possibilities implied in the elasticity estimates do exist (or will exist) in real world production processes, how long would it take to replace existing processes with these new ones, with what required investments in new capital equipment, and with what effects on labour and capital productivity?

These are all critical questions. The productivity question is particularly important because in the past energy developments – the amount of energy used, the form in which it is used, and the use of associated capital equipment – have been critical in the growth of productivity in such significant sectors of the economy as manufacturing, agriculture, and transport.

Because most modellers do not explicitly examine the productivity implications within particular economic sectors or industries of the substitution of labour and capital for energy, they leave us uninformed on such vital matters as the return on capital investments, and the growth of labour productivity that is essential to the rise in the earnings of labour. To neglect these matters is to leave unanswered such questions as the feasibility of achieving the transformations implied in the elasticity assumptions, the nature of some of the stresses and strains that would accompany them, and the length of time it would take to achieve them, if they can be achieved at all. Such factors are sometimes viewed as no more than transitional problems along the road to a new equilibrium, and therefore of no real long-run significance. But to adopt this position is to overlook the fact that such transitional problems, and society's attempts to cope with them, are the stuff of which the future will be made.

There are several major matters which the modellers must consider to produce results meriting greater confidence:

- They should address the energy–productivity connection directly, by modelling the impact of changes in energy use on the productive efficiency of labour and capital in specific industries.
- They should encourage specialists in empirical questions to intensify their efforts to obtain dependable estimates of energy demand in relation to price increases, drawing upon recently generated data, and upon cross-sectional data for different regions and countries.
- They should attempt to bridge the gap between estimates derived from economic statistics of the elasticity of substitution between energy and other factors of production, which are abstractions at best, and the real technological possibilities that can be established from engineering and industrial data.
- Finally, they should undertake, as soon as possible, to test their models by applying them to the replication of those energy–economy interactions that have already taken place in response to higher energy prices and supply constraints. The dichotomy between short-run and long-run responses inherent in such an undertaking will pose a serious analytical problem, but to face this difficulty might also help improve our understanding of the transitional problems involved in reaching the long-run future.

If the modellers do these things, they will be increasingly able to build new generations of models of energy–economy interactions for use in policy making with much greater confidence than is presently justified.

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Input-output techniques and energy cost of commodities

This article quantifies the desirability of several methods of calculating energy cost of commodities from input-output data, using an *a priori* technique based on exact treatment of an artificially 'homogenized' **A**-matrix. The use of an average energy price (the same to all consuming sectors) is much inferior to the use of different prices, even if full attention is paid to indirect economic effects through use of the inverted matrix $(\mathbf{I} - \mathbf{A})^{-1}$. However, there is relatively little advantage in inserting actual energy use data into the **A**-matrix instead of merely premultiplying $(\mathbf{I} - \mathbf{A})^{-1}$ by an energy use vector. The latter conclusion has one important exception, pertaining to comparison of primary and secondary energy types, which is not predicted by the analysis here.

In using input–output data to calculate free energy intensities of commodities, it seems natural that an energy analyst would prefer a physical, rather than monetary, input–output data base. The premise is that physical data are less prone to the effects of price discrimination and economies of scale, which both seem to distort the picture of the 'underlying physical reality'.

I know of little proof that introducing physical data really helps. There are three fragmentary indications:

- Herendeen[1] found that using monetary data for transactions between energy sectors for the US data base for 1963 led to a physical absurdity – that some commodities required more refined petroleum than crude oil for production. Inclusion of actual energy data cured the problem.
- A comparison was made between the work of Wright[2] and of Bullard and Herendeen.[3] Wright used only one average energy price for all sectors, while Bullard and Herendeen used a different one for each sector. For the 350-sector US data base for 1963, Wright's free energy intensities averaged 12% lower, with a mean deviation of 23%. 34 of Wright's energy

- intensities differed by more than 50% from those of Bullard and Herendeen.
- Bullard and Herendeen found that in updating free energy intensities from 1963 to 1967, the major part of the task could be accomplished by updating only the energy consumption data in each sector, without updating the monetary transactions between the sectors.[3]

These three anecdotes are not even directly comparable, and no consistent empirical check, nor a theoretical one, has ever been done. This note presents an *a priori* theoretical argument leading to a quantitative measure of desirability, which is given as a function of the technique actually used and of W, the weight placed on energy data compared with economic data.

