:5$  generates the row vector  $w = [2\ 3\ 4\ 5]$  with unit steps

$u = 1:2:7$  takes steps of 2 to give  $u = [1\ 3\ 5\ 7]$

## How to input a matrix (a row at a time)

$A = [1\ 2\ 3; 4\ 5\ 6]$  has two rows (always a semicolon between rows)

$A = [1\ 2\ 3$   
4 5 6] also produces the matrix  $A$  but is harder to type

$B = [1\ 2\ 3; 4\ 5\ 6]'$  is the *transpose* of  $A$ . Thus  $A^T$  is  $A'$  in MATLAB

## How to create special matrices

**diag**( $v$ ) produces the diagonal matrix with vector  $v$  on its diagonal

**toeplitz**( $v$ ) gives the symmetric *constant-diagonal* matrix with  $v$  as first row and first column

**toeplitz**( $w, v$ ) gives the constant-diagonal matrix with  $w$  as first column and  $v$  as first row

**ones**( $n$ ) gives an  $n \times n$  matrix of ones

**zeros**( $n$ ) gives an  $n \times n$  matrix of zeros

**eye**( $n$ ) gives the  $n \times n$  identity matrix

**rand**( $n$ ) gives an  $n \times n$  matrix with random entries between 0 and 1 (uniform distribution)

**randn**( $n$ ) gives an  $n \times n$  matrix with normally distributed entries (mean 0 and variance 1)

**ones**( $m, n$ )    **zeros**( $m, n$ )    **rand**( $m, n$ )    give  $m \times n$  matrices

**ones**(**size**( $A$ ))    **zeros**(**size**( $A$ ))    **eye**(**size**( $A$ )) give matrices of the same shape as  $A$

## How to change entries in a given matrix $A$

$A(3, 2) = 7$     resets the (3, 2) entry to equal 7

$A(3, :) = v$     resets the third row to equal  $v$

$A(:, 2) = w$     resets the second column to equal  $w$

The colon symbol  $:$  stands for *all* (all columns or all rows)

$A([2\ 3], :) = A([3\ 2], :)$  exchanges rows 2 and 3 of  $A$

## How to create submatrices of an $m \times n$ matrix $A$

$A(i, j)$             returns the  $(i, j)$  entry of the matrix  $A$  (scalar =  $1 \times 1$  matrix)

$A(i, :)$             returns the  $i$ th row of  $A$  (as row vector)

$A(:, j)$             returns the  $j$ th column of  $A$  (as column vector)

$A(2 : 4, 3 : 7)$     returns rows from 2 to 4 and columns from 3 to 7 (as  $3 \times 5$  matrix)

$A([2\ 4], :)$         returns rows 2 and 4 and all columns (as  $2 \times n$  matrix)

$A(:)$                 returns one long column formed from the columns of  $A$  ( $mn \times 1$  matrix)

**triu**( $A$ )    sets all entries below the main diagonal to zero (upper triangular)

**tril**( $A$ )    sets all entries above the main diagonal to zero (lower triangular)

## Matrix multiplication and inversion

$A * B$             gives the matrix product  $AB$  (if  $A$  can multiply  $B$ )

$A .* B$             gives the entry-by-entry product (if **size**( $A$ ) = **size**( $B$ ))

**inv**( $A$ )            gives  $A^{-1}$  if  $A$  is square and invertible

**pinv**( $A$ )          gives the pseudoinverse of  $A$

$A \setminus B$           gives **inv**( $A$ ) \*  $B$  if **inv**( $A$ ) exists: *backslash* is left division

$x = A \setminus b$       gives the solution to  $Ax = b$  if **inv**( $A$ ) exists

See *help slash* when  $A$  is a rectangular matrix!

## Numbers and matrices associated with $A$

$\det(A)$  is the *determinant* (if  $A$  is a square matrix)

$\text{rank}(A)$  is the *rank* (number of pivots = dimension of row space and of column space)

$\text{size}(A)$  is the pair of numbers  $[m \ n]$

$\text{trace}(A)$  is the *trace* = sum of diagonal entries = sum of eigenvalues

$\text{null}(A)$  is a matrix whose  $n - r$  columns are an orthogonal basis for the nullspace of  $A$

$\text{orth}(A)$  is a matrix whose  $r$  columns are an orthogonal basis for the column space of  $A$

## Examples

$E = \text{eye}(4); E(2, 1) = -3$  creates a  $4 \times 4$  elementary elimination matrix

$E * A$  subtracts 3 times row 1 of  $A$  from row 2.

$B = [A \ b]$  creates the augmented matrix with  $b$  as extra column

$E = \text{eye}(3); P = E([2 \ 1 \ 3], :)$  creates a permutation matrix

Note that  $\text{triu}(A) + \text{tril}(A) - \text{diag}(\text{diag}(A))$  equals  $A$

## Built-in M-files for matrix factorizations (all important!)

$[L, U, P] = \text{lu}(A)$  gives three matrices with  $PA = LU$

$e = \text{eig}(A)$  is a vector containing the eigenvalues of  $A$

$[S, E] = \text{eig}(A)$  gives a diagonal eigenvalue matrix  $E$  and eigenvector matrix  $S$  with  $AS = SE$ . If  $A$  is not diagonalizable (too few eigenvectors) then  $S$  is not invertible.

