18.085/18.0851 Computational Science and Engineering I	**Summer 2020**

Homework 7

Homework: Richard Zhang   *Scribes: Richard Zhang*

7.1 Laplace's Equation in a Square: Analytic

Solve $\Delta u = 0$ INSIDE a square defined by $x = 0$, $x = 1$, $y = 0$, and $y = 1$, subject to the following boundary condition

$$
\begin{cases}
u(x, 0) = 0 \\
u(x, 1) = x(1 - x) \\
u(0, y) = 0 \\
u(1, y) = 0
\end{cases}
\tag{7.1}
$$

7.2 Laplace's Equation in a Square: Numeric

Solve the problem above numerically by downloading the attached MATLAB files and filling in the missing lines. Submit the heat map and the convergence plot, namely the L^2 error vs. N on the log-log scale.

7.3 Comparison between 6.3 and 7.2

If we were to numerically solve Problem 6.3 from the last homework and compare its convergence plot with that of 7.2, which one will have a larger slope in absolute value? Explain your reasoning.

7.4 Finite Element Basis in 1D

Let ϕ_i be a finite element basis in $1D$ over the interval $[a, b]$, ie. it is a hat function with the tip of the hat centered at $x_i = a + ih$, where h is the grid size, and with its base spanning between x_{i-1} and x_{i+1}. Verify that it is indeed true that

$$
\int_a^b \phi_i'(x)\phi_{i+1}'(x)dx = -\frac{1}{h}
\tag{7.2}
$$

$$
\int_a^b \phi_i'^2(x)dx = \frac{2}{h}
\tag{7.3}
$$

7.5 Fourier Transform

Compute the Fourier transform of the following function

$$
f(x) = e^{-a|x|}, \text{ where } a > 0
\tag{7.4}
$$

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