Theory

I assume the existence of a monetary input–output data base which can be used to calculate a square direct-requirements matrix **A**. I assume also that each of the N commodities is produced by only one sector. (This is not actually required, since the data can be transformed to this form.)[3] To calculate free energy intensity one can use the following techniques (listed in order of increasing complexity):

1. Compute only direct energy use from economic data and an average energy price.
2. Obtain actual direct energy use only.
3. Multiply the monetary input–output inverted matrix $(\mathbf{I}-\mathbf{A})^{-1}$ by an average energy price.[2] This and methods 4 and 5 account in some way for indirect effects.
4. Multiply $(\mathbf{I}-\mathbf{A})^{-1}$ by a vector (**R**) of direct energy use.[1,4,5,6]
5. Invert a 'mixed' matrix in which n rows of **A** have been replaced by actual energy data.[3] I denote this as $(\mathbf{I}-\mathbf{B})^{-1}$ where **B** is the mixed matrix.

How desirable are these methods? I will assume that in the calculations, an energy transaction is worth W times as much as an economic one. That is, knowing (and *using*) the fact that the steel industry bought x joules of electricity is W times more desirable than either using the fact that steel bought y dollars worth of electricity or using the fact that the auto industry bought z dollars worth of steel. 'Desirability' is particularly directed to comparing the free energy intensities of different commodities. The value of W is doubtless controversial – certainly $W > 1$. Price discrimination in the sale of energy is of order 5 times, which suggests to me that W is of order 5. In any case, W remains an independent variable in the results below.

The desirability of methods 1-5 is as follows:

1. Direct use only, no energy data except average price. Desirability = 1.
2. Direct use only, actual energy data. Desirability = W.
3. Inverted monetary matrix, average energy price. One measure of how much better $(\mathbf{I}-\mathbf{A})^{-1}$ is than **A**, is the rate of convergence of the expansion:

$$(\mathbf{I}-\mathbf{A})^{-1} = \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \ldots.$$

I now assume that each term of **A** is of order f/N, where $f < 1$. This treats **A** as a homogenous blob and ignores the detailed structure, including the fact that **A** is actually rather sparse – it is done for ease of calculation. Then desirability $= (1 + f + f^2 + \ldots) = 1/(1-f)$.

4. Inverted monetary matrix multiplied by physical energy use vector. This multiplies the weighting of method by W. Desirability $= W/(1-f)$.

The mixed matrix (method 5). This is more complex. I use the expression:

$$(\mathbf{I}-\mathbf{B})^{-1} = \mathbf{I} + \mathbf{B} + \mathbf{B}^2 + \mathbf{B}^3 + \ldots$$

The desired free energy intensities are just the entries in the energy rows of $(\mathbf{I}-\mathbf{B})^{-1}$.[7] In the expansion, **I** corresponds to the physical free energy content of free energy itself. For non-energy commodities, the leading term in the expansion is in **B**. **B** is expressed schematically:

$$\mathrm{B}: \quad \begin{matrix} n \\ N-n \end{matrix} \begin{pmatrix} W \\ 1 \end{pmatrix}$$

where the first n rows are the energy rows and contain only physical data, and the remaining $N-n$ rows contain only monetary data. $\mathbf{B}^2$ is then:

$$\mathrm{B}^2: \quad \begin{matrix} n \\ N-n \end{matrix} \overset{N}{\begin{pmatrix} nW^2 + (N-n)W \\ nW + N - n \end{pmatrix}}$$

This means that in the energy rows of $\mathbf{B}^2$, a typical term is the sum of n products of energy term times energy term, and $N-n$ products of energy term times monetary term. One can verify that the energy rows of $\mathbf{B}^n$ are of the form $[nW + N\text{-}n]^{n-1}W$. To weight these terms in the expansion, I first reintroduce the convergence factor f/N from before, and use one of two weightings:

- The weight of pW^q is pW^r (multiplicative).
- The weight of pW^q is pqW (linear).

Multiplicative weighting gives:

$$\text{Weight (mult) } (\mathrm{B}^n) = \left[\frac{n}{N}W + 1 - \frac{n}{N}\right]^{n-1} f^{n-1}\, W$$

and for the whole expansion

$$\mathrm{B} + \mathrm{B}^2 + \mathrm{B}^3 + ---,$$

$$\text{Desirability (mult)} = W \sum_{n=0}^{\infty} f^n \left[\frac{n}{N}W + 1 - \frac{n}{N}\right]^n$$

$$= \frac{W}{1 - f\left[\frac{n}{N}W + 1 - \frac{n}{N}\right]}$$

$$0 \leqslant f\left[\frac{n}{N}W + 1 - \frac{n}{N}\right] < 1$$

Linear weighting gives:

$$\text{Weight (lin) } (\mathrm{B}^{n+1}) = Wf^n \sum_{m=0}^{n} \left(\frac{n}{N}\right)^m \left(1 - \frac{n}{N}\right)^{n-m} \frac{n!}{m!\,(n-m)!}(m+1)$$

and

$$\text{Desirability (lin)} = W \sum_{n=0}^{\infty} f^n \sum_{m=0}^{n} \left(\frac{n}{N}\right)^m \left(1 - \frac{n}{N}\right)^{n-m} \frac{n!}{m!\,(n-m)!}(m+1)$$

$$0 < f < 1$$

The multiplicative weighting is limited in its choice of W – a large W will cause the desirability to go to infinity. The linear weighting converges for all W.