$[Q, R] = \text{qr}(A)$  gives an  $m \times m$  orthogonal matrix  $Q$  and  $m \times n$  triangular  $R$  with  $A = QR$

## Creating M-files

M-files are text files ending with `.m` which MATLAB uses for functions and scripts. A script is a sequence of commands which may be executed often, and can be placed in an m-file so the commands do not have to be retyped. MATLAB's demos are examples of these scripts. An example is the demo called *house*. Most of MATLAB's functions are actually m-files, and can be viewed by writing `type xxx` where *xxx* is the name of the function.

To write your own scripts or functions, you have to create a new text file with any name you like, provided it ends with `.m`, so MATLAB will recognize it. Text files can be created, edited and saved with any text editor, like *emacs*, *EZ*, or *vi*. A script file is simply a list of MATLAB commands. When the file name is typed at the MATLAB prompt, the contents of the file will be executed. For an m-file to be a function it must start with the word *function* followed by the output variables in brackets, the function name, and the input variables.

## Examples

```
function [C]=mult(A)
r=rank(A);
C = A' * A;
```

Save the above commands into a text file named `mult.m`. Then this function will take a matrix  $A$  and return only the matrix product  $C$ . The variable  $r$  is not returned because it was not included as an output variable. The commands are followed by `;` so that they will not be printed to the MATLAB window every time they are executed. It is useful when dealing with large matrices. Here is another example:

```
function [V,D,r]=properties(A)
% This function finds the rank, eigenvalues and eigenvectors of A
[m,n]=size(A);
if m==n
[V,D]=eig(A);
r=rank(A);
else
disp('Error: The matrix must be square');
end
```

Here the function takes the matrix  $A$  as input and only returns two matrices and the rank as output. The `%` is used as a comment. The function checks to see if the input matrix is square and then finds the rank, eigenvalues and eigenvectors of a matrix  $A$ . Typing `properties(A)` only returns the first output,  $V$ , the matrix of eigenvectors. You must type `[V,D,r]=properties(A)` to get all three outputs.

## Keeping a diary of your work

The command **diary**('file') tells MATLAB to record everything done in the MATLAB window, and save the results in the text file named 'file'. Typing **diary on** or **diary off** toggles the recording. Old diary files can be viewed using a text editor, or printed using *lpr* in unix. In MATLAB, they can be viewed using the **type file** command.

## Saving your variables and matrices

The command **diary** saves the commands you typed as well as MATLAB's output, but it does not save the content of your variables and matrices. These variables can be listed by the command **whos** which also lists the sizes of the matrices. The command **save 'xxx'** will save the matrices and all variables listed by the **whos** command into the file named *xxx*. MATLAB labels these files with a .mat extension instead of .m which are scripts or functions. *xxx.mat* files can be read by MATLAB at a later time by typing **load xxx**.

## Graphics

The simplest command is **plot**( $x, y$ ) which uses two vectors  $x$  and  $y$  of the same length. The points  $(x_i, y_i)$  will be plotted and connected by solid lines.

If no vector  $x$  is given, MATLAB assumes that  $x(i) = i$ . Then **plot**( $y$ ) has equal spacing on the  $x$ -axis: the points are  $(i, y(i))$ .

The type and color of the line between points can be changed by a third argument. The default with no argument is a solid black line “-”. Use *help plot* for many options, we indicate only a few:

MATLAB 5: **plot**( $x, y, 'r+ :'$ ) plots in  $r$  = red with + for points and dotted line

MATLAB 4: **plot**( $x, y, '- -'$ ) is a dashed line and **plot**( $x, y, '. '$ ) is a dotted line

You can omit the lines and plot only the discrete points in different ways:

**plot**( $x, y, 'o'$ ) gives circles. Other options are  $'+'$  or  $'\times'$  or  $'*'$

For two graphs on the same axes use **plot**( $x, y, X, Y$ ). Replace **plot** by **loglog** or **semilogy** or **semilogx** to change one or both axes to logarithmic scale. The command

**axis** (*a b c d*) will scale the graph to lie in the rectangle  $a \leq x \leq b$ ,  $c \leq y \leq d$ . To title the graph or label the  $x$ -axis or the  $y$ -axis, put the desired label in quotes as in these examples:

*title* ('height of satellite')    *xlabel* ('time in seconds')    *ylabel* ('height in meters')

The command **hold** keeps the current graph as you plot a new graph. Repeating **hold** will clear the screen. To print, or save the graphics window in a file, see *help print* or use

`print -Pprintername    print -d filename`

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18.085 Computational Science and Engineering I  
Summer 2020

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