These results are listed in Table 1.

Table 1. Expressions for desirability of different methods of calculating free energy intensity, expressed as function of relative weighting of energy data (W), convergence factor (f) and number of energy sectors (n).

Method	Notation	Desirability	Desirability for $n/N \to 1$
1. Economic data, average energy price, direct use only	A	1	Not applicable
2. Actual energy data, direct use only	R	W	W
3. Inverted monetary matrix, average energy price	$(\mathbf{I} - \mathbf{A})^{-1}$	$\frac{1}{1-f}$	Not applicable
4. Inverted monetary matrix, actual energy data	$\mathbf{R}(\mathbf{I} - \mathbf{A})^{-1}$	$\frac{W}{1-f}$	Not applicable
5. Inverted mixed matrix	$(\mathbf{I} - \mathbf{B})^{-1}$		
a. Multiplicative weighting		$W(1 - f\left[\frac{n}{N}W + 1 - \frac{n}{N}\right])^{-1}$	$\frac{W}{1 - Wf}$
b. Linear weighting		$W \sum_{n=0}^{\infty} f^n \sum_{m=0}^{n} (\frac{n}{N})^m (1 - \frac{n}{N})^{n-m} \frac{n!}{m!\,(n-m)!}(m+1)$	$\frac{W}{(1-f)^2}$

Note that in the limit $n/N \to 1$,

$$\text{Desirability (lin)} \to W(1 + 2f + 3f^2 + \text{---} (n+1) f^n + \text{---}) = W(1 + f + f^2 + \text{---})^2 = \frac{W}{(1-f)^2}$$

The reader can evaluate the expressions in Table 1 using his own values for W, f, and n/N. In Table 2 I do so for three values of W (2, 5, 10), three values of f (0·22, 0·40, 0·63), and several values of n/N, including 5/350 and 4/150, appropriate to the US and Norwegian data bases.[8]

Discussion

I will limit discussion to $f < 0{\cdot}40$ (corresponding to $f^5 \simeq (0{\cdot}01)$ and $W < 5$ ($f = 0{\cdot}40$ represents much slower convergence than in a typical economic system). These results are listed in Table 3. According to the weighting scheme

Table 2. Desirability of different methods of calculating free energy intensity, calculated from expressions in Table 1.

		f=0·22 ($f^3 \simeq$ 0·01)			f=0·40 ($f^5 \simeq$ 0·01)			f=0·63 ($f^{10} \simeq$ 0·01)		
Method	**Notation**	w=2	w=5	w=10	w=2	w=5	w=10	w=2	w=5	w=10
1.	A	1	1	1	1	1	1	1	1	1
2.	R	2	5	10	2	5	10	2	5	10
3.	$(\mathbf{I} - \mathbf{A})^{-1}$	1·282	1·282	1·282	1·667	1·667	1·667	2·703	2·703	2·703
4.	$\mathbf{R}(\mathbf{I} - \mathbf{A})^{-1}$	2·564	6·410	12·821	3·333	8·333	16·667	5·405	13·514	27·027
5.	$(\mathbf{I} - \mathbf{B})^{-1}$									
	n/N = 5/350									
	a. Multiplicative	2·574	6·515	13·303	3·365	8·663	18·229	5·540	14·970	34·602
	b. Linear	2·574	6·436	12·872	3·364	8·413	16·825	5·537	13·842	27·684
	n/N = 4/150									
	a. Multiplicative	2·584	6·609	13·751	3·394	8·971	19·841	5·663	16·513	45·704
	b. Linear	2·583	6·458	12·917	3·393	8·481	16·963	5·651	14·127	28·254
	n/N = 0·25									
	a. Multiplicative	2·759	8·929	35·088	4·000	25·000	∞	9·412	∞	∞
	b. Linear	2·745	6·862	13·725	3·889	9·722	19·444	7·706	19·265	38·530
	n/N = 0·50									
	a. Multiplicative	2·985	14·706	∞	5·000	∞	∞	36·364	∞	∞
	b. Linear	2·926	7·314	14·629	4·444	11·111	22·222	10·007	25·017	50·035
	n/N = 1									
	a. Multiplicative	3·571	∞	∞	10·000	∞	∞	∞	∞	∞
	b. Linear	3·287	8·218	16·437	5·556	13·889	27·778	14·609	36·523	73·046

Note: Methods 1-4 are independent of n.

and other assumptions made here, method 3 (use of an inverted monetary matrix and an average energy price) is the least desirable method – less desirable, in fact, than looking only at direct energy requirements in physical terms (method 2). Methods 4 and 5, both of which use inversion and actual energy use data, are seen to be about W times as desirable as method 3 (the factor is exactly W for method 4). However, for $n/N = 5/350$ or $4/150$, there is relatively little difference between methods 4 and 5.

This is true with either multiplicative or linear weighting, and is an apparent consequence of the fact that 4/150 is still a small fraction – most of matrix **B** is still monetary data. The largest difference occurs for $n/N = 4/150, f = 0{\cdot}40$, $W = 5$. Method 5 is 7·7% more desirable (multiplicative weighting) or 1·8% more desirable (linear weighting) than method 4, which itself is five times more desirable than method 3.

These results support the claim that the use of an average energy price (method 3) is much inferior to use of actual energy use by sector. They do not support the use of the mixed approach, which seems to yield only small improvements. Of course this discussion is changed for large values of W or of n/N.

These conclusions are based upon a simple picture. First, the assumption of homogeneity of **A** is a likely general source of error. Second, there is a systematic defect in method 4 which is not covered by my definition of desirability, and which is cured by method 5. The defect relates to primary and secondary energies. For example, it is theoretically possible that method 4 can lead to the physically impossible result that a commodity has a refined petroleum energy intensity exceeding its crude oil energy intensity. This has happened in analysis of the 1963 US economic data. The problem is a consequence of price discrimination in the sale of refined petroleum. In method 4 the crude and refined sectors communicate only in monetary terms, and the physical consequence of price discrimination is not passed back to the crude sector. In method 5 they communicate in physical terms, and the problem does not arise.

My definition of desirability gives some dependence on this communication in that, for method 5, n (the number of energy sectors), appears explicitly (it is absent from methods 1-4). However, it does not give strong enough dependence to avoid the primary-secondary problem, which remains as an exception to the general conclusions above.

Table 3. Selected results from Table 2 showing desirability of different methods of calculating free energy intensity.

Method	Notation	$f = 0{\cdot}22$		$f = 0{\cdot}40$	
		$(f^3 \simeq 0{\cdot}01)$		$(f^5 \simeq 0{\cdot}01)$	
		$w=2$	$w=5$	$w=2$	$w=5$
2.	R	2	5	2	5
3.	$(\mathbf{I} - \mathbf{A})^{-1}$	1·282	1·282	1·667	1·667
4.	$\mathbf{R}(\mathbf{I} - \mathbf{A})^{-1}$	2·564	6·410	3·333	8·333
5.	$(\mathbf{I} - \mathbf{B})^{-1}$				
	$n/N = 5/350$ [a]				
	a. Multiplicative	2·574	6·515	3·365	8·663
	b. Linear	2·574	6·436	3·364	8·413
	$n/N = 4/150$ [b]				
	a. Multiplicative	2·584	6·609	3·394	8·971
	b. Linear	2·583	6·458	3·393	8·481

a US data base.
b Norwegian data base.

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[1] R. Herendeen, *An Energy Input-Output Matrix for the United States, 1963: User's Guide*, Document No 69, Center for Advanced Computation, University of Illinois, Urbana, March 1973.

[2] D. Wright, 'Goods and services: an input-output analysis', *Energy Policy*, Vol 2, No 4, December 1974, pp 307-315.

[3] C. Bullard and R. Herendeen, 'The energy costs of goods and services', *Energy Policy*, Vol 3, No 4, December 1975, pp 268-278.

[4] R. Denton, 'The energy costs of goods and services in the Federal Republic of Germany', *Energy Policy*, Vol 3, No 4, December 1975, pp 279-284.

[5] *Energi, 1985, 2000*, Document 1974:65, Department of Industry, Stockholm, Sweden, 1974.

[6] P. Longva, *Krysslopskorrigerte Energikoeffisientar Utrekna ved Hjelp av MODIS IV* (Input-Output energy coefficients derived with the help of MODIS IV), Document 10 76/38, Central Bureau of Statistics, Oslo, Norway, December 1976.

[7] Bullard and Herendeen, *op cit*, Ref 3, p 270.

[8] US data base: *Input-Output Structure of the US Economy: 1967*, Vols I-III, US Department of Commerce, 1974. This has 357 sectors producing 357 commodities. Norwegian data base: Various editions of the 1973 data base for MODIS IV, one of the economic models of the Central Bureau of Statistics in Oslo. This is derived from the Norwegian National Accounts, and has 157 sectors producing 176 commodities. (Some manipulation is required to reduce this to about 150 sectors producing 150 commodities).